STATISTICAL MODELING OF BIT-ERROR-RATES IN ASYNCHRONOUS MULTICARRIER CDMA AND DIRECT-SEQUENCE CDMA SYSTEMS

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ABSTRACT

This paper presents a method for modeling the bit-error-rate (BER) probability density functions (pdf) of asynchronous Multicarrier Code Division Multiple Access (MC-CDMA) and Direct-Sequence Code Division Multiple Access (DS-CDMA) systems. An uplink channel is considered and it is assumed that the only channel distortion introduced by the channel is caused by the timing misalignments. Deterministic spreading sequences are used and the pdfs of each interferer's multiple access interference (MAI) are determined as a function of timing offset. A Nakagami-m distribution is fitted to the pdf of the total MAI power and the BER pdf is obtained directly from this Nakagami-m pdf. Both Walsh-Hadamard (WH) and Gold sequences are analyzed and the mean BERs are compared amongst the two multiple access systems for both sets of spreading sequences of varying lengths. The results suggest a higher resistance to MAI in the MC-CDMA technique for the considered environment.

1. INTRODUCTION

Code Division Multiple Access (CDMA) is a very popular multiple access technique for which Direct-Sequence CDMA (DS-CDMA) has been one of the prominent multiple access schemes for the air interface for 3rd Generation cellular systems normalization (W-CDMA, UMTS, cdma2000) [1]. With the proposed combination of multicarrier modulation (MCM) with CDMA, Multicarrier CDMA (MC-CDMA) [2] [3] is emerging as a possible candidate for the air interface multiple access scheme for 4th Generation cellular systems.

Multiple access interference (MAI) is one of the limiting factors of CDMA systems in terms of bit-error-rate (BER) performance. Ensuring orthogonal code properties minimizes the effects of MAI over a synchronous channel, however, when users are asynchronous, as is the case over the uplink channel, code orthogonality is lost and MAI produces significant performance degradation.

In [4], asynchronous MC-CDMA was shown to outperform DS-CDMA over frequency-selective channels with additive white Gaussian noise (AWGN) when the subcarrier bandwidths are such that each subcarrier is subject to frequency nonselective fading. However, in [5] it was shown that in a synchronous frequency-selective channel with AWGN the performance of DS-CDMA and MC-CDMA is similar. This suggests the difference between the two systems could lie in the asynchronous channel. This paper is a contribution towards investigating the effect of the asynchronous uplink channel on the performance of MC-CDMA and DS-CDMA. We propose a statistical model for the MAI power and BER pdfs for both MC-CDMA and DS-CDMA systems where the only distortion introduced by the channel is caused by the random timing misalignments amongst users.

The paper is organized as follows: Section 2 gives a description of the DS-CDMA and MC-CDMA systems considered. In Section 3 we propose a statistical model for the MAI power based on the Nakagami-m distribution and derive subsequently the pdf of the BER. Section 4 confirms the results of Section 3 obtained through Monte Carlo simulations. Finally, Section 5 concludes the paper.

2. SYSTEM DESCRIPTION

A system of K asynchronous users $(j = 1, ..., K)$ transmitting over the uplink channel is considered. The ith data symbol of user j is denoted as b_{ji}; all data is BPSK modulated and $i.i.d$ such that $Pr\{b_{ji} = -1\} = Pr\{b_{ji} = 1\} = 0.5$. Both WH and Gold spreading sequences of spreading factor N are considered and thus data symbols are represented by N samples in each system. With the timing offsets producing the only channel distortion, the channel is represented by (1), where the timing offset of each user, τ_j, is uniformly distributed over one symbol duration, T_s. The timing offsets are quantized to integer multiples of T_c making $\tau_j \in [0, N-1]$. Timing offsets are made with respect to the reference user, denoted as user x, for which $\tau_x = 0$.

$$c_j(t) = \delta(t - \tau_j) \tag{1}$$

The DS-CDMA system structure considered is conventional, employing spreading and despreading operations at the transmitter and receiver respectively, followed by a matched filter adapted to the rectangular shape of the data and a decision device at the receiver. The MC-CDMA system considered is described in [3] wherein the number of subcarriers is equal to the spreading factor, N. The multicarrier modulation is implemented such that single chips are modulated onto successive subcarriers and transmitted in parallel. This is achieved by an N-point IFFT operation at the transmitter [6][7] and the received signal is demodulated by an N-point FFT operation at the receiver. No Cyclic Prefix is added in the MCM due to the assumptions made on the channel.

3. PROBABILITY DENSITY FUNCTION APPROXIMATIONS

3.1 MAI Analysis

The MAI is characterized by a symbol by symbol recovery method, where the MAI contribution of user j in the recovery of b_{xi} is denoted by MAI_{jxi}. Expressions for MAI_{jxi} can be seen in [8]. In a system of $K-1$ interferers, the total MAI incurred by the reference user, MAI_{xi}, is represented by the expression in (2) for the offset vector $\underline{\tau} = [\ \tau_1 \quad \cdots \quad \tau_K\ ]$, with $\tau_x = 0$.

$$MAI_{xi}(\underline{\tau}) = \sum_{j \neq x}^{K} MAI_{jxi}(\tau_j) \tag{2}$$

As the interferers are assumed to produce independent MAI contributions, the total MAI is Gaussian by the Central Limit Theorem for a large number of interferers. For the asynchronous environment presented, the values of MAI_{jxi} are dependent upon the data symbols $b_{j(i-1)}$ and b_{ji}. Taking the expected value of MAI_{jxi} with respect to these data symbols gives $E[MAI_{jxi}] = 0, \forall j$. With the mean MAI produced by each interferer being zero, the interference power produced by each interferer is equal to the variance of the MAI, given by

$$\sigma_{jx}^2 = E\left[MAI_{jxi}^2\right] \tag{3}$$

This interference power is offset dependent due to the cross-correlation properties of the assigned spreading sequences. An expression for the MAI power produced by each interferer for a given offset, τ_j, is represented by

$$\sigma_{jx}^2 = \sum_{n=0}^{N-1} \alpha_n \delta(\tau_j - n) \tag{4}$$

for all possible offset values in the range $[0, N-1]$. Here $\alpha_n = E\left[MAI_{jxi}^2(\tau_j = n)\right]$ represents the MAI power at $\tau_j = n$ and is calculated computationally.

Figure 1 illustrates an example of such an MAI power distribution over possible timing offsets represented by the expression in (4). This particular distribution considers Gold sequences of spreading factor $N = 63$. From the expression in (4), the discrete pdf of the interference power contributed by each interferer, $p\left(\sigma_{jx}^2\right)$, is obtained by the expression in (5) where $Pr\{\cdot\}$ denotes probability. The resulting pdf can take various forms depending on the distribution of the timing offsets, τ_j. In this paper, τ_j is uniformly distributed over one symbol duration and it follows that the expression in (5) reduces to that in (6).

$$Pr\left\{\sigma_{jx}^2 = \alpha_k\right\} = \sum_{n=0}^{N-1} Pr\{\tau_j = n\}\, \delta(\alpha_n - \alpha_k) \tag{5}$$

$$Pr\left\{\sigma_{jx}^2 = \alpha_k\right\} = \sum_{n=0}^{N-1} \frac{1}{N} \delta(\alpha_n - \alpha_k) \tag{6}$$

As previously stated, the only distortion introduced by the channel is caused by the timing offsets. The signal-to-noise ratio (SNR) is therefore characterized by the total interference power, σ_x^2, incurred by the recovering user.

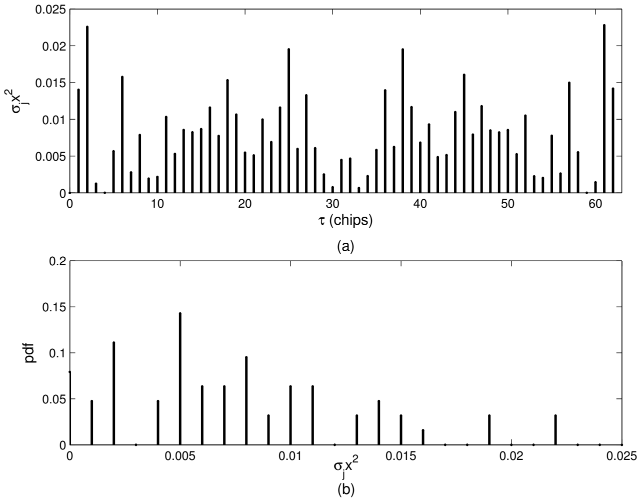

Figure 1: (a) MAI Power Distribution Over Offset; (b) Discrete pdf of Individual Interferer MAI Power

An expression for the total interference power is given in (7), where interferers are assumed to contribute independent MAI powers. Equation (7) is therefore a sum of independent random variables and the pdf of σ_x^2, denoted as $p\left(\sigma_x^2\right)$, can be computed by $K-1$ convolutions. This is represented in (8).

$$\sigma_x^2 = \sum_{j \neq x}^{K} \sigma_{jx}^2 \tag{7}$$

$$p\left(\sigma_x^2\right) = p\left(\sigma_{1x}^2\right) * p\left(\sigma_{2x}^2\right) * \ldots * p\left(\sigma_{Kx}^2\right) \tag{8}$$

3.2 Statistical Modeling of Interference Power pdf

In this section, the pdfs obtained by the convolutions in (8) are fitted to the Nakagami-m distribution. In [9], the Nakagami-m pdf for a random variable r was given as

$$p(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\,\Omega^m} exp\left(-\frac{mr^2}{\Omega}\right) \tag{9}$$

where $m = E\left[r^2\right]^2 / var\left(r^2\right)$, $\Omega = E\left[r^2\right]$ and $\Gamma(m)$ is the Gamma function given in (10). As in [9], by putting $w = r^2$, we can make use of the Nakagami-m pdf of the power w. This pdf is obtained through the transformation $p(w)|dw| = p(r)|dr|$ and is given in (11), where Ω is replaced by $\bar{w} = E[w]$.

$$\Gamma(m) = \int_0^\infty x^{m-1} exp(-x)\, dx \tag{10}$$

$$p(w) = \left(\frac{m}{\bar{w}}\right)^m \frac{w^{m-1}}{\Gamma(m)} exp\left(-\frac{mw}{\bar{w}}\right) \tag{11}$$

Substituting our power variable, σ_x^2, into (11) as $w = \sigma_x^2$ and replacing $\bar{w}$ by $\zeta = E\left[\sigma_x^2\right]$, gives the expression of $p\left(\sigma_x^2\right)$ in (12).

$$p\left(\sigma_x^2\right) = \left(\frac{m}{\zeta}\right)^m \frac{\left(\sigma_x^2\right)^{m-1}}{\Gamma(m)} exp\left(-\frac{m\sigma_x^2}{\zeta}\right) \tag{12}$$

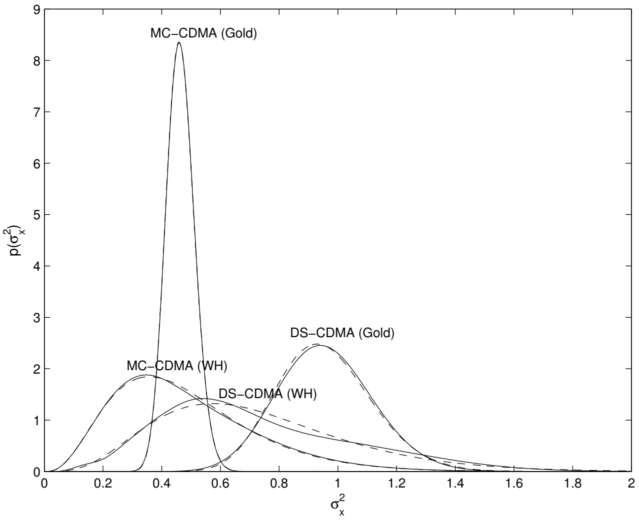

Figure 2: Nakagami-m pdfs of Total MAI Power; K=64, N=64 (WH) & N=63 (Gold)

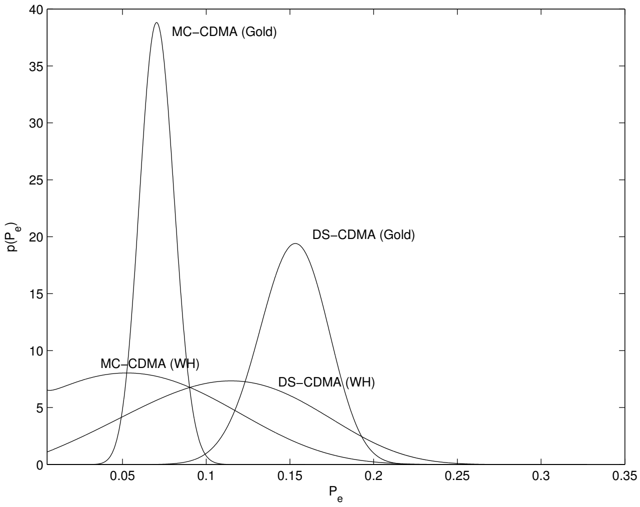

Figure 3: BER pdfs; K=64, N=64 (WH) & N=63 (Gold)

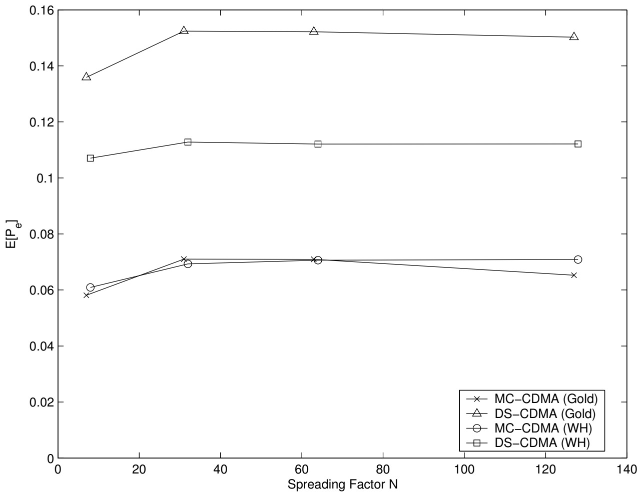

Figure 4: Mean BER as a Function of Spreading Factor

The results of the fittings for both systems are illustrated in Fig. 2 for the use of WH sequences of $N = 64$ and Gold sequences of $N = 63$. The solid curves represent the pdfs obtained by the convolutions in (8) and the Nakagami-m pdfs of the power (12) are represented by the dashed curves. The fitting is very accurate for both sets of spreading sequences, and is perfect for the MC-CDMA system using Gold sequences.

3.3 BER Analysis

Assuming perfect power control at the receiving base station, the SNR, denoted by γ, is given by $\gamma = 1/\sigma_x^2$. For BPSK modulated data, the probability of bit-error is directly related to the SNR. From the Complementary Gaussian Cumulative Distribution Function, the BER is obtained by the expression in (13) [10].

$$P_e = Q(\sqrt{\gamma}) = Q\left(\frac{1}{\sigma_x}\right) \tag{13}$$

The pdf of the BER can be obtained through the transformation $p(P_e)|dP_e| = p(\sigma_x^2)|d\sigma_x^2|$. This transformation is represented in (14), which becomes (15) after the substitution of the expression in (12).

$$p(P_e) = p(\sigma_x^2)\frac{2\left|(\sigma_x^2)^{3/2}\right|}{\left|Q'\left(\frac{1}{\sqrt{\sigma_x^2}}\right)\right|} \tag{14}$$

$$p(P_e) = \frac{2\left(\frac{m}{\zeta}\right)^m \frac{(\sigma_x^2)^{m-1}}{\Gamma(m)} exp\left(-\frac{m\sigma_x^2}{\zeta}\right)\left|(\sigma_x^2)^{3/2}\right|}{\left|Q'\left(\frac{1}{\sqrt{\sigma_x^2}}\right)\right|} \tag{15}$$

The BER pdfs of both systems for WH and Gold sequences of $N = 64$ and $N = 63$ respectively can be seen in Fig. 3. As is evident from the figure, the pdfs of the MC-CDMA system are distributed over the lower end of the BER scale for both sets of spreading sequences, in contrast to those of the DS-CDMA system. The systems using Gold sequences exhibit lower standard deviations of BER values in comparison to the systems using WH sequences for which the pdf is more dispersed about the mean.

The mean BER values are easily calculated from the BER pdfs. Figure 4 shows the mean BER values as a function of spreading factor for WH and Gold sequences of $N = 8, 32, 64, 128$ and $N = 7, 31, 63, 127$ respectively. After the initial increase in the mean BER value, the curves show little performance dependence on the spreading factor. The curves show a clear distinction between the BER performances of the MC-CDMA and DS-CDMA systems, for which the MC-CDMA systems offer a better performance. Moreover, there is a negligible difference between the mean BER performances under the use of WH and Gold sequences for the MC-CDMA system considered. The results suggest a higher resistance to MAI in the MC-CDMA technique considered in comparison to the DS-CDMA system for the considered transmission scenario.

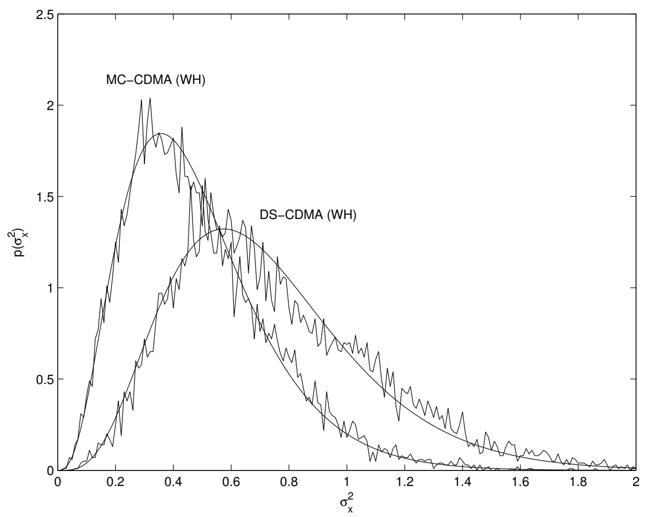

Figure 5: Pdf of Total MAI Power; K=64, N=64 (WH)

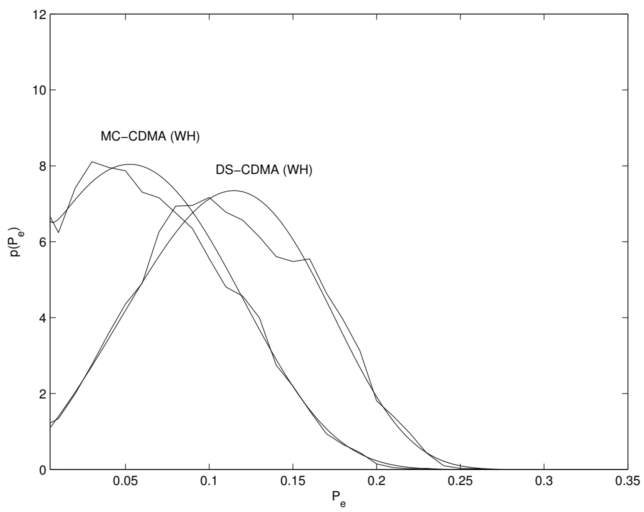

Figure 6: Pdf of BER; K=64, N=64 (WH)

4. SIMULATION RESULTS

In this section we confirm the results of the previous section through Monte Carlo simulations. The DS-CDMA and MC-CDMA systems considered in this paper have been simulated for WH and Gold codes with spreading factors $N = 8, 32, 64, 128$ and $N = 7, 31, 63, 127$ respectively. To ensure a fair comparison between the two multiple access techniques, the number of users in the system, K, was taken as N and $N+1$ under the use of WH and Gold sequences respectively.

Histograms of σ_x^2 values were produced through Monte Carlo simulations for each of the systems to observe the accuracy of the previous calculations. Each histogram consisted of 10000 different randomly generated offset vectors, $\underline{\tau}$, for which τ_j was uniformly distributed over the interval $[0, N-1]$. Figure 5 demonstrates the close approximation of the histogram to the total interference power pdf, $p\left(\sigma_x^2\right)$, for both the MC-CDMA and DS-CDMA systems using WH sequences of $N = 64$. The smooth curves represent the approximations of $p\left(\sigma_x^2\right)$ obtained from the convolutions in (8), while the jagged curves represent the histograms produced by the Monte Carlo simulations.

Similar histograms were produced for the BER pdfs through Monte Carlo simulations. Figure 6 shows the BER histograms for both systems using WH codes of $N = 64$. Again, the smooth curves represent the analytical approximation, while the jagged curves represent the Monte Carlo histograms. The results show a good approximation for both of the multiple access techniques. Similar plots to Figures 5 and 6 are available for the use of Gold sequences but have been omitted in this paper.

5. CONCLUSION

A statistical model for the MAI power and the BER pdfs of asynchronous MC-CDMA and DS-CDMA was proposed. The total MAI power was fitted to the Nakagami-m distributed for both WH and Gold sequences. The mean BERs obtained by the BER pdfs suggest a higher resistance to MAI in the MC-CDMA technique for the considered environment. The results also show negligible differences in the BER performance for the use of WH and Gold sequences in the MC-CDMA system.

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