DESIGN OF PERFECT RECONSTRUCTION MODULATED FILTER BANKS WITH ARBITRARILY HIGH NUMBER OF SUBBANDS

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ABSTRACT

We propose a design method for perfect reconstruction modulated filter banks that can be used for an arbitrarily high number of subbands. Its principle is based on the introduction of a function, named prototype, that is defined over the real field and is built in order to satisfy orthogonality or biorthogonality properties. By a uniform sampling of the prototype function, depending upon the number of subbands, we get afterwards perfect reconstruction prototype filters. The method is presented in the case of an optimization of the prototype function based on an energy criterion. The quality of the resulting prototype filters is illustrated by design examples in the orthogonal case.

1. INTRODUCTION

Critically decimated modulated filter banks present many advantages. They can be implemented thanks to fast algorithms and various design techniques are already available. Moreover, their design reduces to the optimization of only two discrete-time filters, $h_d[n]$and $f_d[n]$, called prototype filters, that will be assumed to be real-valued. In this paper, we will also assume that the two prototype filters are identical, $h_d[n] = f_d[n]$, but not necessarily linear-phase[1]. These prototypes can be used in different contexts, *e.g.* for subband coding using cosine modulation [1] or modified discrete Fourier transform (MDFT) [2], or even for multicarrier modulation [3], the perfect reconstruction conditions being the same. We just illustrate our method for modulated filter banks using a type IV discrete cosine transform (DCT-IV).

We denote by N the number of subbands of such a filter bank and the length L of the filters is assumed to be a multiple of twice the number of subbands, *i.e.* $L = 2mN$. The N analysis and synthesis filters are given respectively by [2]

$$h_k[n] = 2h_d[n]\cos\left[(2k+1)\frac{\pi}{2N}\left(n-\frac{D}{2}\right)+\theta_k\right], \qquad (1)$$

$$f_k[n] = 2f_d[n]\cos\left[(2k+1)\frac{\pi}{2N}\left(n-\frac{D}{2}\right)-\theta_k\right], \qquad (2)$$

with $\theta_k = (-1)^k\frac{\pi}{4}$, $0 \le n \le L-1$, $0 \le k \le N-1$ and D being the reconstruction delay, assumed to be such that $D = 2(s+1)N-1$, with s an integer parameter. For an orthogonal prototype filter, $D = L-1$.

In order to get a perfectly orthogonal or biorthogonal transformation, the coefficients $h_d[n]$ have to satisfy some perfect reconstruction conditions [1, 2]. Thus, usual criteria used for their optimization lead to some non-linear problems which can not easily be solved. That is why, if there exist various efficient techniques in the case of relatively reduced lengths (cf. for example [4, 5, 6]), it is difficult to provide orthogonal or biorthogonal optimized prototypes with many coefficients. Recently, a method described in [7] has nevertheless made it possible to attain 2048 subbands and 16384 coefficients. Nonetheless, the computational complexity of this method, that is still related to the evaluation of the optimization criterion, increases when the length of the filters increases, and in particular when the number of subbands increases. That is why we propose here a new method which avoids this problem and allows the optimization of filters for a number of subbands that can be absolutely arbitrarily high. We illustrate it for an optimization criterion that is the maximization of the weighted energy of the prototype filter $H_d(z)$, measured by the expression

$$J_{W_d}(h_d) = \frac{\int_{-\frac{1}{2}}^{\frac{1}{2}}\left|H_d\left(e^{j2\pi\nu}\right)\right|^2 W_d(\nu)\,d\nu}{\int_{-\frac{1}{2}}^{\frac{1}{2}}\left|H_d\left(e^{j2\pi\nu}\right)\right|^2 d\nu}, \qquad (3)$$

where $W_d(\nu)$ is a weighting positive function restricted to the interval $\left[-\frac{1}{2},\frac{1}{2}\right]$. It is worthwhile mentioning that the denominator term is a constant which simply allows us to get an energy measure independent of any amplitude normalization of the filters. Note also that this work gives an extension in the biorthogonal case of our reference [8].

2. PROTOTYPE FUNCTIONS

2.1 Definition and properties

For an integer parameter $m \ge 1$, we designate by $\mathbf{E}_m$ the set of functions h defined on the real interval $]-m,+m[$ such that on each sub-interval of $]-m,+m[$ of the form $]\frac{l}{2},\frac{l+1}{2}[$, with $l = -2m, -2m+1, \ldots, 2m-1$, the restriction of h is continuous and h has a limit on the bounds of this sub-interval. In all the following, we will say that a function of $\mathbf{E}_m$ is a *prototype function* [2] with parameter m. For h belonging to $\mathbf{E}_m$ and N an even integer parameter ≥ 2, we denote $H_N(z)$ the $2mN$-length filter with $h_N[n]$ coefficients given by

$$h_N[n] = h\left(\frac{2n+1-2mN}{2N}\right),\ n = 0,\ldots,2mN-1\ , \qquad (4)$$

[1] In the linear-phase case, we then get orthogonal prototypes.

[2] We will carefully distinguish the prototype functions from the prototype filters, the first ones being continuous whereas the second ones are discrete.

and we say that $H_N(z)$ is generated by h. The points for which the values of the function h are computed are never some semi-integers and they constitute a regular subdivision of the interval $]-m,+m[$. If the function h is even (respectively odd), the filters $H_N(z)$ are symmetrical (respectively anti-symmetrical).

We now introduce a weighting function $W(\nu)$, real-valued, symmetrical and positive, *i.e.* $W(\nu) \geq 0, \forall \nu \in]-\infty,\infty[$, and which has an inverse Fourier transform denoted $w(t)$. We consider the case of a prototype filter such that $h_d[n] = h_N[n]$ and we set $W_d(\nu) = W_N(\nu) = W(N\nu)$. We denote $J_{W_N}(h_N)$ the weighted energy of this filter $h_N[n]$.

Theorem 1.– The weighted energy of the filter $H_N(z)$ generated by the prototype function $h(t)$ has a limit when N tends towards infinity. It is denoted $J_W(h)$ and given by

$$\lim_{N\to\infty} J_{W_N}(h_N) = J_W(h) = \frac{\int_{-\infty}^{+\infty} |H(f)|^2 W(f)\, df}{\int_{-\infty}^{+\infty} |H(f)|^2\, df} \quad (5)$$

$$= \frac{\int_{-m}^{+m}\int_{-m}^{+m} h(t)h(u)w(t-u)\, dt\, du}{\int_{-m}^{+m} h^2(t)\, dt}, \quad (6)$$

where $H(f)$ is the Fourier transform of $h(t)$ given by

$$H(f) = \int_{-\infty}^{+\infty} h(t)e^{-j2\pi ft}\, dt. \quad (7)$$

Proof.– The denominator of equation (3) can be written

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} |H_N(e^{j2\pi\nu})|^2 d\nu = \sum_{n=0}^{2mN-1} h^2\left(\frac{2n+1-2mN}{2N}\right). \quad (8)$$

The function h is integrable in the Riemann sense on the interval $[-m,+m]$. The latest expression in (8) is therefore equivalent to $N\int_{-m}^{+m} h^2(t)\, dt$. The numerator of $J_{W_N}(h_N)$ is equal to

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} |H_N(e^{j2\pi\nu})|^2 W_N(\nu)\, d\nu = \frac{1}{N}\int_{-\frac{N}{2}}^{\frac{N}{2}} |H_N(e^{j2\pi\frac{f}{N}})|^2 W(f)\, df. \quad (9)$$

The expression of $H_N(z)$ allows us to consider $H_N(e^{j2\pi\frac{f}{N}})$ as a Riemann sum and we have

$$H_N(e^{j2\pi\frac{f}{N}}) \sim Ne^{j2\pi f(m-\frac{1}{2N})}\int_{-m}^{+m} h(t)e^{-j2\pi ft}\, dt. \quad (10)$$

Since h is real-valued, we also obtain the conjugate relation and the numerator of $J_{W_N}(h_N)$ is equivalent to

$$N\int_{-\frac{N}{2}}^{\frac{N}{2}} |H(f)|^2 W(f)\, df\, . \quad (11)$$

And we get (5) when N tends towards infinity.

The numerator of (5) is equal to

$$\int_{-\infty}^{+\infty}\left(\int_{-m}^{+m} h(t)e^{j2\pi ft}\, dt \int_{-m}^{+m} h(u)e^{-j2\pi fu}\, du\right) W(f)\, df$$

and it also writes

$$\int_{-m}^{+m}\int_{-m}^{+m} h(t)h(u)w(t-u)\, dt\, du. \quad (12)$$

This property shows that, when the number of subbands is sufficiently high, it is possible to maximize the weighted energy $J_W(h)$ of the prototype function rather than the one of the generated prototype filters $H_N(z)$.

2.2 Orthogonal and biorthogonal prototype function

Definition .– A prototype function h is orthogonal if, for all N, the filter $H_N(z)$ is an orthogonal linear-phase prototype filter.

Theorem 2.– Let $h \in \mathbf{E}_m$ be a prototype function with parameter m. We denote $G(t,z) = \sum_{n=0}^{m-1} h(-m+2n+t)z^{-n}$, for $0 < t < \frac{1}{2}$. Then h is an orthogonal prototype function if and only if h is symmetrical and if there exist m functions $\theta_i(t)$, $0 \leq i \leq m-1$, continuous on $[0,\frac{1}{2}]$ such that for $0 < t < \frac{1}{2}$, $G(t,z)$ and $G(t+1,z)$ satisfy the matrix equality

$$[G(t,z)\ G(t+1,z)] = [G^{\text{init}}(t,z)\ G^{\text{init}}(t+1,z)] \prod_{i=1}^{m-1} \mathbf{\Lambda}(z)\mathbf{\Theta}(\theta_i(t)) \quad (13)$$

with

$$\mathbf{\Lambda}(z) = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}, \mathbf{\Theta}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \quad (14)$$

and

$$[G^{\text{init}}(t,z)\ G^{\text{init}}(t+1,z)] = \alpha\,[\cos\theta_0(t)\sin\theta_0(t)] \quad (15)$$

where α is a non zero constant. When $\alpha = 1$, we also have $\int_{-m}^{+m} h^2(t)dt = 1$.

Definition .– A prototype function h is biorthogonal if, for all N, the filter $H_N(z)$ is a biorthogonal prototype filter.

Theorem 3.– Let $h \in \mathbf{E}_m$ be a prototype function with parameter m. We denote $G(t,z) = \sum_{n=0}^{m-1} h(-m+2n+t)z^{-n}$, for $0 < t < \frac{1}{2}$. Then h is a biorthogonal prototype function if and only if there exist $2m+1$ functions $\mu_i(t)$, $0 \leq i \leq 2m$, continuous on $[0,\frac{1}{2}]$ such that for $0 < t < \frac{1}{2}$, $G(t,z)$, $G(t+1,z)$, $G(1-t,z)$ and $G(2-t,z)$ satisfy the matrix equality

$$[G(t,z)\ G(t+1,z)] = [G^{\text{init}}(t,z)\ G^{\text{init}}(t+1,z)] \times \prod_{i=1}^{i_1} \mathbf{A}(\mu_{2i+i_0}(t))\mathbf{B}(\mu_{2i+i_0+1}(t)) \times \prod_{j=1}^{j_1} \mathbf{C}(\mu_{2j+2i_1+i_0}(t))\mathbf{D}(\mu_{2j+2i_1+i_0+1}(t)) \quad (16)$$

$$\begin{bmatrix} G(1-t,z) \\ G(2-t,z) \end{bmatrix} = \prod_{j=j_1,-1}^{1} z^{-2}\mathbf{D}^{-1}(\mu_{2j+2i_1+i_0+1}(t))\mathbf{C}^{-1}(\mu_{2j+2i_1+i_0}(t)) \times \prod_{i=i_1,-1}^{1} \mathbf{B}^{-1}(\mu_{2i+i_0+1}(t))\mathbf{A}^{-1}(\mu_{2i+i_0}(t)) \begin{bmatrix} G^{\text{init}}(1-t,z) \\ G^{\text{init}}(2-t,z) \end{bmatrix} \quad (17)$$

with

$$\mathbf{A}(\mu) = \begin{bmatrix} 1 & 0 \\ \mu z^{-1} & 1 \end{bmatrix}, \quad \mathbf{B}(\mu) = \begin{bmatrix} 1 & \mu \\ 0 & 1 \end{bmatrix} \tag{18}$$

$$\mathbf{C}(\mu) = \begin{bmatrix} z^{-1} & 0 \\ \mu & 1 \end{bmatrix}, \quad \mathbf{D}(\mu) = \begin{bmatrix} 1 & \mu \\ 0 & z^{-1} \end{bmatrix} \tag{19}$$

and

$$[G^{\text{init}}(t,z)\ G^{\text{init}}(t+1,z)] = \alpha[1\ 1]\mathbf{F}_0(\mu_0(t),\mu_1(t),\mu_2(t))\mathbf{F}_1 \tag{20}$$

$$\begin{bmatrix} G^{\text{init}}(1-t,z) \\ G^{\text{init}}(2-t,z) \end{bmatrix} = \alpha\hat{\mathbf{F}}_1\mathbf{F}_0^{-1}(\mu_0(t),\mu_1(t),\mu_2(t)) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{21}$$

with

$$\mathbf{F}_0(\mu_0,\mu_1,\mu_2) = \begin{bmatrix} 1 & 0 \\ \mu_0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \mu_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \mu_2 & 1 \end{bmatrix}, \tag{22}$$

and

$$\mathbf{F}_1 = \mathbf{C}(\mu_3)\mathbf{B}(\mu_4) \quad \text{and} \quad \hat{\mathbf{F}}_1 = z^{-1}\mathbf{F}_1^{-1} \quad \text{if } s \text{ is odd,} \tag{23}$$

$$\mathbf{F}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \hat{\mathbf{F}}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{if } s \text{ is even,} \tag{24}$$

where α is still a non-zero constant and s is defined by $D = 2(s+1)N-1$, D being the reconstruction delay. We assume therefore that $i_0 = 1$ if s is even, and $i_0 = 3$ if s is odd.

The proofs of theorems 2 and 3 directly result from the definitions of the orthogonal and biorthogonal prototype functions and from the lattice and lifting representation of the orthogonal and biorthogonal prototype filters associated to cosine modulated filter banks, cf. for example [1, 2].

3. PROTOTYPE FUNCTIONS OPTIMIZATION

3.1 Computation of the energy

Let us now focus on the numerical computation of the weighted energy of the prototype function h. It is obtained from the quantity

$$E_W(h) = \int_{-m}^{+m}\int_{-m}^{+m} h(t)h(u)w(t-u)\,du. \tag{25}$$

To numerically compute the integral (25), we divide the integration domain in squares with length $\frac{1}{2}$ and with semi-integer valued extremities. On each of these $16m^2$ small squares, we compute the integral thanks to the n-order Gauss-Legendre integration scheme, with fixed n, on each of the directions u and t.

We recall that the n-order Gauss-Legendre integration scheme allows us to evaluate the numerical value of a function f on the interval $[-1,+1]$ thanks to the formula [9]

$$\int_{-1}^{+1} f(t)\,dt \approx \sum_{k=1}^{n} p_k f(x_k)\,, \tag{26}$$

where $x_k,\ k=1,\ldots,n$ are the n roots of the n-degree Legendre polynomial, denoted $P_n(x)$, and $p_k,\ k=1,\ldots,n$ some weights computed thanks to the formula

$$p_k = \frac{2(1-x_k^2)}{n^2P_{n-1}^2(x_k)}\,. \tag{27}$$

In practice, we choose $n=6$ or $n=10$. After a slight transformation of formula (26) to adapt it to an interval with length $\frac{1}{2}$, and thanks to the symmetry, the integral writes

$$E_W(h) \approx \sum_{i=0}^{2mn-1}\sum_{j=0}^{2mn-1} h(u_i)h(u_j)w_{i,j} \tag{28}$$

where $w_{i,j}$ are the coefficients of a symmetrical $2mn$-order matrix $\mathbf{W}$ which does not depend on h but only on n and on the weighting function W. Therefore, we only evaluate one time the value of its coefficients before optimizing the cost function. The vector $\mathbf{V} = (h(u_0),h(u_1),\ldots,h(u_{2mn-1}))$, with length $2mn$ is evaluated for some given values of the parameters of the optimization problem and the function $E_W(h)$ writes $E_W(h) \approx \mathbf{V}^T\ \mathbf{W}\ \mathbf{V}$.

3.2 Energy optimization

We now impose the orthogonality conditions defined by theorem 2 and we write the prototype function with m functions $\theta_i(t)$ chosen such that

$$\theta_i(t) = \sum_{l=0}^{K-1}\theta_{i,l}t^l\ i=0,\cdots,m-1, \tag{29}$$

where $K-1$ is the considered degree of the interpolation polynomial and $\theta_{i,l},\ l=0,\ldots,K-1$ the K corresponding coefficients. It is worthwhile noting that this formulation is equivalent, in continuous-time, with the so-called compact representation that we have already presented in [7] in the discrete-time case. The optimization problem then consists to determine the coefficients $\theta_{i,l}$ minimizing the limit energy, *i.e.* $\min_{\theta_{i,l}} \mathbf{V}^T\ \mathbf{W}\ \mathbf{V}$.

This is a non-linear optimization problem that we solve with the "feasible sequential quadratic programming" method (CFSQP) described in reference [10] and taking care of choosing different initialization points. It is clear that a similar procedure can be used in the case of biorthogonal prototypes replacing $\theta_i(t)$ by $\mu_i(t)$, but, to be more concise, we just illustrate our method in the orthogonal case.

3.3 Complexity gain

A direct computation of $J_{W_N}(h_N)$ imposes mN evaluations of the coefficients $h_N[n]$ using some angular parameters whose expression is similar to the one given in (29) [7], and the use of fast Fourier transforms of size greater than $4mN$.

Using the limit allows us to avoid that the complexity grows with N. Indeed, only $2mn$ evaluations of the prototype functions with the help of relations (29) and (13) are necessary, whereas the computation of the quadratic form leading to $E_W(h)$ has an arithmetic complexity in $\mathscr{O}(m^2n^2)$.

4. METHOD EVALUATION

We have applied our optimization technique to the computation of prototype functions generating low-pass prototype filters with cutoff frequency $f_{c,N} = \frac{1}{2N}$. The function W we use is therefore defined by

$$W(\nu) = \begin{cases} 1 & \text{if } 0 \le |\nu| < f_c = \frac{1}{2} \\ 0 & \text{else.} \end{cases} \tag{30}$$

The optimization of the prototype function for $m = 4$ yields the results given in figures 1 and 2. On the positive part of the time axis, we notice the presence of discontinuities for $t = 5/2$ and $t = 7/2$. The frequency representation of figure 2 shows the result of the maximization of the energy of the prototype function in the frequency interval $[-\frac{1}{2}, \frac{1}{2}]$.

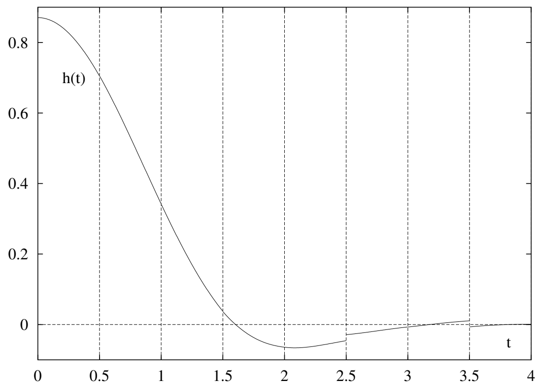

Figure 1: Time response of the prototype function for $m = 4$ and $f_c = \frac{1}{2}$.

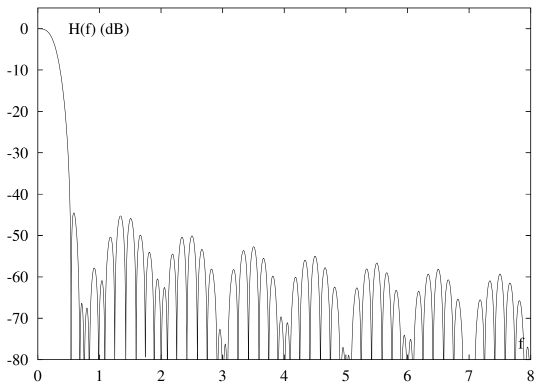

Figure 2: Frequency response of the prototype function for $m = 4$ and $f_c = \frac{1}{2}$.

The optimal results for $K = 6$ are given in table 1 where $J_\infty = 1 - J_W(h)$ corresponds to the optimal out-of-band energy for the prototype function and J_{128} the out-of-band energy of the generated filters for $N = 128$ subbands.

Indeed, the knowledge of an optimized orthogonal prototype function $h(t)$ allows us to instantaneously compute, thanks to a simple sampling (cf. eq. (4)), the prototype filters $H_N(z)$ for any number of subbands N. We notice that, as soon as $N \geq 128$, the value of the out-of-band energy of the generated filters is very close to the limit value J_∞.

For $N = 128$ a new optimization is then made thanks to the method presented in [7] and using as starting points the filters obtained by sampling. The resulting optimum J^*_{128} provides a relative energy gain $\varepsilon = (J_{128} - J^*_{128})/J_{128}$ reported in the last column of table 1. The very low values of ε show the quasi-optimality of the results obtained by sampling. These results can also be favorably compared to those provided by lapped transforms (LT) [6] that for $m \leq 2$ have closed-form PR expressions for any value of N. When $N = 128$, with the modulated LT, based on [6, page 178] we get an out-of-band energy equal to 2.99×10^{-2} for $m = 1$, and for the extended one [6, page 184], we get 7.76×10^{-3} for $m = 2$.

m	J_∞	J_{128}	ε
1	1.898334×10^{-2}	1.898132×10^{-2}	1.10×10^{-10}
2	2.037405×10^{-3}	2.037340×10^{-3}	2.37×10^{-11}
3	3.183674×10^{-4}	3.182363×10^{-4}	8.01×10^{-6}
4	8.636129×10^{-5}	8.634476×10^{-5}	5.46×10^{-6}
5	1.344354×10^{-5}	1.380102×10^{-5}	7.60×10^{-4}
6	2.927395×10^{-6}	3.018592×10^{-6}	1.48×10^{-2}
7	1.229321×10^{-6}	1.373668×10^{-6}	4.70×10^{-3}
8	2.773781×10^{-7}	2.739358×10^{-7}	4.54×10^{-3}

Table 1: Out-of-band energy optimization for some m-parameter orthogonal prototype functions and $K = 6$.

5. CONCLUSION

We have proposed a design method that can lead to PR filter banks for an arbitrarily high number of subbands (N). A comparison with LT [6] illustrate its efficiency in terms of the minimization of the out-of-band energy. Compared to the design method we proposed in [7], another advantage is, for a given N, to significantly reduce the CPU design time.

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