

NEARLY-PERFECT RECONSTRUCTION COSINE-MODULATED FILTER BANK APPLIED TO ECG SIGNAL CODING

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ABSTRACT

This work considers the design of a electrocardiogram (ECG) signal coder based on nearly perfect reconstruction (N-PR) cosine modulated filter banks. The target of the coder is the quality of the reconstructed signal, which must remain within predetermined accuracy limits. We employ as quality criterion the percentage root-mean-square difference (PRD). The tests have been done for the twelve principal cardiac leads, and the amount of compression is evaluated by means of the mean number of bits per sample (MBPS) and the compression ratio (CR). The results confirm the validity of the proposed system.

1. INTRODUCTION

ECG processing using subband and wavelet transforms is a subject of great interest and plenty of data compression techniques have been developed to encode digitized ECG signals (see the introduction of [1]).

In this work, we deal with the design of another easy and efficient ECG signal coder that is able to obtain a low bit rate whilst maintaining the quality of the reconstructed signal. The overall block diagram of the proposed coder is shown in Fig. 1, and consists of three different stages. The first stage is based on N-PR cosine-modulated filter bank. That is the novel and main contribution of this work: the application of N-PR cosine-modulated filter banks to ECG signal coding. The algorithm implemented in the second and third stages of Fig. 1 is extremely easy: it is based on thresholding and has been developed to continuously process the signal without QRS detection and heartbeat segmentation. Its simplicity can be exploited in order to implement the coder in a real system. As there is some information loss, the quality of the reconstructed signal must be assured. Our objective is the quality of the recovered signal. The quality measurement must be obtained by objective means, as the waveform of the retrieved signal must be as close as possible to the original in order to guarantee that important information (such as certain kinds of pathologies) is not lost. PRD has been widely used (see as example [1]) as an objective measurement that preserves the quality of the original waveform to within an acceptable degree:

$$PRD = \sqrt{\frac{\sum (x[n] - \hat{x}[n])^2}{\sum (x[n])^2}} \cdot 100, \quad (1)$$

where $x[n]$ and $\hat{x}[n]$ are the original and the reconstructed signals, respectively. We compare the proposed method with the wavelet packet (WP) techniques, usually adopted by the scientific community.

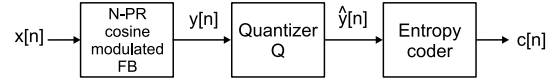


Figure 1: Overall block diagram of the compressor.

The outline of the work is as follows. Section 2 presents a brief review of N-PR cosine-modulated filter banks and the implementation cost is studied. In Section 3 the compression algorithm is explained, and in Section 4 several examples are shown. Finally, in Section 5 our conclusions are presented.

2. M-CHANNEL N-PR COSINE-MODULATED FILTER BANKS

M -channel maximally decimated filter bank with a parallel structure have been extensively studied and they are used in many applications to solve biomedical problems, such as ECG beat detection [2] and analysis and classification of infarcted myocardial tissue [3].

An important subclass of M -channel filter banks is the modulated filter bank group – those for which all the analysis and synthesis filters are obtained by the modulation of lowpass prototype filters. In this work, we use conventional N-PR cosine-modulated filter banks [4] to divide the incoming signal into separate subband signals. These systems offer almost but not true perfect reconstruction (PR). They do however offer an alternative to PR systems avoiding the highly nonlinear optimization that is needed to obtain the coefficients of the filters in PR systems. We will show how this type of filter bank can be used for compressing ECG signals.

The scheme of the cosine modulation is the corresponding to conventional N-PR cosine-modulated filter banks: the real coefficients impulse response of the analysis $h_k[n]$ and synthesis filters $f_k[n]$, $0 \leq n \leq N$, $0 \leq k \leq M-1$, are obtained as

$$\begin{aligned} h_k[n] &= 2 \cdot p[n] \cdot \cos \left((2k+1) \frac{\pi}{2M} \left(n - \frac{N}{2} \right) + (-1)^k \frac{\pi}{4} \right), \\ f_k[n] &= 2 \cdot p[n] \cdot \cos \left((2k+1) \frac{\pi}{2M} \left(n - \frac{N}{2} \right) - (-1)^k \frac{\pi}{4} \right), \end{aligned} \quad (2)$$

where $p[n]$ is the N -order prototype filter.

2.1 Prototype Filter Design Techniques

Several methods have been proposed that facilitate the design of the filter to be modulated (see, for example, refs. [5, 6]). In this work, we employ the prototype filter design method proposed in [7]. The problem can be stated in several

different ways, but the purpose consists in minimizing

$$\phi = \left| \left| P \left(e^{j\pi/(2M)} \right) \right| - 1/\sqrt{2} \right|, \quad (3)$$

where $P(e^{j\pi/(2M)})$ is the frequency response of the prototype filter at $\omega = \pi/(2M)$.

When we use an appropriate finite impulse response (FIR) filter design technique (by windowing or by means of the Parks-McClellan algorithm), we can guarantee that the frequency response of the prototype filter approximately satisfies the power complementary property. In other words, this technique controls the position of the 3dB cutoff frequency of the prototype filter and sets it approximately at $\pi/(2M)$. This condition ensures that $p[n]$ is approximately a spectral factor of a $2M - th$ band filter. In this way, it is possible to reduce the amplitude distortion and aliasing errors introduced in the filter bank.

2.2 Fast Algorithm of Implementation

One of the reasons why cosine-modulated filter banks are widely used is due to the efficient implementations of the analysis and synthesis banks that can be obtained. In order to obtain one of these efficient implementations, we express the prototype filter as

$$P(z) = \sum_{\ell=0}^{2M-1} z^{-\ell} \cdot G_{\ell}(z^{2M}), \quad (4)$$

where $G_{\ell}(z)$ is the $2M$ type 1 polyphase components of the prototype filter $P(z)$ [4].

A simplified implementation of this bank can be derived from the polyphase matrices when the prototype filter length $(N + 1)$ and the number of channels M are related as

$$N + 1 = 2mM. \quad (5)$$

If we accept the above restriction, and assume that m is an even number, it has been shown that the analysis filters can be expressed as

$$\mathbf{h}^T(z) = \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \hat{\mathbf{C}}_A \cdot \begin{bmatrix} \mathbf{g}_0(-z^{2M}) \\ z^{-M} \mathbf{g}_1(-z^{2M}) \end{bmatrix} \cdot \mathbf{e}(z), \quad (6)$$

where

$$\mathbf{g}_0(z) = \text{diag}[G_0(z) \quad G_1(z) \quad \cdots \quad G_{M-1}(z)], \quad (7)$$

$$\mathbf{g}_1(z) = \text{diag}[G_M(z) \quad G_{M+1}(z) \quad \cdots \quad G_{2M-1}(z)], \quad (8)$$

and

$$\mathbf{e}(z) = [1 \quad z^{-1} \quad \cdots \quad z^{-(M-1)}]^T. \quad (9)$$

The cosine modulation matrix $\hat{\mathbf{C}}_A$ can be expressed as [4]

$$\hat{\mathbf{C}}_A = \sqrt{M} \cdot \Lambda_C \cdot \mathbf{C} \cdot \begin{bmatrix} (\mathbf{I} - \mathbf{J}) & -(\mathbf{I} + \mathbf{J}) \end{bmatrix}. \quad (10)$$

Matrices \mathbf{I} and \mathbf{J} denote, respectively, the $k \times k$ identity matrix, and the $k \times k$ reverse operator. Λ_C is a diagonal matrix with elements

$$[\Lambda_C]_{k,k} = \cos(\pi \cdot (0.5 + k) \cdot m), \quad (11)$$

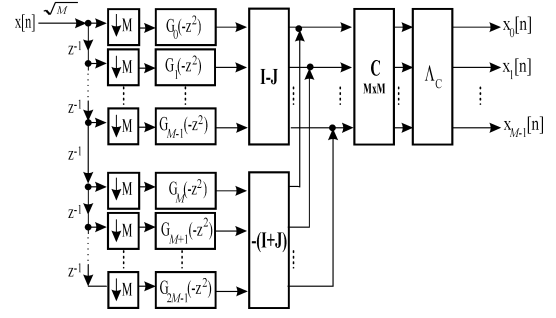


Figure 2: Efficient polyphase implementation of the analysis bank.

and \mathbf{C} is the Type 4 Discrete Cosine Transform (DCT) matrix defined as [8]

$$[\mathbf{C}]_{k,n} = \sqrt{\frac{2}{M}} \cos\left(\frac{\pi}{M} \cdot (0.5 + k) \cdot (0.5 + n)\right). \quad (12)$$

Therefore, based on eq. 6, the analysis bank structure can be drawn as in Fig. 2.

2.3 Implementation Cost

In this subsection, we study the implementation costs – taking into consideration the number of multiplications and additions – of the N-PR cosine-modulated filter bank. We only consider the computational cost of the analysis stage, as the cost of the corresponding synthesis bank is similar for both systems.

The direct polyphase implementation of the analysis bank requires N multiplications and N additions per input sample (see [4] for more details). On the other hand, the implementation cost of the M -channel cosine-modulated analysis filter bank using the structure in Fig. 2 is roughly the following:

1. $(N + 1)/M$ multiplications and N/M additions per input sample in the polyphase filters stage [4].
2. $((M/2) \log_2 M + M)$ multiplications and $(3M/2) \log_2 M$ additions to compute the M -point Type 4 DCT [8].
3. As m is an even number, Λ_C only changes the signs of the subband signals in special cases. We do not consider these operations to be multiplications.
4. $(\mathbf{I} - \mathbf{J})$ matrix requires M additions per input sample if M is an even number, and $(M - 1)$ additions per input sample if M is an odd number.
5. $-(\mathbf{I} + \mathbf{J})$ matrix requires M additions per input sample if M is an even number, and $(M - 1)$ additions and 1 multiplication per input sample if M is an odd number.
6. 1 multiplication and M additions per input sample for the rest of the operations.

Thus, the total cost of the fast implementation of the analysis bank is the following. For M even, $(M/2) \log_2 M + 2M + (N + 1)$ multiplications and $(3M/2) \log_2 M + N + 3M^2$ additions per M input samples. For M odd, $(M/2) \log_2 M + 3M + (N + 1)$ multiplications and $(3M/2) \log_2 M + N + 3M^2 - 2M$ additions per M input samples.

3. COMPRESSION SCHEME

The algorithm implemented in this work is the following. When the above scheme of subband decomposition is applied to ECG signals, most of the energy is concentrated in a few at low band, so a thresholding technique can be applied. Samples with an amplitude below a threshold value are discarded. Only the largest are maintained thus assuring that the quality of the reconstructed signal is as close as possible to the original. The quality of the reconstructed signal is selected before the compression is made as a predetermined PRD value. The algorithm begins by fixing an initial threshold value to check the target PRD, which is the same for all subbands. If it is not reached, a new threshold is chosen iterating the previous procedure until the target PRD is accomplished. The preceding technique is applied to each input segment.

For the entropy coder stage, a run-length coding is used as a means of joining the void samples. The non-discarded samples of each processed segment are sent or stored without varying the original precision (16 bits). In each set of processed samples there will be unused codes called escape codes. The threshold value can be used as an escape code to indicate the zero position. The next sample will be the number of consecutive zeros.

Fig. 3 shows the flowchart of the run-length coding algorithm. Two samples must be included as a header in every segment: the first will be a word indicating the beginning of the segment and the second the escape code of the current segment. The next samples are the informative content of the segment. Non-discarded samples are coded until a zero stream appears, which is indicated by the escape code. Then the number of consecutive zeros is coded with different precision. As in the first step with 4 bits, when the number of zeros is less than sixteen. The bit number '1111' marks an overflow (more than fifteen consecutive zeros). In this case, the stream of zeros is coded with the previous 4 bits plus an adequate number of bits to complete the length of the segment. Note that, an isolated zero is coded without an escape code.

4. RESULTS

In this section, we show the behaviour of the N-PR filter bank based scheme. The database¹ used to carry out the test contains two sets of twelve standard leads. Each lead is sampled at 360 Hz and each sample is coded in pulse code modulation (PCM) with 16 bits per sample. Each set has a different length. Every lead of the first set lasts 10 minutes, whereas the signals of the second last 2 minutes. The signals included in the database are cleaned from high frequency noise and some leads have baseline fluctuations. Atrial fibrillation is the pathology contained in the database.

We have used two compression degree measurements in order to adequately show the results. The first is the MBPS that evaluates the number of bits by means of each encoded sample. The second is the CR, which is the ratio between the number of bits of the original signal and the number of bits of the retrieved signal.

The tests were carried out with a set of 16-channel N-PR cosine modulated filter banks. They were obtained using

¹This database has been supplied by the Electro-physiology Laboratory (Cardiology floor) group of the General Hospital Universitario Gregorio Marañón of Madrid.

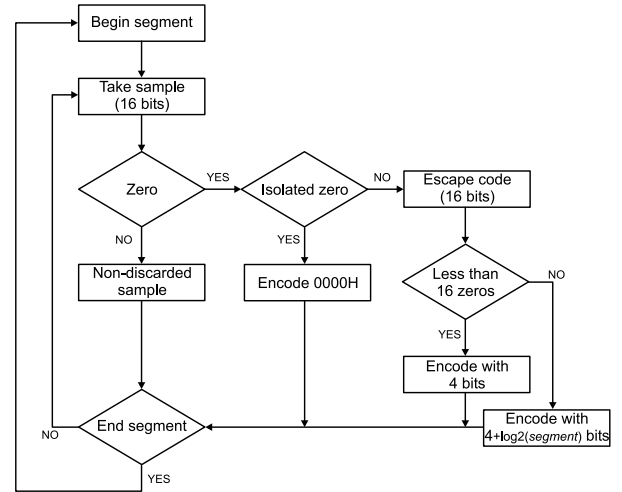


Figure 3: Flowchart of the run-length coding algorithm.

192-length prototype filters designed by the technique explained in Section 2 and using the Blackman window. Apart from the length of the filters and the number of channels, there are still two free parameters: the segment length that splits the input signal and the PRD value to select the quality of the recovered signal.

As the objective is quality, the recovered signal waveform must remain as close as possible to the original signal. However, the PRD as a performance measure is not sufficient to decide whether the retrieved signal is suitable or not. As a clinical expert will make the diagnosis, a clinical expert must also validate the compression algorithm after visually inspecting the waveforms. High PRD values are unsuitable for ensuring that the retrieved waveform will be within an acceptable error margin of the original. On the other hand, as far as noise is concerned, thresholding-based compressors behave as a low-pass filter so that high CR values could be obtained by smoothing the ECG, which is not our case as the signals have got no high frequency noise added. So to ensure that the retrieved signal will remain close to the original signal, the target PRD selected must be low. For these reasons, we have decided to select only PRD values from 0.5% to 5%.

As far as segment length is concerned, the CR improves by increasing the segment length. We have come to the conclusion that by using the proposed N-PR filter bank, good results are obtained for segment lengths from as little as 512 samples. Thus, the tests have been made using block lengths of incoming signals from 512 to 4096 samples.

Fig. 4 shows the global results, for both WP and the N-PR cosine-modulated filter bank designed by the proposed method (using a Blackman window) for a 10 minute-long AVR lead, as a three-dimensional representation. The transparent surface represents the results of a WP-based compressor explained in reference [9]. The MBPS is represented as a function of PRD and the segment length. This graphic representation clearly shows the behaviour of the compression method. The results presented are the mean compression values of all the segments processed. In order not to falsify the performance, the last segment has been removed, as it was zero padded before compression to lengthen the segment to the corresponding power of two. If we compare both sheets,

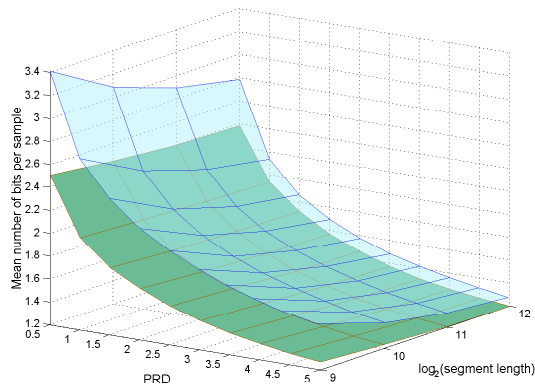


Figure 4: Results for the 10 minute-long AVR lead using WP (transparent surface), and N-PR cosine-modulated filter bank (opaque surface).

it can be seen that the one corresponding to the N-PR filter bank (the opaque surface in Fig. 4) is always lower than the WP sheet (the transparent surface in Fig. 4). This clearly demonstrates that the compression achieved by using the N-PR cosine-modulated filter bank, provides good results when used with the compression scheme applied in this work, even better than a WP scheme.

The compressor retrieves the incoming signal for a previously fixed quality value. As the input signal is processed using two power length blocks, the quality of each block will be within a 5% margin of the chosen PRD value ($PRD \pm 5\%$), which is specified by the algorithm. The target PRD is quickly reached with just a few iterations of the compression algorithm. For those cases that do not converge to the specified quality value, the number of iterations is restricted to 25. To appreciate this behaviour, Fig. 5 shows the histograms of both the PRD and the CR for three different target PRD: 0.5%, 1% and 1.5%. As is shown, the PRD spreads around the target PRD selected being wider as the target PRD increases. Nevertheless, it does not overcome the 5% of the target PRD chosen.

5. CONCLUSIONS

In this work, a new compression algorithm based on N-PR CMFB scheme is proposed, whose objective is the quality of the retrieved signal. The algorithm is thresholding-based, so it is very easy to implement in real time. The novel and the main contribution of this work is the application of N-PR CMFB to code the ECG signal. Moreover, two signal processing tools, WP and a N-PR cosine-modulated filter bank have been compared in order to show the performance of the proposed method. In conclusion, the scheme based on cosine-modulated filter banks provides a good degree of compression, particularly when a small target PRD value (0.5%) is requested. Increasing the segment length, does not significantly improve the CR for an N-PR filter bank, which makes it of interest for use in real time implementations. The tests were done for the twelve cardiac leads and the system behaved the same for all of them, obtaining similar results under the same conditions of compression.

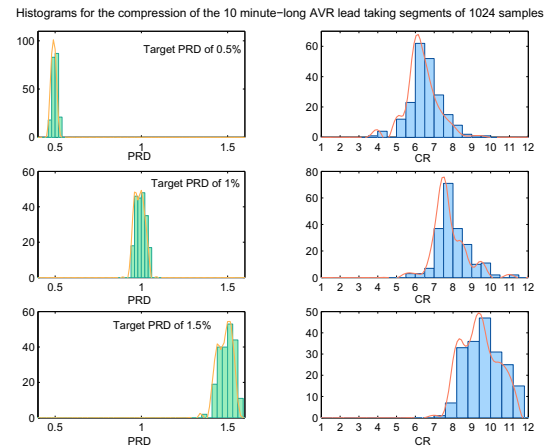


Figure 5: Histograms of both the PRD (left) and CR (right) for several target PRD.

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