BIT RATE MAXIMIZING WINDOW AND EQUALIZER DESIGN FOR DMT-SYSTEMS

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ABSTRACT

The classical equalizer structure in a discrete multi tone (DMT) receiver, as e.g. in asymmetric digital subscriber line (ADSL), consists of a (real) time domain equalizer (TEQ) combined with complex 1-tap frequency domain equalizers (FEQ's). In addition, receiver windowing can be added in front of the demodulating DFT to improve the spectral containment of the DFT-filters. In this paper, a framework is developed for the combined design of an equalizer and a window that maximizes the achievable bit rate. This general framework allows to treat equalizer-only and window-only designs as well, which appear as special cases in a natural way. This bit rate maximizing design can serve either as a practical design method, or as an upperbound for existing (suboptimal) methods. For the same achievable bit rate it will also be shown that equalizer taps can be exchanged for window coefficients to obtain a complexity reduced receiver.

1. INTRODUCTION

In a DMT-transmitter the available frequency bandwidth is divided into parallel subchannels or tones by means of an inverse discrete Fourier transform (IDFT). After IDFT modulation, a guard time sequence of ν samples - called a cyclic prefix (CP) - is inserted between two successive symbols to cope with inter-symbol-interference (ISI) and inter-carrier-interference (ICI). At the receiver, the CP is removed and demodulation is performed by means of a DFT.

The CP is only effective if the channel impulse response length is smaller that the CP length plus one. In this case equalization can easily be done by means of a 1-tap frequency domain equalizer (FEQ) for each tone. If the channel exceeds the CP duration a T-tap time domain equalizer (TEQ) is typically inserted in front of the DFT to shorten the channel, see e.g. [7] and references therein. In ADSL the ultimate goal is to design the TEQ to maximize the achievable bit rate for a predefined bit error rate. Although many algorithms have been developed to design the TEQ, only in [7] the bit rate is effectively maximized.

Apart from ISI and ICI, DMT transmission is also impaired by noise such as narrowband radio frequency interference (RFI) (emerging from AM broadcast and HAM radio [1]). Due to the block based transmission, the signal at the receiver is effectively *windowed* in time by a rectangular window, leading to sinc shaped sidelobes of the DFT filters. The bad spectral containment of the sinc shaped filters will cause the RFI to be spread over many tones, thereby reducing the number of bits that can be loaded onto these tones. The DFT sidelobes can be reduced by windowing the received DMT symbols with a non-rectangular window prior to DFT demodulation. If the channel impulse response length is shorter than $\nu-\mu+1$ samples, receiver windowing (with μ window coefficients) is used to mitigate RFI [4], while preserving the DMT orthogonality, required for ISI/ICI-free operation. In [2] research is focused on how to design the window coefficients, without optimizing the bit rate and without considering the equalization problem.

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The goal of this paper is to provide a general framework where a combined window and TEQ can be designed to truly maximize the bit rate for a given number of TEQ and window taps. The outcome will be referred to as the bit rate maximizing window and TEQ (BM-WinTEQ). Special cases where only a TEQ or only a window is desired can then straightforwardly be obtained. The resulting framework will also be used to demonstrate the relation with existing equalizer and window designs.

The organization of the paper is as follows: the data model and the BM-WinTEQ are introduced in Sections 2 and 3 respectively. Special design cases will be treated in Section 4. Complexity issues, simulations and conclusions can be found in Sections 5, 6 and 7 respectively.

2. DATA MODEL AND NOTATION

Here, we summarize our notation, which is mostly based on [5].

- N is the (I)DFT size; ν is the prefix length; $s=N+\nu$ equals the symbol size; N_u is the number of used tones; $\mathscr{S}$ is the set of used tones; i and k are the tone index and DMT symbol index.
- $\mathscr{F}_N$ and $\mathscr{I}_N$ are a DFT and IDFT matrix of size N respectively; $\mathscr{F}_N(i,:)$ is the i-th row of $\mathscr{F}_N$.
- The transmitted QAM frequency domain symbol at time k on tone i is X_i^k.
- D_i is the 1-tap (complex) frequency domain equalizer (FEQ) for tone i; $\mathbf{w}=[w_0\ \dots\ w_{T-1}]^{\mathrm{T}}$ is the T-taps TEQ and $\mathbf{u}=[u_0\ \dots\ u_{\mu-1}]^{\mathrm{T}}$ are the μ window coefficients; diag($\mathbf{u}$) represents a diagonal matrix with the elements of $\mathbf{u}$ on its diagonal.
- $\mathbf{Y}^{ext,k}$ is a Toeplitz matrix (size $(N+\mu)\times T$) of received time domain samples, which is completely defined by its first column $[y^k_{-\mu}\ \dots\ y^k_{N-1}]^{\mathrm{T}}$ and its first row $[y^k_{-\mu}\ \dots\ y^k_{-\mu-T+1}]$. The first column of $\mathbf{Y}^{ext,k}$ and the last N rows are denoted as $\mathbf{y}^k$ and $\mathbf{Y}^k$ respectively. The elements of the first row and the first column of $\mathbf{Y}^{ext,k}$ stacked in a vector are denoted as $\mathbf{y}^{ext,k}$.
- $\mathbf{O}_{P\times Q}$ is the all zero matrix of size $P\times Q$; $\mathbf{I}_Q$ is the identity matrix of size $Q\times Q$.

A convolution of the received samples with a TEQ can now easily be represented by $\mathbf{Y}^{ext,k}\mathbf{w}$. Receiver windowing, as proposed in [4], multiplies the last μ samples of the part of $\mathbf{y}^k$ that corresponds to the CP with the window coefficients u_l, $l=0,...,\mu-1$ and adds them to the last μ samples of $\mathbf{y}^k$ multiplied by $1-u_l$ (fold operation). These window and fold operations are equivalent to a multiplication of the received samples with a matrix $\mathbf{U}$ of size $N\times(N+\mu)$:

$$\mathbf{U} = \underbrace{\begin{bmatrix} \mathbf{O} & \mathbf{I}_{N-\mu} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{I}_\mu \end{bmatrix}}_{\mathbf{U}_1} + \underbrace{\begin{bmatrix} \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathrm{diag}(\mathbf{u}) & \mathbf{O} & -\mathrm{diag}(\mathbf{u}) \end{bmatrix}}_{\mathbf{U}_2} \quad (1)$$

The time domain vector of length N at time k that is fed to the N-point DFT after TEQ filtering and windowing is now the result of the product $\mathbf{U}\mathbf{Y}^{ext,k}\mathbf{w}$. Windowing without TEQ filtering is represented by $\mathbf{U}\mathbf{y}^k=\mathbf{U}\mathbf{Y}^{ext,k}[1\ 0\dots\ 0]^{\mathrm{T}}$, while TEQ filtering without windowing is denoted by $\mathbf{Y}^k\mathbf{w}=\mathbf{U}_1\mathbf{Y}^{ext,k}\mathbf{w}$.

3. BIT RATE MAXIMIZING WINDOW AND EQUALIZER DESIGN (BM-WINTEQ)

In the derivation towards a bit rate maximizing window and TEQ design cost function, we start from the bit rate expression[1], where the total number of bits transmitted in one DMT symbol is given by

$$\boxed{b_{DMT} = \sum_{i \in \mathscr{S}} \log_2 \left(1 + \frac{SNR_i}{\Gamma_i}\right). \qquad (2)}$$

Here, SNR_i represents the signal-to-noise ratio (SNR) on tone i and Γ_i is the SNR gap between the actual SNR_i and the SNR required to achieve the Shannon capacity. The SNR on tone i can be written in a simple form as:

$$SNR_i = \frac{\text{energy in (desired signal}_i)}{\text{energy in (received signal}_i - \text{desired signal}_i)}, \qquad (3)$$

where the denominator contains all possible noise contributions. The overall received signal on tone i is determined by the i-th FEQ output, Z_i^k, which can be obtained as

$$Z_i^k = D_i \underbrace{\mathscr{F}_N(i,:)\mathbf{U}\mathbf{Y}^{ext,k}\mathbf{w}}_{\text{FEQ input } Y^k_{uw,i}}. \qquad (4)$$

Based on Z_i^k the slicer will then make a decision to estimate the QAM symbol that was transmitted. W.l.o.g. Z_i^k is also equivalent to

$$Z_i^k = \alpha_i X_i^k + E_i^k, \qquad (5)$$

where α_i is a scale factor, $\alpha_i X_i^k$ is the desired signal component and E_i^k denotes the overall noise component on tone i. Based on (4) and (5), the SNR_i in (3) then becomes

$$SNR_i = \frac{\mathscr{E}\{|\alpha_i X_i^k|^2\}}{\mathscr{E}\{|D_i Y^k_{uw,i} - \alpha_i X_i^k|^2\}}, \qquad (6)$$

where $\mathscr{E}\{\cdot\}$ is the statistical expectation. When the traditional *unconstrained* MMSE FEQ is used, given by the Wiener solution

$$D_i = \frac{\mathscr{E}\{Y^{k*}_{uw,i} X_i^k\}}{\mathscr{E}\{|Y^k_{uw,i}|^2\}}, \qquad (7)$$

one can easily prove that the desired signal part at the FEQ output is *biased*, i.e. $\alpha_i \neq 1$ [7]. Consequently, the slicer at the FEQ output requires a scaling with α_i when making decisions.

A simpler form of (6) is obtained when a *constrained minimum mean square error (MMSE)* FEQ is used. A constrained MMSE FEQ gives an unbiased desired signal part at the FEQ output, i.e. $\alpha_i = 1$. In the case of a one-tap equalizer, the *zero forcing (ZF)* and constrained MMSE equalizer yield exactly the same solution:

$$\boxed{D_i = \frac{\mathscr{E}\{|X_i^k|^2\}}{\mathscr{E}\{Y^k_{uw,i} X_i^{k*}\}}. \qquad (8)}$$

The ZF FEQ for tone i is not merely equal to the inverse of the i-th frequency component of the convolution of the channel with the TEQ. Clearly, the windowing operation has to be taken into account as well. Whereas a multi-taps ZF equalizer may cause noise enhancement, a *single-tap* ZF equalizer scales both the desired and noise components in the same way and hence leaves the SNR unaltered. With a ZF FEQ the slicer does not require scaling and (6) can be simplified to

$$\boxed{SNR_i = \frac{\mathscr{E}\{|X_i^k|^2\}}{\mathscr{E}\{|E_i^k|^2\}} = \frac{\mathscr{E}\{|X_i^k|^2\}}{\mathscr{E}\{|D_i Y^k_{uw,i} - X_i^k|^2\}}. \qquad (9)}$$

[1] To keep mathematics tractable, we do not consider integer bit loading.

Throughout this paper ZF FEQ's will be assumed. In practice, any other FEQ may be applied, which will not change the SNR_i obtained, provided that a scaled slicer is used.

Before constructing the BM-WinTEQ cost function, it is useful to write (8) and (9) explicitly as a function of the TEQ and windowing coefficients on the one hand and the signal statistics on the other hand. Therefore, when combining the definition of $Y^k_{uw,i}$ in (4) with (1), we can write

$$Y^k_{uw,i} = \mathbf{Y}_i^{sl,k}\mathbf{w} + \mathbf{u}_i^{\mathrm{T}} \Delta\mathbf{Y}^{ext,k}\mathbf{w}, \qquad (10)$$

where $\mathbf{Y}_i^{sl,k} = \mathscr{F}_N(i,:)\mathbf{Y}^k$ represents the i-th row of a sliding DFT on the received signal vector (i.e. the DFT of the T columns of the Toeplitz matrix $\mathbf{Y}^k$). Furthermore, we have

$$\mathbf{u}_i^{\mathrm{T}} = [u_{i,0} \ \dots \ u_{i,\mu-1}] = \mathscr{F}_N(i, N{-}\mu{+}1{:}N)\mathrm{diag}(\mathbf{u}) = \mathbf{u}^{\mathrm{T}}\mathbf{D}_{\mathscr{F},i}, \quad (11)$$

with $\mathbf{D}_{\mathscr{F},i} = \mathrm{diag}(\mathscr{F}_N(i, N{-}\mu{+}1{:}N))$ and the Toeplitz matrix

$$\begin{aligned} \Delta\mathbf{Y}^{ext,k} &= \begin{bmatrix} \mathbf{I}_\mu & \mathbf{O}_{\mu\times(N-\mu)} & -\mathbf{I}_\mu \end{bmatrix} \mathbf{Y}^{ext,k}, & (12) \\ &= \begin{bmatrix} \Delta y_0^k & \dots & \Delta y^k_{-T+1} \\ \vdots & \ddots & \\ \Delta y^k_{\mu-1} & & \Delta y^k_{\mu-T} \end{bmatrix}. & (13) \end{aligned}$$

If we define $\Delta\mathbf{y}^{ext,k} = [\Delta y^k_{-T+1} \ \dots \ \Delta y^k_{\mu-1}]^{\mathrm{T}}$ of length $\mu + T - 1$, the second term in (10) is equivalent to

$$\mathbf{u}_i^{\mathrm{T}} \Delta\mathbf{Y}^{ext,k}\mathbf{w} = \Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\mathbf{U}_i\mathbf{w} = \Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\mathbf{W}_i\mathbf{u}, \qquad (14)$$

with $\mathbf{U}_i$ and $\mathbf{W}_i$ tone-dependent matrices of size $(\mu + T - 1) \times T$ and $(\mu + T - 1) \times \mu$ resp.

$$\mathbf{U}_i = \begin{bmatrix} 0 & \cdot^{\cdot^{\cdot}} & \boxed{\mathbf{u}_i} \\ \boxed{\mathbf{u}_i} & & 0 \end{bmatrix}, \mathbf{W}_i = \begin{bmatrix} \boxed{\bar{\mathbf{w}}} & \ddots & 0 \\ 0 & & \boxed{\bar{\mathbf{w}}} \end{bmatrix} \mathbf{D}_{\mathscr{F},i}, \quad (15)$$

where $\bar{\mathbf{w}}$ contains the TEQ coefficients in reversed order, i.e. $\bar{\mathbf{w}} = [w_{T-1} \ \dots \ w_0]^{\mathrm{T}}$. Furthermore, define for each tone i the signal statistics at the receiver as

$$\begin{aligned} r_{XX,i} &= \mathscr{E}\{|X_i^k|^2\}, & \mathbf{r}_{X\Delta,i} &= \mathscr{E}\{X_i^{k*}\Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\}, \\ \mathbf{r}_{XY,i} &= \mathscr{E}\{X_i^{k*}\mathbf{Y}_i^{sl,k}\}, & \mathbf{R}_{YY,i} &= \mathscr{E}\{\mathbf{Y}_i^{sl,k^{\mathrm{H}}}\mathbf{Y}_i^{sl,k}\}, \quad (16) \\ \mathbf{R}_{Y\Delta,i} &= \mathscr{E}\{\mathbf{Y}_i^{sl,k^{\mathrm{H}}}\Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\}, & \mathbf{R}_{\Delta\Delta} &= \mathscr{E}\{\Delta\mathbf{y}^{ext,k}\Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\}. \end{aligned}$$

Based on (8), (10) and (14), the ZF FEQ can now be expressed as

$$D_i = \frac{\mathscr{E}\{|X_i^k|^2\}}{\mathscr{E}\{Y^k_{uw,i}X_i^{k*}\}} = \frac{r_{XX,i}}{\mathbf{r}_{XY,i}\mathbf{w} + \mathbf{r}_{X\Delta,i}\mathbf{U}_i\mathbf{w}}. \qquad (17)$$

Likewise, the SNR of tone i can be written compactly as

$$SNR_i = \frac{\mathscr{E}\{|X_i^k|^2\}}{\mathscr{E}\{|D_i Y^k_{uw,i} - X_i^k|^2\}} = \frac{1}{\rho_i^{-2}(\mathbf{w},\mathbf{u}) - 1}, \qquad (18)$$

with $\rho_i^2(\mathbf{w},\mathbf{u})$ a tone-dependent normalized correlation function:

$$\begin{aligned} \rho_i^2(\mathbf{w},\mathbf{u}) &= \frac{|\mathscr{E}\{Y^k_{uw,i}X_i^{k*}\}|^2}{\mathscr{E}\{|Y^k_{uw,i}|^2\}\mathscr{E}\{|X_i^k|^2\}} = \qquad (19) \\ & \frac{\left|\mathbf{r}_{XY,i}\mathbf{w} + \mathbf{r}_{X\Delta,i}\mathbf{U}_i\mathbf{w}\right|^2}{\left(\mathbf{w}^{\mathrm{T}}(\mathbf{R}_{YY,i} + \mathbf{R}_{Y\Delta,i}\mathbf{U}_i + \mathbf{U}_i^{\mathrm{H}}\mathbf{R}^{\mathrm{H}}_{Y\Delta,i} + \mathbf{U}_i^{\mathrm{H}}\mathbf{R}_{\Delta\Delta}\mathbf{U}_i)\mathbf{w}\right) r_{XX,i}}. \end{aligned}$$

A complete proof can be found in [8]. When combining (2) and (18), the BM-WinTEQ can be obtained by maximizing the following cost function with respect to $\mathbf{w}$ and $\mathbf{u}$:

$$\boxed{b_{DMT}(\mathbf{w},\mathbf{u}) = \sum_{i\in\mathscr{S}} \log_2\left(1+\frac{1}{\Gamma_i(\rho_i^{-2}(\mathbf{w},\mathbf{u})-1)}\right). \qquad (20)}$$

Maximizing (20) is an *unconstrained non-linear* optimization problem and calls for non-linear optimization techniques.

Although channel shortening is the original underlying idea for using a TEQ, the BMWin-TEQ criterion does not explicitly impose channel shortening any more. Likewise, the window and fold operations, as originally proposed in [4], assume that the last μ samples of the part of $\mathbf{y}^{ext,k}$ corresponding to the CP are effectively ISI-free. By incorporating windowing into a joint bit rate maximizing window and TEQ design, this assumption can be omitted since the truly optimal combination of window and TEQ will be obtained. Traditionally, equalization is performed by a TEQ-FEQ combination only. Here, simulations will show that receiver windowing can indeed help the equalizer in reducing the ICI by lowering the sidelobe level of the demodulating DFT.

4. SPECIAL CASES

4.1 Per tone equalizer and window design

The BM-WinTEQ cost function (20) specifies the design of an optimal equalizer and window common for all the used tones ($i\in\mathscr{S}$). Alternatively, we can divide the set of all used tones, $\mathscr{S}$, in multiple subsets or groups of tones and design an equalizer and window for each group. A special case of the per group approach occurs when an equalizer and a window are designed for each tone separately. In this case, the cost function (20) (or (2)-(8)-(9)) for a single tone reduces to

$$\arg\max_{\mathbf{w}_i,\mathbf{u}_i} SNR_i = \arg\min_{\mathbf{w}_i,\mathbf{u}_i} \mathscr{E}\{|D_i Y_{uw,i}^k - X_i^k|^2\},\ \ \forall i\in\mathscr{S}, \qquad (21)$$

i.e. bit rate maximization is equivalent to maximizing the SNR or minimizing the overall noise for each tone separately, where $\mathbf{w}_i$ and $\mathbf{u}_i$ denote the tone-dependent equalizer and window coefficients. In the following we will show that the solution for (21) is equivalent to *per tone equalization (PTEQ)* [5][6], i.e. the μ-taps window operation can effectively be incorporated in the equalizer operation after increasing the number of per tone equalizer coefficients from T to $\mu+T$ for each tone i.

From (10) and (14), we know that

$$Y_{uw,i}^k = \mathbf{Y}_i^{sl,k}\mathbf{w} + \Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\mathbf{U}_i\mathbf{w}. \qquad (22)$$

As $\mathbf{Y}_i^{sl,k}$ is equal to $\mathscr{F}_N(i,:)\mathbf{Y}^k$, the first term in (22) involves the computation of T successive DFT's on the received time domain samples. Since $\mathbf{Y}^k$ has a Toeplitz structure, this computation can be done in terms of one full DFT and $T-1$ difference terms, i.e.

$$\mathbf{Y}_i^{sl,k}\mathbf{w} = \bar{\mathbf{w}}^{\mathrm{T}} \begin{bmatrix} 1 & \alpha & \dots & \alpha^{T-1} \\ & \ddots & & \vdots \\ & & 1 & \alpha \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{y}^k \\ Y_i^k \end{bmatrix}, \qquad (23)$$

with $\alpha = e^{-j2\pi(i-1)/N}$, $Y_i^k = \mathscr{F}_N(i,:)[y_0^k\ \dots\ y_{N-1}^k]^{\mathrm{T}}$, $\Delta\mathbf{y}^k = [\Delta y_{\mu-T+1}^k \dots \Delta y_{\mu-1}^k]^{\mathrm{T}}$ and where $\bar{\mathbf{w}}$ denotes the TEQ coefficients in reversed order [5]. Hence, the first term of (22) can be written as a linear combination of Y_i^k and $T-1$ difference terms.

When $\mathbf{U}_i\mathbf{w}$ is replaced by one tone-dependent vector of unknowns, it is easy to see that the second term in (22) also comprises a linear combination of the difference terms $\Delta y_{-T+1}^k,\dots,\Delta y_{\mu-1}^k$. As a consequence, (22) can be written in general as a linear combination of the DFT output, Y_i^k, and $\mu+T-1$ real valued difference term, $\Delta\mathbf{y}^{ext,k}$, i.e.

$$Y_{uw,i}^k = \bar{\mathbf{v}}_i^{\mathrm{T}}\left[\Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\ Y_i^k\right]^{\mathrm{T}}, \qquad (24)$$

where $\bar{\mathbf{v}}_i$ are now the unknown coefficients. With (24), (21) can be modified to

$$\boxed{\arg\min_{\bar{\mathbf{v}}_i} \mathscr{E}\left\{\left|D_i\bar{\mathbf{v}}_i^{\mathrm{T}}\left[\Delta\mathbf{y}^{ext,k^{\mathrm{T}}}\ Y_i^k\right]^{\mathrm{T}} - X_i^k\right|^2\right\},\ \ \forall i\in\mathscr{S}, \qquad (25)}$$

with D_i still given by (8). Hence, $D_i\bar{\mathbf{v}}_i$ corresponds to the *constrained (unbiased) MMSE* solution of a $\mu+T$-taps PTEQ design problem. With $D_i = 1$ the solution of (25) reduces to the design of an *unconstrained (biased) MMSE PTEQ* of $\mu+T$ taps as proposed in [6]. The *constrained MMSE PTEQ* is simply a scaled (by D_i) or unbiased version of this solution. In practice, the constrained (unbiased) MMSE PTEQ is preferred due to its simple decision rule at the receiver output for tone i. Hence, in contrast to [6], an extra FEQ at the PTEQ output is required to obtain the *unbiased* solution.

4.2 Equalizer-only design

Based on (20), a single TEQ to maximize the bit rate can be designed, without receiver windowing, i.e. $\mathbf{u}=\mathbf{0}$. Hence, (10) reduces to $Y_{w,i}^k = \mathbf{Y}_i^{sl,k}\mathbf{w}$. Also, the ZF FEQ for tone i in (8) can be adapted accordingly, i.e.

$$D_i = \frac{\mathscr{E}\{|X_i^k|^2\}}{\mathscr{E}\{Y_{w,i}^k X_i^{k^*}\}} = \frac{r_{XX,i}}{\mathbf{r}_{XY,i}\mathbf{w}}. \qquad (26)$$

It can be shown that the ZF FEQ is not merely equal to the inverse of the i-th frequency component of the channel convolved with the TEQ. Although the latter is an assumption that is often made in literature, it only holds when the channel is perfectly shortened to the CP length plus one. Plugging (26) into (9) and (2), the bit rate maximizing TEQ (BM-TEQ) cost function, as originally proposed in [7], is obtained:

$$\boxed{\arg\max_{\mathbf{w}} b_{DMT}(\mathbf{w}) = \arg\max_{\mathbf{w}} \sum_{i\in\mathscr{S}} \log_2\left(\frac{\mathbf{w}^{\mathrm{T}}\mathbf{A}_i\mathbf{w}}{\mathbf{w}^{\mathrm{T}}\mathbf{B}_i\mathbf{w}}\right), \qquad (27)}$$

where $\mathbf{A}_i$ and $\mathbf{B}_i$ are independent of $\mathbf{w}$:

$$\mathbf{A}_i = \Gamma_i r_{XX,i}\mathbf{R}_{YY,i} + (1-\Gamma_i)\mathbf{r}_{XY,i}^{\mathrm{H}}\mathbf{r}_{XY,i}, \qquad (28)$$

$$\mathbf{B}_i = \Gamma_i\left(r_{XX,i}\mathbf{R}_{YY,i} - \mathbf{r}_{XY,i}^{\mathrm{H}}\mathbf{r}_{XY,i}\right). \qquad (29)$$

Although the BM-TEQ is optimizing the bit rate, simulations will show that the addition of a window may reduce equalization complexity for a similar bit rate performance.

4.3 Window-only design

Dual to the equalizer-only problem, we can investigate the windowing-only problem, i.e. $\mathbf{w} = [1\ 0\dots0]^{\mathrm{T}}$. Assume we have a DMT-system where the channel length is shorter than $\nu-\mu+1$ (or shortened to that length)[2], as e.g. in a Zipper VDSL system [3]. The FEQ input for tone i after windowing can then be written as

$$Y_{u,i}^k = \mathscr{F}_N(i,:)\mathbf{U}\mathbf{y}^k = \mathscr{F}_N(i,:)(\mathbf{U}_1+\mathbf{U}_2)\mathbf{y}^k = Y_i^k + \mathbf{u}_i^{\mathrm{T}}\Delta\mathbf{y}^k, \quad (30)$$

where we used (1) and $\Delta\mathbf{y}^k = [\Delta y_0^k\ \dots\ \Delta y_{\mu-1}^k]^{\mathrm{T}}$. Eq. (30) shows that the windowed FEQ input is equal to the unwindowed FEQ input plus a linear combination of tone-independent difference terms and frequency modulated windowing coefficients. When the channel order is limited to $(\nu-\mu)$, $\Delta\mathbf{y}^k$ is zero in the noiseless case. In other words, due to the symmetry of the window function and the folding operation, the window only acts on the external noise without destroying the DMT orthogonality.

[2] Similar derivations are possible when the assumption on the channel order is not met, but will not be given here.

	# real multiplications	memory
BM-WinTEQ	$(N+\mu)T+2\mu$	$T+\mu$
PTEQ	$2N_u(T+\mu+1)$	$2N_u(T+\mu)$
BM-TEQ	NT	T
BM-Win	2μ	μ

Table 1: Complexity figures for windowing and equalizer designs.

Since $\Delta\mathbf{y}^k$ only contains noise which is uncorrelated with the transmitted symbol, (8) will be independent of the window function and will be equal to the ZF FEQ of the unwindowed case, i.e. $D_i = r_{XX,i}/r_{XY,i}$. The resulting ZF FEQ is now simply the inverse of the channel transfer function for tone i. To design a single, *bit rate maximizing window (BM-Win)* we have to solve

$$\boxed{\arg\max_{\mathbf{u}} b_{DMT}(\mathbf{u}) = \arg\max_{\mathbf{u}} \sum_{i\in\mathscr{S}} \log_2\left(1+\frac{r_{XX,i}}{\Gamma_i g_i(\mathbf{u})}\right), \quad (31)}$$

with $g_i(\mathbf{u}) = \mathbf{u}^{\mathrm{T}}\mathbf{A}_i\mathbf{u}+2\mathbf{b}_i^{\mathrm{T}}\mathbf{u}+c_i$, where $\mathbf{A}_i$, $\mathbf{b}_i$ and c_i are independent of $\mathbf{u}$, given by

$$\mathbf{A}_i = |D_i|^2\mathbf{D}^*_{\mathscr{F},i}\mathscr{E}\{\Delta\mathbf{y}^k\Delta\mathbf{y}^{k^{\mathrm{T}}}\}\mathbf{D}_{\mathscr{F},i}, \quad (32)$$

$$\mathbf{b}_i = \mathscr{R}e\{D_i\mathbf{D}_{\mathscr{F},i}\mathscr{E}\{(D_iY_i^k-X_i^k)^*\Delta\mathbf{y}^k\}\}, \quad (33)$$

$$c_i = \mathscr{E}\{|D_iY_i^k-X_i^k|^2\}, \quad (34)$$

and where $\mathscr{R}e\{\cdot\}$ takes the real part of its argument. In [2], a sum (or a weighted sum) of squared errors at the FEQ input is used to design an adaptive window, but, as opposed to (31), the resulting cost function has no direct relation with bit rate optimization and hence results in suboptimal windowing.

5. COMPLEXITY

Complexity figures and memory requirements are given in **Table 1**. To compute these figures, we count the number of *real* multiplications per symbol *during data transmission*. The following conclusions can be drawn:

- Since μ is typically much smaller than N, the extra complexity and memory of adding a single window on top of a single TEQ is almost negligible.
- Since $N_u < N/2$, a single TEQ often leads to a higher processing complexity during data transmission than a PTEQ with the same number of taps, but the memory requirement of a PTEQ is N_u times higher than the memory requirement for a TEQ.

6. SIMULATION RESULTS

The simulation results will be limited to the BM-WinTEQ design. More simulation results are provided in [8]. Consider a downstream ADSL setup, where the channel noise consists of 24 DSL near-end crosstalk (NEXT) disturbers and additive white Gaussian noise of -140 dBm/Hz. The data carrying tones are 38 to 256. The transmission channel includes all digital and analog front-end filters. Furthermore, the following parameters were used: $N = 512$, $\nu = 32$, $F_s = 2.208$ MHz, $\gamma_c = 3$ dB, $\gamma_m = 6$ dB.

In **Fig. 1**, the bit rate is depicted as a function of the number of BM-WinTEQ coefficients for a $T1.601\#7$ downstream loop. Note that the curve for $T = 1$ represents the windowing-only case, since for $T = 1$ no time domain equalization is performed in practice: a TEQ with one tap causes a simple scaling of the time domain samples, which is perfectly compensated by the FEQ's. On the other hand, the curve with $\mu = 0$ shows the performance for the BM-TEQ without windowing. The BM-WinTEQ is obtained by means of the MATLAB® optimization toolbox. The optimization procedure did not have problems with local minima: different local minima all resulted in close to optimal performance. Unfortunately, we do not have a theoretical proof to generalize this observation. From **Fig. 1**, one can clearly see that BM-TEQ taps ($\mu = 0$) can be traded for window taps to obtain the same performance. E.g. a BM-WinTEQ with $T = 3$ and $\mu = 8$ attains the same performance as a BM-TEQ

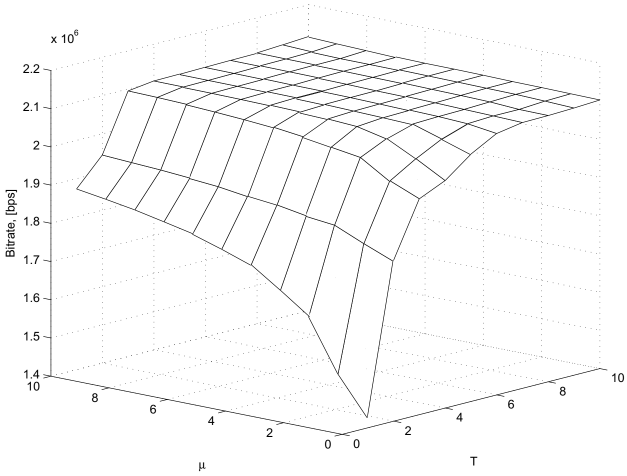

Figure 1: Bit rate as a function of the number of TEQ taps T and windowing coefficients μ (BM-WinTEQ).

with $T = 7$, although the former has a much lower complexity, cf. **Table 1**.

7. CONCLUSIONS

We investigated the *combined* optimization of a TEQ and a window taper (BM-WinTEQ) in order to maximize the bit rate in a DMT-based system. The outcome is a non-linear cost function, based on the traditional bit rate expression. Simulations indicated that windowing, which was originally designed to mitigate RFI, can contribute to solve the equalization problem by lowering the sidelobes of the demodulating DFT. Moreover, the taps of a TEQ can be exchanged for some windowing coefficients, leading to a complexity reduced receiver. We showed how the bit rate maximizing framework also includes some special design cases, such as the PTEQ, BM-TEQ and BM-Win.

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