BLIND PHASE RECOVERY FOR QAM CONSTELLATIONS

G. Panci, P. Campisi**, S. Colonnese*, G. Scarano**

*Dip. INFOCOM, Università "La Sapienza" di Roma
via Eudossiana 18, I-00184 Roma, Italy
Tel:+39.06.44585.500, Fax:+39.06.4873.300

**Dip. Elettronica Applicata, Università degli Studi "Roma Tre"
via Della Vasca Navale 84, I-00146 Roma, Italy
Tel:+39.06.5517.7064, Fax:+39.06.5517.7026

e-mail: (*)(gpanci,colonnese,scarano)@infocom.uniroma1.it
(**) campisi@uniroma3.it

ABSTRACT

In this contribution we present a novel phase estimator that can be employed for both square and cross QAM constellations. It is based on the estimation of the orientation of the concentration ellipses of the bivariate Gaussian distribution having the same second order moments of the two random variables obtained considering the real and the imaginary part of the fourth power of the received data. It does not require knowledge of the transmitted symbol constellation and does not need gain control. Experimental results outline the good performance of the here described estimator, superior to that of well known phase estimation methods.

1. INTRODUCTION

In synchronous systems using high-speed signaling such as QAM modulation, phase recovery is a problem of paramount importance. For efficiency reasons the phase estimation must be performed in a blind manner, that is without using training sequences of known transmitted symbols.

In the recent literature several approaches for blind phase estimation have been proposed. In [1] the blind phase recovery problem has been dealt with using higher order statistics after a gain control stage. In [2] Cartwright presents a modification of the method described in [1], and obtains an estimator based on a set of fourth-order statistics, without needing any gain control. It has been shown in [3] that the estimator in [2] is equivalent to the fourth-power estimator presented in [4], which in turns was demonstrated to approximate the maximum-likelihood estimator in the limit of small signal-to-noise ratio (SNR). In [5] an estimator based on eighth-order statistics gives improved performance for cross QAM systems over the fourth power phase estimator [2]. Moreover, less observed samples are needed. A phase estimator based on a modification of the received constellation is presented in [6]. In [7] a nonlinear filtering is performed in order to retain only constellation points more "reliable" for the phase estimation.

In this paper a new blind phase estimator that does not require any gain control is presented. It is based on the evaluation of the fourth power of the received data and subsequent estimation of the orientation of the concentration ellipses of the bivariate Gaussian distribution having the same second order moments of the two random variables obtained considering the real and the imaginary part of the fourth power the received data.

2. PROBLEM STATEMENT

The problem of phase estimation in baseband QAM systems is here addressed. Let us indicate with

$$Y[n] = e^{j\theta}X[n] + W[n] \quad ; \quad n = 0, \cdots, N-1 \tag{1}$$

the received data sample, of size N, where $X[n]$ is the complex transmitted symbol, and θ is the unknown carrier offset that has to be estimated. It is further assumed that $W[n]$ is a realization of circularly distributed complex noise, statistically independent of $X[n]$.

3. ESTIMATION OF THE PHASE ROTATION

The first step toward the estimation of the unknown phase θ consists in evaluating the fourth-power of the received data $Y = y_r + jy_i$, which gives $Y^4 = R + jI$, being

$$R = y_r^4 - 6y_r^2y_i^2 + y_i^4 \tag{2}$$

$$I = 4(y_r^3y_i - y_ry_i^3). \tag{3}$$

By performing the fourth-power, the constellation points having phase $\phi_k = \pi/4 + k\pi/2$ (with $k = 0, 1, 2, 3$) are mapped into the single point having phase $4\phi_k = \pi$, while the constellation points having phase $\phi_k = \pi/4 + k\pi/2 \pm \gamma$ (with $k = 0, 1, 2, 3$) are mapped into points having phase $4\phi_k = \pi \pm 4\gamma$ dependent of the angular offset γ. In Figs.1 and 2, the fourth-power of the constellation points for both the 16-QAM and 32-QAM constellations are displayed respectively.

The bidimensional random variable (R, I) is described at second order by the following five moments:

$$m_R \stackrel{\text{def}}{=} \mathrm{E}\{R\} \quad ; \quad m_I \stackrel{\text{def}}{=} \mathrm{E}\{I\}$$

$$m_R^{(2)} \stackrel{\text{def}}{=} \mathrm{E}\left\{R^2\right\} \; ; \; m_{R,I}^{(1,1)} \stackrel{\text{def}}{=} \mathrm{E}\{R \cdot I\} \; ; \; m_I^{(2)} \stackrel{\text{def}}{=} \mathrm{E}\left\{I^2\right\}$$

Let us consider the bivariate Gaussian probability density function (pdf) having the same second order moments; the loci of points at equal pdf are the so-called concentration ellipses.
In Figs. 1 and 2 the constellations points of the bidimensional variable (R, I) are plotted for the 16-QAM and the 32-QAM constellations, respectively, for $\theta = 0$.

In these figures we have also reported the concentration ellipse described by the equation

$$\frac{1}{1-\rho_{R,I}^2}\left(\frac{(R-m_R)^2}{\sigma_R^2} - 2\rho_{R,I}\,\frac{(R-m_R)(I-m_I)}{\sigma_R\sigma_I} + \frac{(I-m_I)^2}{\sigma_I^2}\right) = 1$$

where $\rho_{R,I}$ is the correlation coefficient of the random variables R, I, with variance σ_R^2 and σ_I^2, respectively.

The effect of $\theta \neq 0$ is a simple counter-clockwise rotation by $\theta/4$ of all the figures. Hence, the phase estimation can be performed by calculating the orientation of the principal axes of the concentration ellipses above defined. For cross constellations, a

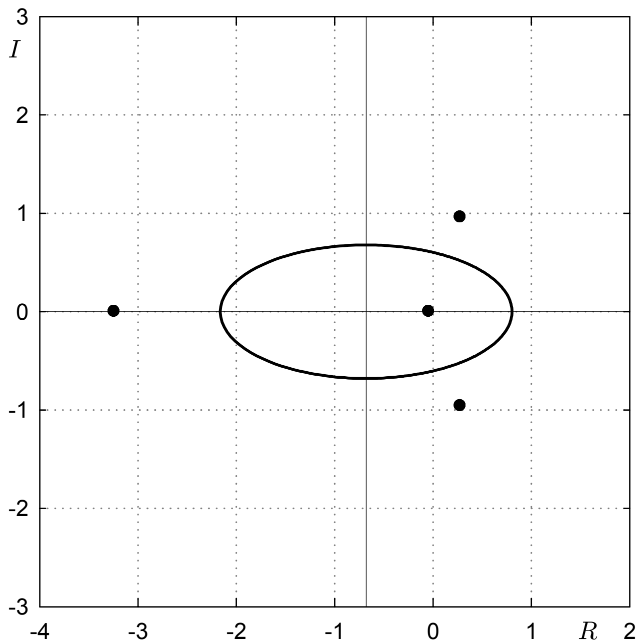

Figure 1: *Fourth-power of the 16-QAM constellation and corresponding concentration ellipse.*

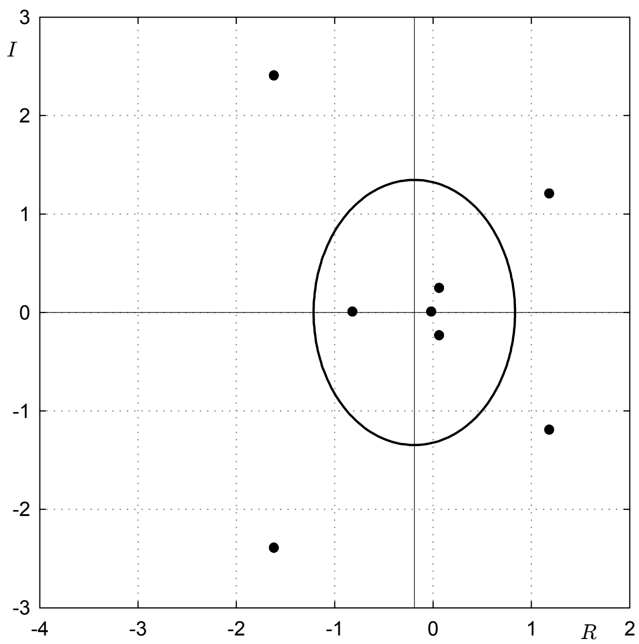

Figure 2: *Fourth-power of the 32-QAM constellation and corresponding concentration ellipse.*

$\pi/2$ correction must be taken into account, since for these constellations the phase rotation $\theta/4$ is measured by the minor principal axis.

The orientation of the principal axis of the concentration ellipses with respect to the horizontal axis is measured by the angle α given by

$$\tan(2\alpha) = \frac{2(m_{R,I}^{(1,1)} - m_R m_I)}{m_R^{(2)} - (m_R)^2 - m_I^{(2)} + (m_I)^2} \tag{4}$$

Since we measure $\tan(2\alpha)$, computation of θ from α is affected by a $\pi/4$ ambiguity, while the $\pi/2$ rotational symmetry of the symbol constellation induces only an unavoidable $\pi/2$ ambiguity.

In order to obtain a correct $\pi/2$ ambiguity we resort, for example, to the additional information provided by

$$\gamma = \arctan\left(\frac{m_I}{m_R}\right)$$

that basically constitutes the fourth-order estimator as shown in [3]. Actually, the angle γ is needed only to determine in which of the four quadrants of the Cartesian plane lies the center of the concentration ellipse so to suitably correct by a $\pm\pi/4$ rotation the value of θ as illustrated in the following. Obviously, γ can be substituted by any parameter able to locate the quadrant where the center of the concentration ellipse lies.

Hence, from the angle

$$\beta_s = \frac{1}{2}\arctan\Big(\tan(2\alpha)\Big)$$

the phase ambiguity can be eliminated as follows:

$$0 \le \beta_s < \frac{\pi}{2}\left\{\begin{array}{lcl} \frac{3}{4}\pi \le \gamma \le \frac{7}{4}\pi & \longrightarrow & \theta = \beta_s/4 \\ -\frac{\pi}{4} \le \gamma \le \frac{3}{4}\pi & \longrightarrow & \theta = (\beta_s - \pi)/4 \end{array}\right.$$

$$-\frac{\pi}{2} \le \beta_s < 0\left\{\begin{array}{lcl} \frac{\pi}{4} \le \gamma \le \frac{5}{4}\pi & \longrightarrow & \theta = \beta_s/4 \\ -\frac{3}{4}\pi \le \gamma \le \frac{\pi}{4} & \longrightarrow & \theta = (\beta_s + \pi)/4 \end{array}\right.$$

thus obtaining the value $\theta \in (-\pi/4, \pi/4)$ now within the above said unavoidable $\pi/2$ ambiguity due to the $\pi/2$ rotational symmetry of the symbol constellation.

For cross constellations, *e.g.* 32-QAM, in lieu of β_s it can be used the angle

$$\beta_c = \beta_s + \pi/2$$

evaluated modulo π such that $\beta_c \in (-\pi/2, \pi/2)$.

4. PHASE ESTIMATION FROM FINITE SIZE SAMPLE

When an observed sample of finite size N is observed, moments have to be estimated by sample averaging, that is, after defining $\mathrm{Av}\{(\cdot)\} \stackrel{\mathrm{def}}{=} \sum_{n=0}^{N-1}(\cdot)/N$:

$$\hat{m}_R \stackrel{\mathrm{def}}{=} \mathrm{Av}\{R[n]\} \quad ; \quad \hat{m}_I \stackrel{\mathrm{def}}{=} \mathrm{Av}\{I[n]\}$$
$$\hat{m}_R^{(2)} \stackrel{\mathrm{def}}{=} \mathrm{Av}\left\{R^2[n]\right\} \quad ; \quad \hat{m}_I^{(2)} \stackrel{\mathrm{def}}{=} \mathrm{Av}\left\{I^2[n]\right\}$$
$$\hat{m}_{R,I}^{(1,1)} \stackrel{\mathrm{def}}{=} \mathrm{Av}\{R[n] \cdot I[n]\}$$

and the estimated rotation $\hat{\alpha}$ can be obtained from

$$\tan(2\hat{\alpha}) = \frac{2(\hat{m}_{R,I}^{(1,1)} - \hat{m}_R m_I)}{\hat{m}_R^{(2)} - (\hat{m}_R)^2 - \hat{m}_I^{(2)} + (\hat{m}_I)^2} \tag{5}$$

It is worth noting that even when the additive noise is absent, the finite sample size will affect the estimation of moments and the subsequent phase estimation, since the constellation points depicted in Figs.1 and 2 will not be equally populated, and this in turn determines a deformation of the concentration ellipse. This latter modifies its eccentricity as well as its orientation, due to the stretching and to the rotation towards the more populated points. This explains the so-called “constellation self-noise” firstly illustrated in [4].

Observe that, since the constellation points located at phases $\phi_k = \pi/4 + k\pi/2$ (with $k = 0,1,2,3$) are mapped into the single point with phase $4\phi_k = \pi$ after raising to the fourth power, they do not contribute to said constellation self-noise. A constellation having only such points will result free from self-noise, e.g. the 4-QAM constellation. On the other hand, self-noise of cross constellations increases since these latter lack some corner points; consequently, performance of phase estimation is expected to deteriorate.

Moreover, improved phase estimation is expected by retaining only the "good" constellations points located at phases $\phi_k = \pi/4 + k\pi/2$ (with $k = 0, 1, 2, 3$), and rejecting the other "bad" constellation points. Note that this requires gain control and knowledge of the constellation at the receiver. The behavior of the optimum nonlinearity obtained in [8] is thus explained as above discussed. Phase estimation using a strategy of points selection at the receiver will be addressed elsewhere.

5. EXPERIMENTAL RESULTS

In this Section, the performance of the here presented estimator, in terms of standard deviation, are discussed for both square and cross QAM constellations. A comparison with the performance of the fourth order Cartwright estimator [2] and with the one of the eighth order Cartwright estimator [5] is conducted for both square QAM constellations and cross QAM constellations, respectively. Specifically, the experimental standard deviation is obtained by performing a number of 500 MonteCarlo trials; the phase is maintained $\theta = \pi/16$ for all the experiments and the additive noise is complex Gaussian distributed.

Moreover, in the same figures we have also reported the variance of our estimator obtained after a theoretical analysis; due to lack of space, analytical details will be reported elsewhere.

In Figs.3, 4, 5, and 6 the standard deviation of the estimates versus the observed sample size N is considered for the Concentration Ellipse Estimator (CEO) and for the Cartwright estimators [2] and [5] in high and low SNR per bit. From Figs.3 and 4 it is evident that our CEO estimator has better performance than the estimator presented by Cartwright in [2]. Specifically, with regard to the 16-QAM constellation our method offers a performance gain of about 3dB at high SNR values, whereas at lower SNRs, the gain decrease to 1dB. In the case of 64-QAM constellation the performance gain is about 2dB, independently of both the SNR level and of the sample size.

In Figs. 5 and 6 the performance of our CEO estimator for cross constellations (32-QAM, 128-QAM) are reported.

It is worth noting that our CEO estimator performs quite similarly to the eighth-order estimator by Cartwright [5].

Therefore, it is evident that, in order to obtain an optimal performance, the Cartwright estimators need to know the constellation in use, whereas our CEO estimator performs even better without the need to know the constellation employed. In Figs. 7 and 8 the standard deviation of our CEO estimator vs. the phase θ has been considered for various sample size N. It is worth noting that the standard deviation remains constant with respect to θ, as predicted from the theoretical analysis.

As far as the bias of the CEO estimator is concerned, it is ideally zero, as confirmed by numerical results shown in Fig. 9.

6. CONCLUSION

In this paper we have presented a new phase estimator that is based on the estimation of the orientation of the concentration ellipses associated to the probability distribution of the fourth-order power of the received data. The estimator does not need any gain control, and simulations show that it either performs better than existing phase estimators for square constellations, or has the same performance for cross constellations. Thus, it presents the advantage that the constellation type does not need to be known, whereas the other estimators do need it. Simulations also confirm a theoretical performance analysis whose details are not reported here.

Finally, we have given an interpretation of the so-called constellation self-noise considering the finite sample size effects on the estimation of moments of random variables belonging to a discrete distribution. This analysis explains why cross constellations have increased self-noise with respect to square constellations. Moreover, stemming from this interpretation, new phase estimators can be developed using a suitable selection of received data. The selection can be operated only if the constellation is known to the receiver and gain control have been already performed as will be discussed in forthcoming works. Furthermore, some analytical conclusion found in [8] are now well understood.

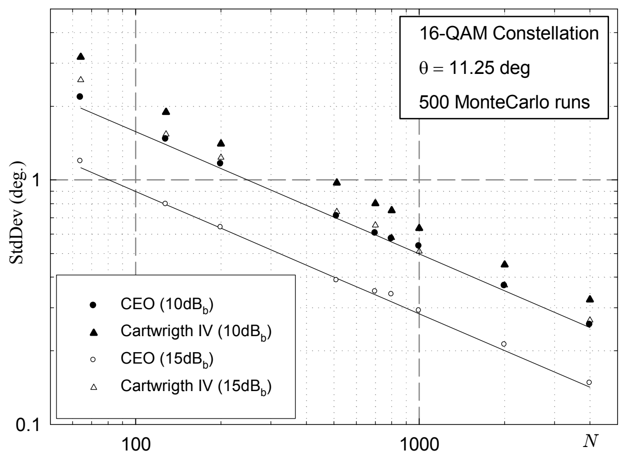

Figure 3: *16-QAM constellation: standard deviation of the CEO estimator (circles) and the fourth order Cartwright estimator [2] (triangles) vs. the observed sample size N in high SNR* $(15dB_b)$ *and low SNR* $(10dB_b)$*. Straight lines are obtained from theoretical analysis.*

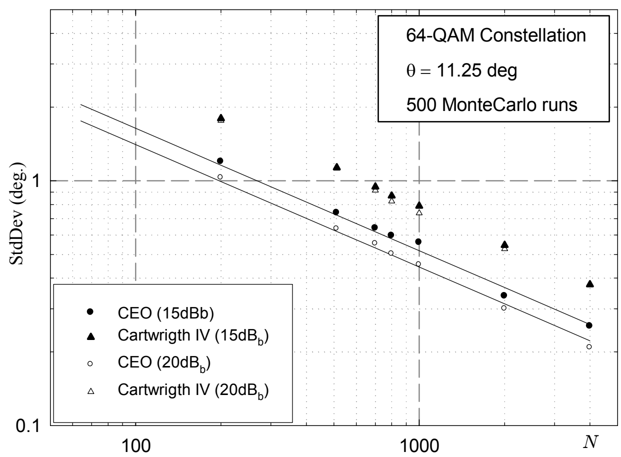

Figure 4: *64-QAM constellation: standard deviation of the CEO estimator (circles) and the fourth order Cartwright estimator [2] (triangles) vs. the observed sample size N in high SNR* $(20dB_b)$ *and low SNR* $(15dB_b)$*. Straight lines are obtained from theoretical analysis.*

REFERENCES

[1] L. Chen, H. Kusaka, and M. Kominami, "Blind Phase Recovery in QAM Communication Systems Using Higher Order Statistics" *IEEE Signal Processing Letters*, vol.3, May 1996.

[2] K. V. Cartwright, "Blind Phase Recovery in General QAM Communication Systems Using Alternative Higher Order Statistics", *IEEE Signal Processing Letters*, vol.6, Dec 1999.

[3] E. Serpedin, P. Ciblat, G. B. Giannakis, and P. Loubaton, "Perfor-

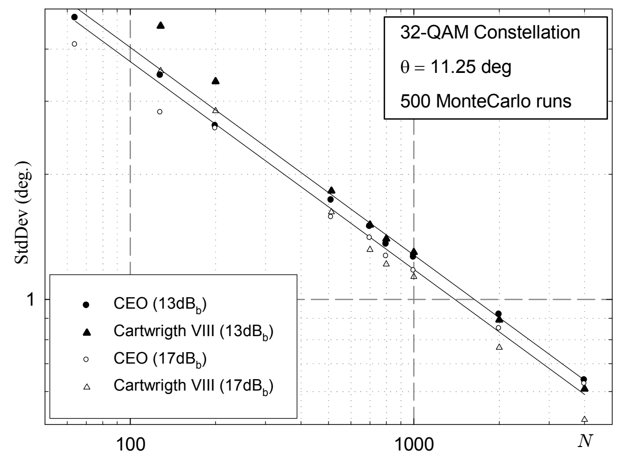

Figure 5: *32-QAM constellation: standard deviation of the CEO estimator (circles) and the eighth order Cartwright estimator [5] (triangles) vs. the observed sample size N in high SNR* $(17dB_b)$ *and low SNR* $(13dB_b)$. *Straight lines are obtained from theoretical analysis.*

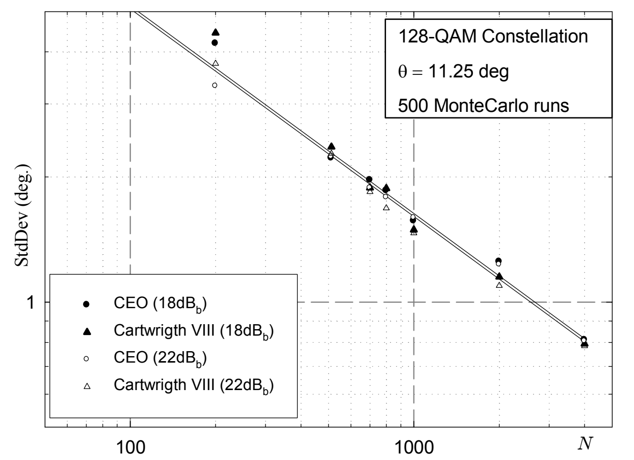

Figure 6: *128-QAM constellation: standard deviation of the CEO estimator (circles) and the eighth order Cartwright estimator [5] (triangles) vs. the observed sample size N in high SNR* $(22dB_b)$ *and low SNR* $(18dB_b)$. *Straight lines are obtained from theoretical analysis.*

mance Analysis of Blind Carrier Phase Estimators for General QAM Constellations", *IEEE Trans. Sig. Proc.*, vol.49, August 2001.

[4] M. Moeneclaey and G. de Jonghe, "ML-Oriented NDA Carrier Syncronization for General Rotationally Symmetric Signal Constellations", *IEEE Transactions on Communications*, vol.42, August 1994.

[5] K. V. Cartwright, "Blind Phase Recovery in Cross QAM Communication Systems with Eighth-Order Statistics", *IEEE Signal Processing Letters*, vol.8, December 2001.

[6] K. V. Cartwright, "Fourth Power Phase Estimation with Alternative Two-Dimensional Odd-Bit Constellations", *IEEE Communications Letters*, vol.4, June 2000.

[7] C. N. Georghiades, "Blind Carrier Phase Acquisition for QAM Constellations", *IEEE Trans. Comm.*, vol.45, Nov. 1997.

[8] Y. Wang and E. Serpedin, "A Class of Blind Phase Recovery Techniques for Higher Order QAM Modulations: Estimators and Bounds", *IEEE Signal Processing Letters*, vol.9, October 2002.

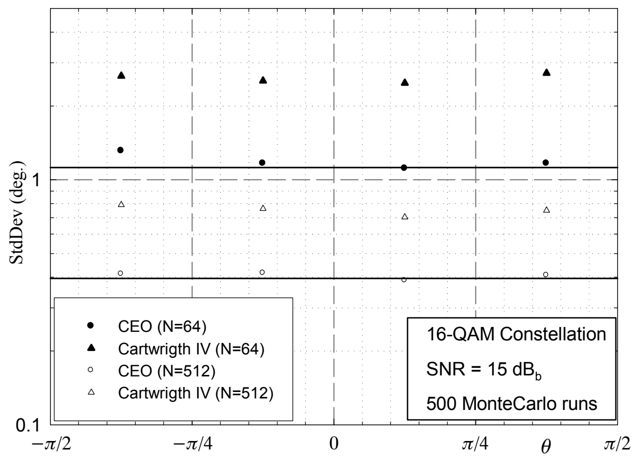

Figure 7: *16-QAM constellation: standard deviation of the CEO estimator (circles) and the eighth order Cartwright estimator [2] (triangles) vs. the phase* θ *for various sample sizes* N. *Straight lines are obtained from theoretical analysis.*

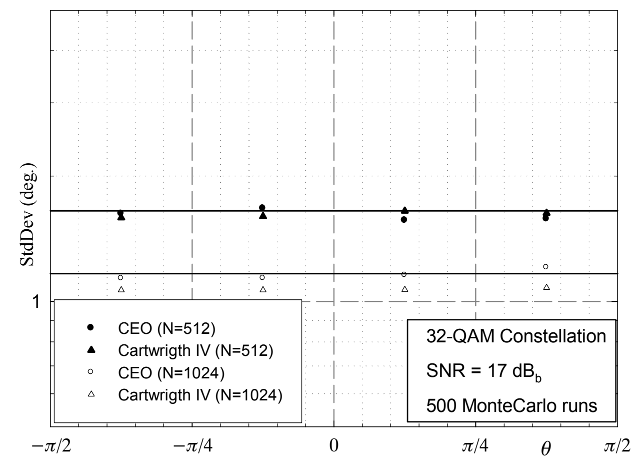

Figure 8: *32-QAM constellation: standard deviation of the CEO estimator (circles) and the eighth order Cartwright estimator [5] (triangles) vs. the phase* θ *for various sample sizes* N. *Straight lines are obtained from theoretical analysis.*

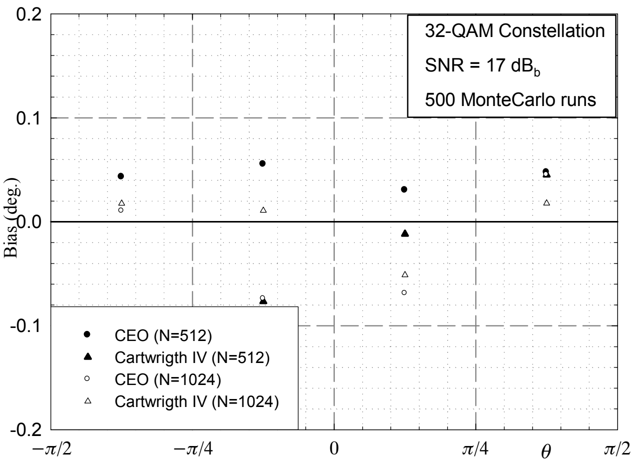

Figure 9: *32-QAM constellation: bias of the CEO estimator (circles) and the eighth order Cartwright estimator [5] (triangles) vs. the phase* θ *for various sample sizes* N.