NOISE REMOVAL ON COLOR IMAGE SEQUENCES USING COUPLED ANISOTROPIC DIFFUSIONS AND NOISE-ROBUST MOTION DETECTION

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ABSTRACT

In this paper, we propose a new approach for noise removal on color image sequences, based on coupled spatial and temporal anisotropic diffusions. This approach consists in smoothing a sequence while keeping all discontinuities caused by edges and moving objects. Our approach is based on a low level detection of moving areas in a noisy sequence, in order to associate temporal diffusion to a spatial diffusion process, thus denoising the sequence more efficiently.

1. INTRODUCTION

Because they allow noise removal while preserving global characteristics such as edges, Partial Differential Equations-based denoising methods have become quite popular in image processing over the last decade [1, 2, 3]. Unfortunately, most of these methods tend to focus on still images, while only a few have been proposed for image sequences processing [4, 5], even less when it comes to color sequences. Of course still image processing techniques could be applied to perform "frame-by-frame" video processing, but more efficient algorithms are needed to better exploit the redundancy between successive frames. A simple idea for PDE-based image sequence denoising would be to consider a sequence as a 3D object, and apply Perona-Malik's PDE [1] in a 3D space, using a 3D gradient to detect discontinuities. In this paper, we propose a color image sequence noise removal method, which is based on this idea, except for one point : in our method, temporal and spatial diffusions both use specific discontinuities detection. While the well-known gradient norm is used to detect spatial discontinuities, we decided to use a better, noise-robust, descriptor for temporal discontinuities detection. To make this descriptor noise-robust, the idea is to look for variations of statistical attributes of a local neighborhood, instead of looking for variations of a single pixel's brightness. More details about this methods will be given in this article, along with experimental results.

In section 2, we present the principles of PDE-based grayscale and color image denoising. In section 3, we will focus on image sequences, before introducing our coupled spatial and temporal anisotropic diffusions method in section 4. Experimental results will be provided in section 5.

2. PDE-BASED NOISE REMOVAL

In this part, we propose a simple approach of noise removal in image processing. This will allow us to introduce the principles of isotropic and anisotropic diffusions for still grayscale and color images.

2.1 Isotropic diffusion

Let $I(x,y) = I$ be a still, grayscale, image, represented by a function of $\Omega \subset \mathbb{R}^2 \to \mathbb{R}$ that associates to a pixel $(x,y) \in \Omega$ its gray level $I(x,y)$; Ω is the support of the image. We define $I_0(x,y) = I_0$ as a noisy version of $I(x,y)$. A well-known solution to recover $I(x,y)$ from $I_0(x,y)$ is to perform Gaussian convolution, which happens to be equivalent to the evolution of the linear heat (or diffusion) equation (see [6]) :

$$\begin{cases} \frac{\partial I(x,y,t)}{\partial t} = div(\nabla I(x,y,t)) \\ I(x,y,0) = I_0(x,y) \end{cases} \tag{1}$$

Equation (1) defines the diffusion process of pixel (x,y)'s brightness around neighboring pixels $(x \pm \partial x, y \pm \partial y)$, during a time t ($t \in [0,T]$) which value is directly linked to the Gaussian kernel variance. This process, called isotropic diffusion, is known to introduce blur, since it operates the same way in all directions, smoothing both noise and edges.

2.2 Anisotropic diffusion

In 1990, Perona and Malik [1] introduce anisotropic diffusion, a nonlinear process in which smoothing is only performed in low gradient areas (homogeneous areas), thus allowing noise blurring with edge preservation :

$$\begin{cases} \frac{\partial I(x,y,t)}{\partial t} = div(c(\|\nabla I(x,y,t)\|)\nabla I(x,y,t)) \\ I(x,y,0) = I_0(x,y) \end{cases} \tag{2}$$

with $c(.)$ a decreasing positive function, called "diffusion fonction", which allows to define the strength of the smoothing process for each gradient norm value. (2) can also be written as follows [7] :

$$\begin{cases} \frac{\partial I}{\partial t} = \Phi''(\|\nabla I\|)I_{\xi\xi} + \frac{\Phi'(\|\nabla I\|)}{\|\nabla I\|}I_{\eta\eta} \\ I(x,y,0) = I_0(x,y) \end{cases} \tag{3}$$

with $I_{\xi\xi}$ and $I_{\eta\eta}$ the second-order directional derivatives respectively along the gradient direction and along its orthogonal direction. This equation actually is Perona-Malik's PDE (2), with $c(\|\nabla I\|) = \frac{\Phi'(\|\nabla I\|)}{\|\nabla I\|}$. Deriche-Faugeras' formulation (3) allows to define conditions on function $\Phi(.)$:

$$\begin{cases} \Phi''(0) \geq 0 \quad et \quad \Phi'(0) \geq 0 \\ \lim_{s\to 0} \frac{\Phi'(s)}{s} = \lim_{s\to 0} \Phi''(s) = \Phi''(0) \\ \lim_{s\to\infty} \Phi''(s) = 0, \quad \lim_{s\to\infty} \frac{\Phi'(s)}{s} = 0 \\ \lim_{s\to\infty} \frac{\Phi''(s)}{\frac{\Phi'(s)}{s}} = 0 \end{cases} \tag{4}$$

Equation (4) means that smoothing is performed in all directions (isotropic diffusion) for low gradient areas, while it is only performed along the gradient's orthogonal direction (anisotropic diffusion) for high gradient areas.

2.3 Extension to color images

In the case of color images, we define a vectorial function $\vec{I}(x,y) : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3$ which associates to pixel $(x,y) \in \Omega$ its 3 component values in the Red-Green-Blue color space. To perform anisotropic diffusion on color images, we can use DiZenzo's gradient norm [8], which is based on differential geometry of surfaces. It consists in defining a multispectral tensor, associated to a vector field, to look for local variations in the image. The highest eigenvalue of the multispectral tensor then corresponds to the square norm of the gradient.

3. IMAGE SEQUENCE DENOISING

3.1 Introduction

Video restoration being a more recent research area than still image restoration, it is no surprise that most papers dealing with PDE-based noise removal tend to focus on still image processing. Of course we could use still image denoising techniques to perform frame-per-frame video restoration. But in this case redundancies between successive frames wouldn't be exploited, and those provide very useful information for noise removal. In 1984, Dubois and Sabri [9] introduce a method in which static scenes of a video are identified. Noise reduction then consists in performing temporal averaging on those scenes, while dynamic scenes are kept unfiltered. Identifying static/dynamic scenes to perform noise removal happens to be very efficient. Kornprobst successfully adapted this idea for PDE-based restoration [5].

3.2 A study of 3D diffusion

Let $I(x,y,z) = I$ be a function of $\Omega \subset \mathbb{R}^3 \to \mathbb{R}$ that associates to a pixel (x,y) localized in frame z its gray level $I(x,y,z)$. $I_0(x,y,z)$ being a noisy version of $I(x,y,z)$, a basic idea to recover $I(x,y,z)$ would be to extend Perona-Malik's PDE (2) to 3D space :

$$\begin{cases} \frac{\partial I(x,y,z,t)}{\partial t} = div(c(\|\nabla I(x,y,z,t)\|)\nabla I(x,y,z,t)) \\ I(x,y,z,0) = I_0(x,y,z) \end{cases} \tag{5}$$

Figure 1 shows results obtained on noisy "Foreman" sequence by this 3D anisotropic diffusion process. Although it seems to give satisfying results, we can find a few drawbacks to this method. Considering a video as a 3D object actually consists in considering the video's temporal coordinate as a third spatial coordinate : in this case, the video's voxels would be evolving in a homogeneous and isotropic medium. This, of course, is a wrong assumption. Taking a closer look at Figure 1.b, we can notice noise residuals on a few areas : although a 3D gradient norm allows to detect discontinuities, its value won't give any information about either spatial or temporal variations. This means homogeneous areas of one frame won't be blurred if pixels of these areas present high temporal variations on several frames.

(a) Noisy sequence (b) Restored sequence

FIG. 1 – 3D diffusion of a noisy sequence (frame n°82)

In the next section, we are presenting a denoising method which is based on this 3D anisotropic diffusion process, except in our method spatial and temporal discontinuities have their own detector.

4. COUPLED SPATIAL AND TEMPORAL DIFFUSIONS

In this section, we consider a color image sequence. $\vec{I}(x,y,z) : \Omega \subset \mathbb{R}^3 \to \mathbb{R}^3$ is a function which associates to pixel (x,y,z) its component values in the RGB color space. Our method consists in performing simultaneously two nonlinear 2D and 1D filtering operations : spatial and temporal anisotropic diffusions. Such process should independently define the diffusion of pixel (x,y,z)'s chromatic RGB spectrum around neighboring pixels $(x \pm \partial x, y \pm \partial y)$ on frame z, as well as its diffusion around neighboring pixels $z \pm \partial z$ on spatial localisation (x,y). In both cases, diffusion speed is determinated by medium-specific (space or time) discontinuities detectors. Spatial discontinuities are measured via DiZenzo's color gradient norm, while we propose our own method to detect temporal discontinuities. This method consists in studying a neighborhood for each pixel and comparing neighborhood statistics to an estimated model. We will explain how this method turns out to be more noise-robust and useful than a simple, easily perturbable, 1D gradient norm.

4.1 Diffusion equation

To differentiate spatial diffusion from temporal diffusion, we write our PDE as follows :

$$\begin{cases} \frac{\partial \vec{I}}{\partial t} = div \begin{pmatrix} c_s(\left\|\nabla \vec{I}\right\|_s)\frac{\partial \vec{I}}{\partial x} \\ c_s(\left\|\nabla \vec{I}\right\|_s)\frac{\partial \vec{I}}{\partial y} \\ c_t(\left\|\nabla \vec{I}\right\|_t)\frac{\partial \vec{I}}{\partial z} \end{pmatrix} \\ \vec{I}(x,y,z,0) = \vec{I}_0(x,y,z) \end{cases} \tag{6}$$

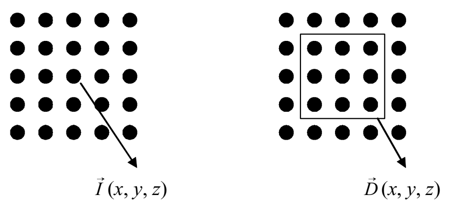
with $c_s(.)$ the spatial diffusion function, $c_t(.)$ the temporal diffusion function, $\| \nabla \vec{I} \|_s$ the vectorial norm representing spatial discontinuities, and $\| \nabla \vec{I} \|_t$ the vectorial norm representing temporal discontinuities.

As we said already, $\| \nabla\vec{I} \|_s$ is calculated using DiZenzo's gradient norm. The calculation of $\| \nabla\vec{I} \|_t$ is explained in next paragraph.

4.2 Temporal discontinuities detection

To detect temporal discontinuities in a noisy sequence, the easiest solution would be to compute a 1D gradient calculated on coordinate z for each pixel of this sequence. Unfortunately, experimental results show that the high noise sensibility of this method makes it unusable.

Suppose now that instead of studying the temporal evolution of a single pixel $\vec{I}$, we focus on a whole set of pixels (a.e. a $M \times M$ neighborhood, as shown on Figure 2), and decide to study statistical attributes of this set instead of studying raw RGB pixel values.



(a) Single-pixel approach (b) Cluster approach

FIG. 2 – Neighborhood study on one z frame

In our cluster approach, statistical attributes of pixel $\vec{I}(x,y,z)$ and surrounding neighborhood are defined in vectorial function $\vec{D}(x,y,z)$:

$$\vec{D}(x,y,z) = \begin{bmatrix} D_1(x,y,z) \\ D_2(x,y,z) \\ \vdots \\ D_L(x,y,z) \end{bmatrix} \tag{7}$$

In practice, statistical attributes D_1, D_2, ..., D_L could be spatial means and variances, median, mimimum or maximum values, etc.

Assuming the presence of motion between two consecutive frames z and $z+dz$ produces brightness intensity variations on pixels of a defined array, statistical attributes of this array may also be subject to variations. This way, both single-pixel value and array statistics-based approaches can be used to detect temporal discontinuities.

Suppose we are working on a noisy sequence, this noise being characterized by a statistical model. In most cases, including natural scenes, this noise model in unlikely to change during sequence-time. This actually makes our statistics-based detection method noise-robust, at least stationary noise-robust. We can find several motion detection methods based on cluster approaches in literature [10, 11]. Ours operates this way :

- on frame z, statistics vector $\vec{D}$ is calculated for each pixel $\vec{I}$ and surrounding neighborhood,
- statistical attributes of a same neighborhood are also calculated on surrounding frames $z \pm 1$, $z \pm 2$, etc. to obtain temporal means and variances/covariances for attributes D_1, D_2, ..., D_L,
- finally, for each pixel $\vec{I}$, a Mahalanobis [12] distance between vector $\vec{D}$ on frame z and its parametric model estimated on neighboring frames is calculated, and corresponds to $\| \nabla\vec{I} \|_t^2$.

Let P be the number of frames used to estimate a parametric model for vector $\vec{D}$ (P is an even value). For each pixel (x,y), we define :

$$\mu_{D_l}(z) = \frac{1}{P} \sum_{\substack{i=-P/2,\\ i\neq 0}}^{+P/2} D_l(z+i), \quad l = 1,...,L \tag{8}$$

$$\sigma_{D_l}{}^2(z) = \frac{1}{P} \sum_{\substack{i=-P/2,\\ i\neq 0}}^{+P/2} (D_l(z+i) - \mu_{D_l}(z))^2 \tag{9}$$

$$\mathrm{cov}(D_k, D_l)(z) = \frac{1}{P} \sum_{\substack{i=-P/2,\\ i\neq 0}}^{+P/2} \begin{array}{l} (D_k(z+i) - \mu_{D_k}(z)) \\ \times\, (D_l(z+i) - \mu_{D_l}(z)) \end{array} \tag{10}$$

The Mahalanobis distance between vector $\vec{D}$ and its parametric model is defined by :

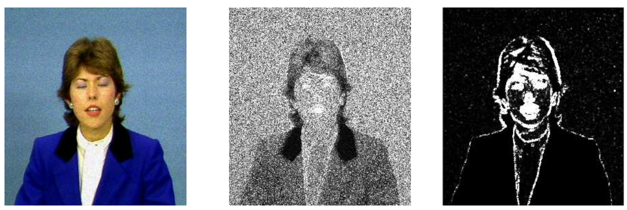
$$\left\| \nabla\vec{I} \right\|_t^2 = (\vec{D} - \mu_{\vec{D}})^t . C_{\vec{D}}^{-1} . (\vec{D} - \mu_{\vec{D}}) \tag{11}$$

with $\mu_{\vec{D}}$ and $C_{\vec{D}}$ respectively the mean and covariance matrices of vector $\vec{D}$'s parametric model.

5. EXPERIMENTAL RESULTS

5.1 Motion detection

Figure 3 shows temporal discontinuities detection results obtained on noisy "Claire" color sequence (additive gaussian noise) using a 1D gradient (Fig 3.b) and our statistics-based method, using $P = 4$ and $M = 3$ as neighborhood parameters (Fig 3.c). In this example, we used spatial means (D_1) and variances (D_2) of brightness values $Y(x,y,z)$ ($Y = 0.299R + 0.587V + 0.114B$) to define statistics vector $\vec{D}$. As we can see, noise perturbates the temporal gradient calculation and makes it almost unusable for detection, but our methods seems to present more robustness : Fig 3.c only shows actual motion elements, while noise hasn't been detected as such.



(a) Noisy sequence (b) 1D gradient (c) our method

FIG. 3 – Motion detection on a noisy sequence (frame n°16)

5.2 Noise removal

In this part, we present experimental results of our noise removal method. Temporal discontinuities detection statistical attributes and parameters remain the same as in 5.1. We decided to use the following diffusion function :

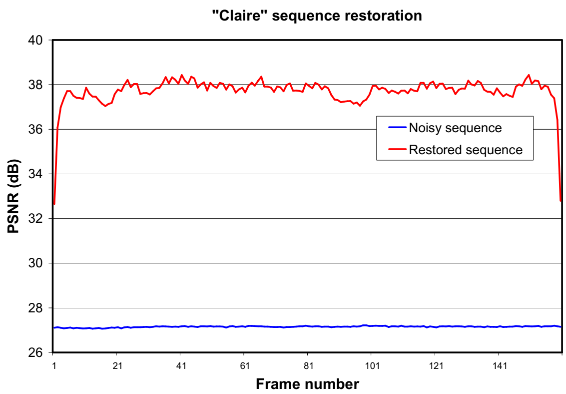
$$c_j(x) = c_j(x) = (1 + (x/k_j)^2)e^{-(x/k_j)}, \quad j \in \{s,t\} \qquad (12)$$

The purpose of using this diffusion function instead of Perona-Malik's is to focus on noise removal properties only (Perona-Malik's function performs both noise removal and edge enhancement).

Figures 4.a and 4.b show results obtained on noisy "Claire" color sequence, using diffusion parameters $k_s = 2.5$, $k_t = 0.25$, and $t = 15$. We can notice serious enhancement between Fig 4.a and Fig 4.b. This enhancement is confirmed by PSNR values provided on Fig 4.c, which increase from about 27dB to about 38dB.

(a) Noisy sequence

(b) Restored sequence



(c) PSNR per frame

FIG. 4 – Restoration of noisy "Claire" sequence

6. CONCLUSION

In this paper, we've presented a noise removal method for color image sequences, based on coupled spatial and temporal anisotropic diffusions. To make it more efficient, we've also introduced a new, noise-robust, temporal discontinuities detection technique. The main advantages of our method is that it will work for most types of stationary noise (besides additive Gaussian noise, it also gave good results on compression noise), and the fact that PDE formalism allows to provide fast yet stable algorithms. Promising experimental results have been achieved and shown on this paper, and open new perspectives in PDE-based image sequence processing. Future works will include automatic diffusion/detection parameters estimation, that would lead to an unsupervised restoration algorithm.

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