Peak Constrained Two Dimensional Quadrantally Symmetric Eigenfilter Design without Transition Band Specification

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Abstract— **The design of a two dimensional (2D) quadrantally symmetric FIR filter with peak constrained magnitude response is considered. We further considered the design specification without explicitly specifying the transition band. A novel iterative algorithm without transition band specification is proposed to design FIR filters with various design constraints. The eigenfilter formulation further allows the proposed algorithm to incorporate time domain constraints simultaneously. Various design examples are presented to illustrate the versatility of the proposed 2D filter design method. Although we have not proven the convergence of the proposed algorithm, it is found to converge efficiently in all the simulations.**

I. Introduction

The design of two dimensional (2D) FIR digital filters is not a trivial task because the very nature of multidimensional processing introduces special considerations and cross-dimensional dependencies that do not exist in the one dimensional (1D) cases [4]. Previous work in the design of 2D FIR digital filters can be roughly classified into two categories, (i) indirect and (ii) direct methods. The indirect design methods obtain the 2D digital filters through 1D to 2D spectral transformations of pre-designed 1D digital filters. The McClellan transformation [7], [8] is one of the most popular 1D to 2D spectral transformation. Such design method has very low computational complexity. However, the obtained 2D digital filters are not guaranteed to inherit the spectral property of the 1D prototypes. As a result, it is not suitable to be used to design 2D digital filters with tight spectral specifications.

The direct approach designs the 2D digital filter by optimizing the filter coefficients to approximate a desired spectral response. Various approximation criteria have been considered in literature. The L_2 and L_∞ are the most popular approximation criteria used in 2D digital filter design. It should be noted that the Remez exchange method, which has been used widely in 1D L_∞ optimal filter design, cannot be generalized to 2D L_∞ filter design problem. This is because the Remez exchange algorithm is based on the alternation theorem, which is not available in the 2D case [5]. Instead, most of the 2D L_∞ optimal filter design methods found in the literature are formulated as an iterative reweighted least squares optimization problem [11].

The objective of the L_∞ optimal filter design problem is to minimize the peak of the approximation error. However, the L_2 approximation error is assumed to be irrelevant. Similarly, the L_2 filter design criterion is based on the assumption that the size of the peak errors can be ignored. Both the L_2 and L_∞ criteria are important. Therefore, Adams argued that the filter design methods that only minimize either the L_2 error or L_∞ error alone are inefficient [12]. In a very different perspective, Adams [12] proposed the peak constrained least squares (PCLS) design criterion, which minimizes the L_2 filter design error subject to constraints on the L_∞ filter design error. Such design criterion is shown to be effective by Adams [12], because the peak errors of the L_2 optimal 1D filter can be significantly reduced with only a slight increase in the squares error. Similarly, the squares error of the L_∞ optimal 1D filter can be significantly reduced with only a slight increase in the peak error of the L_2 optimal 1D filter.

The same properties are also observed in the design of 2D digital filters. In the literature, the only design method for designing PCLS 2D FIR digital filters is given in [9]. However, the design method in [9] exhibits several deficits. First, it applies Lagrange multiplier method iteratively to solve a constrained optimization problem, which involves solving the normal equations by matrix inversion. It is well known that the matrix inversion is numerically unstable and inefficient, especially when the matrix size is large. Second, the algorithm based on the single exchange procedure is computationally inefficient when compared to that of the multiple exchange procedure [5]. Furthermore, the iterative method of [9] may easily be trapped in local optimal solution in the case of multiband filter design. This problem is explained in [14], which showed that the iteration method of [13] is trapped by local optimal solution in the case of bandpass filter design. Note that the iterative method in [9] for 2D filter design is a direct extension from the iterative method of [13] for 1D filter design. A method to remedy this deficit is proposed in [14] for 1D filter design. However, it may not be effective for 2D filter design problems because of the possible existence of multiple local minimal saddle points in the optimization problems for 2D filter designs. As a result, it is a rare case to be able to obtain a good 2D digital filter with multiband spectral support using the extension of [14].

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It is the purpose of this paper, which proposes a novel iterative algorithm for designing 2D peak constrained digital filters. In addition, the proposed algorithm is the first algorithm in the literature that does not require the specification of transition band bandedges, and thus avoids problems related to the ambiguous transition band definition for 2D digital filters. The proposed algorithm is similar to the multiple exchange algorithm which is known to be computational efficient when compared to the design method based on the single exchange algorithm [9]. The proposed design method can be used to design multiband filters and high performance multiband filters are obtained in our simulations presented in the later part of the paper. Although the proposed algorithm is similar to the Remez exchange algorithm 1, there are fundamental differences between the two algorithms. First, the proposed design algorithm can be used to solve peak constrained filter design problems which includes the L_∞ design problem as a special case. However, the Remez exchange algorithm can only be used to solve the L_∞ filter design problem. Second, the number of extremals involved in each iteration varies, whereas in the case of the Remez exchange algorithm is maintained to has a constant number of extremals involved in each exchange. After all, the alternation theorem does not exists for 2D L_∞ optimization problem. As a result, it is fair to say that the proposed algorithm is not a variants of the Remez exchange algorithm for 2D filter design problems.

II. 2D Quadrantally Symmetric Eigenfilter Design

The amplitude response of a causal 2D filter with even symmetric and even order (N_1, N_2) is given by [2]

$$A(\omega_1,\omega_2) = \sum_{n_1=0}^{M_1}\sum_{n_2=0}^{M_2} a(n_1,n_2)\cos(n_1\omega_1)\cos(n_2\omega_2), \tag{1}$$

where

$$a(n_1,n_2) = \begin{cases} h(M_1,M_2) & \text{for} \quad n_1=n_2=0, \\ 2h(M_1-n_1,M_2-n_2) & \text{for} \quad n_1=0 \text{ or } n_2=0, \\ 4h(M_1-n_1,M_2-n_2) & \text{otherwise.} \end{cases}$$

Define

$$\begin{aligned}\mathbf{a} &= [\ a(0,0) \quad a(0,1) \quad \cdots \quad a(0,M_2)\ , \\ &\qquad a(1,0) \quad a(1,1) \quad \cdots \quad a(1,M_2)\ ,\cdots, \\ &\qquad a(M_1,0) \quad a(M_1,1) \quad \cdots \quad a(M_1,M_2)\ ]^t, \qquad (2) \\ \mathbf{c}(\omega_1,\omega_2) &= [\ 1 \quad \cos(\omega_2) \quad \cdots \quad \cos(M_2\omega_2)\ , \\ &\qquad \cos(\omega_1) \quad \cdots \quad \cos(\omega_1)\cos(M_2\omega_2)\ ,\cdots, \\ &\qquad \cos(M_1\omega_1) \quad \cdots \quad \cos(M_1\omega_1)\cos(M_2\omega_2)\ ]^t,\end{aligned}$$

where the superscript t denotes matrix transpose. The amplitude response in (1) can be simplified by using vector-matrix representation as

$$A(\omega_1,\omega_2) = \mathbf{a}^t\mathbf{c}(\omega_1,\omega_2). \qquad (3)$$

Similar expressions can be written for odd order and odd symmetric filters [2].

The 2D filter design problem is to find a set of impulse responses $h(n_1,n_2)$ such that the associated amplitude response $A(\omega_1,\omega_2)$ approximates a given frequency response $D(\omega_1,\omega_2)$. It is the same as finding the coefficient $a(n_1,n_2)$ in (2) to satisfy the given approximation problem. The approximation error in the frequency domain of the filter design problem is given by [1]

$$\begin{aligned}\epsilon(\omega_1,\omega_2) &= \frac{D(\omega_1,\omega_2)}{D(\omega_{10},\omega_{20})}A(\omega_{10},\omega_{20}) - A(\omega_1,\omega_2) \\ &= \frac{D(\omega_1,\omega_2)}{D(\omega_{10},\omega_{20})}\mathbf{a}^t\mathbf{c}(\omega_{10},\omega_{20}) - \mathbf{a}^t\mathbf{c}(\omega_1,\omega_2). \qquad (4)\end{aligned}$$

Various optimization criterion can be applied to the problem. The method in [1] proposed to minimize the weighted L_2 error given by

$$\begin{aligned}\epsilon_{L_2} &= \int\int_{\mathcal{R}} W(\omega_1,\omega_2)[\epsilon(\omega_1,\omega_2)]^2 d\omega_1 d\omega_2, \\ &= \mathbf{a}^t\mathbf{P}\mathbf{a}, \qquad (5)\end{aligned}$$

where $W(\omega_1,\omega_2)$ is a nonnegative weighting function that controls the relative importance of the spectral response in different frequencies and $\mathcal{R}$ is the spectral domain of concern. The matrix $\mathbf{P}$ is a real, symmetric and positive definite matrix given by

$$\begin{aligned}\mathbf{P} &= \int\int_{\mathcal{R}} W(\omega_1,\omega_2)\left(\frac{D(\omega_1,\omega_2)}{D(\omega_{10},\omega_{20})}\mathbf{c}(\omega_{10},\omega_{20}) - \mathbf{c}(\omega_1,\omega_2)\right) \\ &\quad \times\left(\frac{D(\omega_1,\omega_2)}{D(\omega_{10},\omega_{20})}\mathbf{c}(\omega_{10},\omega_{20}) - \mathbf{c}(\omega_1,\omega_2)\right)^t d\omega_1 d\omega_2. \qquad (6)\end{aligned}$$

Obviously, ϵ_{L_2} is minimized when $\mathbf{a} = 0$. To avoid this trivial solution, the above optimization problem is constrained at the reference frequency $(\omega_{10},\omega_{20})$ so that the obtained filter approximates the design specification $D(\omega_{10},\omega_{20})$ in the passband. As a result, the filter design problem is formulated as the following,

$$\begin{aligned}\min_{\mathbf{a}} \mathbf{a}^t\mathbf{P}\mathbf{a} \quad \text{subject to} \quad & \mathbf{a}^t\mathbf{c}(\omega_{10},\omega_{20})\mathbf{c}^t(\omega_{10},\omega_{20})\mathbf{a} \\ & = D(\omega_{10},\omega_{20}). \qquad (7)\end{aligned}$$

The dc response, i.e. $(\omega_{10},\omega_{20}) = (0,0)$, is commonly chosen for $D(\omega_{10},\omega_{20})$ in lowpass filter design, and the value 1 is usually used for $D(\omega_{10},\omega_{20})$. By Rayleigh Principle, the eigenvector corresponds to the minimum eigenvalue of the matrix $\mathbf{P}$ in (7) is the solution vector for the filter design problem. Such filter is known as the eigenfilter in [1], [2], [3].

III. A Novel Iterative 2D Eigenfilter Design Method

An unconstrained 2D eigenfilter $\mathbf{a}_{L_2}$ with specification as shown in Fig.1(a) and $W(\omega_1,\omega_2) = 1\ \forall(\omega_1,\omega_2)$ is designed using the method of [2]. The passband and stopband error, $\epsilon_{L_2}(\omega_1,\omega_2)$ in (4) are shown in Fig.1(b). A peak constrained filter $\mathbf{a}_{PC}$, which the ripples fit within a predefined upper and lower bound (δ_U and δ_L), can be obtained by adding a second filter $\Delta\mathbf{a}_k$ to $\mathbf{a}_{L_2}$, such that $\Delta\mathbf{a}_k$ is designed to reduce the peak error of the unconstrained eigenfilter, i.e.

$$\mathbf{a}_{PC} = \mathbf{a}_{L_2} + \Delta\mathbf{a}_k. \qquad (8)$$

The specification defined in Fig.1(a) is an example used to simplify our discussions. The proposed design method can be used to design 2D eigenfilters with other design specifications, which is demonstrated by the design examples in the later part of the paper. To satisfy (8), $\Delta\mathbf{a}_k$ should be designed in a way that the peak error of the unconstrained eigenfilter decreases to fit within the upper and lower ripple bound (δ_U and δ_L). In other words, the spectral response of $\Delta\mathbf{a}_k$ should be designed to has spectral peaks at $v_k(i,j)$ with magnitude $\Delta_{v_k}(i,j)$, and zero response at all other spectral locations. Note that $\Delta_{v_k}(i,j)$ at (ω_i,ω_j) are observed from Fig.1(d) and are defined as

$$v_k(i,j) = \epsilon_k(\omega_{1i},\omega_{2j}) = \frac{D(\omega_1,\omega_2)}{D(\omega_{10},\omega_{20})}A_k(\omega_{10},\omega_{20}) - A_k(\omega_1,\omega_2),$$

$$\Delta_{v_k}(i,j) = \begin{cases} [\delta_U - \epsilon_k(\omega_{1i},\omega_{2j})] & \text{if } \epsilon_k(\omega_{1i},\omega_{2j}) > \delta_U \\ & \text{and } (\omega_{1i},\omega_{2j}) \in \Phi_k \\ [\epsilon_k(\omega_{1i},\omega_{2j}) - \delta_L] & \text{if } \epsilon_k(\omega_{1i},\omega_{2j}) < \delta_L \\ & \text{and } (\omega_{1i},\omega_{2j}) \in \Phi_k \end{cases} \qquad (9)$$

respectively. Further note that, Φ_k is defined as the collection of all extremal frequencies of $\epsilon_k(\omega_1,\omega_2)$ such that $\epsilon_k(\omega_{1i},\omega_{2j})$ ($i = 1,2,3,\ldots$ and $j = 1,2,3,\ldots$) are not bounded by δ_U and δ_L. *To simplify the 2D peak search routine in the implementation of the proposed algorithm, we will search extremal frequencies $(\omega_{1i},\omega_{2j})$ in a 2D plane such that all $\epsilon_k(\omega_{1i-1},\omega_{2j})$, $\epsilon_k(\omega_{1i},\omega_{2j-1})$, $\epsilon_k(\omega_{1i+1},\omega_{2j})$ and $\epsilon_k(\omega_{1i},\omega_{2j+1})$ are smaller than or bigger than $\epsilon_k(\omega_{1i},\omega_{2j}) = v_k(i,j)$ for maximum or minimum extermals respectively.*

This paper proposes to use eigenfilter method, which is an efficient filter design method, to design $\Delta\mathbf{a}_k$. The squares error function ($\Delta\epsilon_k$) for the eigenfilter design problem can be written as,

$$\Delta\epsilon_k = \Delta\mathbf{a}_k{}^t\mathbf{Q}_k\Delta\mathbf{a}_k, \qquad (10)$$

where

$$\begin{aligned}\mathbf{Q}_k &= \sum_{\substack{(i,j)\ \text{s.t} \\ (\omega_{1i},\omega_{2j})\in\Phi_k}} \left[\frac{\Delta_{v_k}(i,j)}{D(\omega_{10},\omega_{20})}\mathbf{c}(\omega_{10},\omega_{20}) - \mathbf{c}(\omega_{1i},\omega_{2j})\right] \\ &\quad \times\left[\frac{\Delta_{v_k}(i,j)}{D(\omega_{10},\omega_{20})}\mathbf{c}(\omega_{10},\omega_{20}) - \mathbf{c}(\omega_{1i},\omega_{2j})\right]^t \\ &\quad + \sum_{\substack{(i,j)\ \text{s.t} \\ (\omega_{1i},\omega_{2j})\notin\Phi_k}} \mathbf{c}(\omega_{1i},\omega_{2j})\mathbf{c}(\omega_{1i},\omega_{2j})^t, \qquad (11)\end{aligned}$$

with $(\omega_{10},\omega_{20}) = \arg\max_{(\omega_{1i},\omega_{2j})}\{\Delta_k(i,j)\}$, $D(\omega_{10},\omega_{20}) = \max\{\Delta_k(i,j)\}$ and $\Delta\mathbf{a}_k$ is half of the filter coefficients defined similar to (2). By Rayleigh principle [3], the vector $\Delta\mathbf{a}_k$ that minimizes $\Delta\epsilon_k$ is given by the eigenvector of the matrix $\mathbf{Q}_k$ corresponding to the smallest eigenvalue. Fig.1(c) shows the actual spectral response of $\Delta\mathbf{a}_k$ obtained by eigenfilter design method with the discussed design specification defined in (9) and shown in Fig.1(d).

However, it is almost impossible to design a filter with several spectral peaks and a large region of zero response. Fig.1(c) shows the actual spectral response of $\Delta\mathbf{a}_k$ obtained by eigenfilter design method with the design specification that we have just discussed. Obviously, adding $\Delta\mathbf{a}_k$ in Fig.1(c) to $\mathbf{a}_{L_2}$ will not reduce all the peak errors in Fig.1(b) and will result in a $\mathbf{a}_{PC}$ that satisfies the given bound. New peaks may appear because the eigenfilter design method designs $\Delta\mathbf{a}_k$ to minimize the difference between the actual spectral response and the desired spectral response. In addition, it is difficult to design a filter with several spectral peaks and a large region of zero response. To remedy this problem, we formulated the design problem of $\Delta\mathbf{a}_k$ in a recursive way such that we will design a new $\Delta\mathbf{a}_k$ to compensate for any discrepancy until $\mathbf{a}_{PC}$ satisfies the design specification. The proposed method can be summarized as follows.

1) Obtain $\mathbf{a}_0$ by designing $\mathbf{a}_{L_2}$ with the design specifications N, $(\omega_{1c}, \omega_{2c})$, δ_U and δ_L using eigenfilter. Set iteration index $k = 1$.
2) Search for all the extremal frequencies $(\omega_{1i}, \omega_{2j})$ and calculate $\Delta_{v_k}(i, j)$ by using (9).
3) Set $\Delta\mathbf{a}_k$ equals to the eigenvector of the smallest eigenvalue of $\mathbf{Q}_k$ in (11).
4) Update the filter coefficient (i.e. $\mathbf{a}_k$) by $\mathbf{a}_k = \Delta\mathbf{a}_k + \mathbf{a}_{(k-1)}$.
5) Stop when all the peaks are bounded within the upper and lower bound (δ_U and δ_L), when $k > 100$; otherwise set $k = k + 1$ and go to Step 2.

Although we have not proven the convergence of the proposed algorithm, it is found to converge efficiently for the large number of design examples considered. Indeed, the algorithm converges rapidly for all the design examples, and so far we have not encountered a non-convergent design.

IV. Interpretations and Extensions

As discussed in the Introduction, the proposed algorithm looks like that it is a variant of the Remez exchange algorithm 1 in [10] at first glance. However, there are fundamental differences between the two algorithms. For simplicity, we compared the Harris and Mersereau (HM) algorithm [6], a variant of the Remez exchange algorithm for 2D filter design, with the proposed algorithm. First, the number of external frequencies involved in the proposed algorithm varies from one iteration loop to another, whereas the number of external frequencies involved in HM algorithm are maintained to be the same throughout the algorithm. Furthermore, due to the lack of 2D alternation theorem [5], the external frequencies may be perturbated in HM algorithm when the algorithm does not converge. However, no perturbation of the external frequencies is required for the proposed algorithm.

Second, there does not exist a minimum δ_U and δ_L such that the proposed algorithm fails to converge. When δ_U and δ_L are small, the transition band between the passband and the stopband of the filter obtained by the proposed algorithm simply becomes wider. In contrast, the transition band bandwidth of the filter obtained by HM algorithm is fixed and is equal to a predefined value. As a result, there exists an optimal δ_U and δ_L, beyond which, the HM algorithm does not converge. Instead, the proposed algorithm offers a design tradeoff between δ_U and δ_L with the transition band bandwidth. Similar design tradeoff for 1D digital filter designs is considered in [13], [15], and is shown to be an important design feature.

In addition, the transition band in 2D filters is ill-defined. Traditionally, the transition band bandwidth definition used in 2D filters is a direct extension of that defined for 1D filters, which is applicable for circular symmetric 2D filters only. It is inappropriate to use the same definition for 2D filter designs with other spectral support. As an example, when it is used for a 2D filter with square shaped spectral support to achieve constant transition band bandwidth, the actual transition band bandwidth of the corner frequency of the 2D filter will be wider than that of the horizontal and vertical frequencies. As a result, it will be inappropriate to design optimal 2D filter with such transition band specifications. Further note that there does not exist a simple formulation that can be used to determine the transition band bandwidth of an optimal 2D filter with a given δ_U and δ_L. As a result, it will be difficult, if not impossible to design 2D filters with optimal transition band bandedges with a given δ_U and δ_L using the HM algorithm.

In addition, the proposed algorithm does not preclude the specification of a transition band. If both the transition band bandedges, δ_U and δ_L are specified simultaneously, it is possible that no solution exists. This is because the transition band cannot be arbitrary sharp. Note that in here a distinction is being made between the cut-off frequency $(\omega_{1c}, \omega_{2c})$ and the bandedges frequencies (e.g. $(\omega_{1p}, \omega_{2p}) \leq (\omega_{1c}, \omega_{2c}) \leq (\omega_{1s}, \omega_{2s})$ in lowpass 2D digital filter design). This is similar to that of the 1D digital filter discussed in [15].

Finally, note that the proposed algorithm can be initialized with different filters which will affect the convergence of the algorithm. We further propose to initialize the algorithm with the filter obtained by the 2D eigenfilter design method proposed in [1], such that there is no ambiguity about the initial filter used in the iterative procedure.

V. Design Examples

All the filters presented in this section are designed with the weighting factor $W(\omega_1, \omega_2)$ set to 1 for all ω. In practice, the weighting factor can be set to different values to control the relative importance of the spectral response for the filter under concern.

A. Example 1 - Peak Constrained 2D Quadrantally Symmetric Circular Eigenfilter

An order $(N_1, N_2) = (16, 16)$ linear phase 2D circular lowpass filter was designed using the proposed algorithm. The design specification is shown in Fig.2(a) , where the passband is circularly shaped in the (ω_1, ω_2) plane with the radius equal to 0.4π. The upper and lower bound constraints on the amplitude response are equal to $\delta_U(\omega_1, \omega_2) = -\delta_L(\omega_1, \omega_2) = 0.02 = -33.9794dB$ for $(\omega_1, \omega_2) \in ([0, \pi], [0, \pi])$. That is the peak ripple sizes for both passband and stopband are the same and are equal to $-33.9794dB$. The converged magnitude response is shown in Figs.2(d) and (e) with different view angles. Compared to the initial magnitude response as shown in Figs.2(b) and (c), we can conclude that the proposed method provides an efficient method for designing peak constrained eigenfilters.

B. Example 2 - Peak Constrained 2D Diamond Shaped Multiband Eigenfilter

An order $(N_1, N_2) = (22, 22)$ linear phase 2D diamond shaped multiple passband filter was designed using the proposed algorithm. The design specification is shown in Fig.3(a). The upper and lower bound constraints on the amplitude response are equal to $\delta_U(\omega_1, \omega_2) = -\delta_L(\omega_1, \omega_2) = 0.01 = -40dB$ for $(\omega_1, \omega_2) \in ([0, \pi], [0, \pi])$. The initial and the converged magnitude responses are shown in Fig.3(b) and Fig.3(c) respectively. The magnitude response at $\omega_1 = 0$ is shown in Fig.3(d). High performance multiple passband filter is observed from Fig.3(c) and Fig.3(d), which demonstrate the effectiveness of designiging multiple passband filters with the proposed design method.

VI. Conclusions

We have proposed a constrained two dimensional quadrantally symmetric eigenfilter design algorithm, which can be used to design peak constrained two dimensional FIR filters. The algorithm has also exploited the design of two dimensional FIR filters without explicit specification of the transition bands. The proposed algorithm can be used to design 2D filters with optimal transition band bandwidth from the design specification. Various design examples have been presented to demonstrate the performance of the proposed design method. Although we have not proven the convergence of the proposed algorithm in the paper, the algorithm is found to converge efficiently for the large amount of design examples considered. We have not encountered a non-convergent design using the proposed design method. Indeed, the algorithm converges rapidly for all the design examples presented in the paper.

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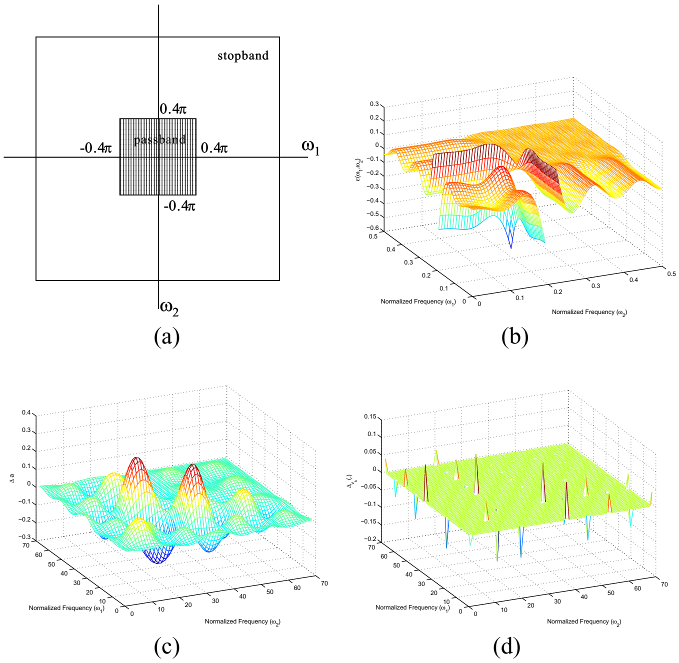

Fig. 1. (a) Design specifications (b) Actual error response of a lowpass filter at k-th iteration. (c) Spectral response of $\Delta\mathbf{a}$ at k-th iteration. (d) $\Delta_{v_k}(.)$ as defined in (9)

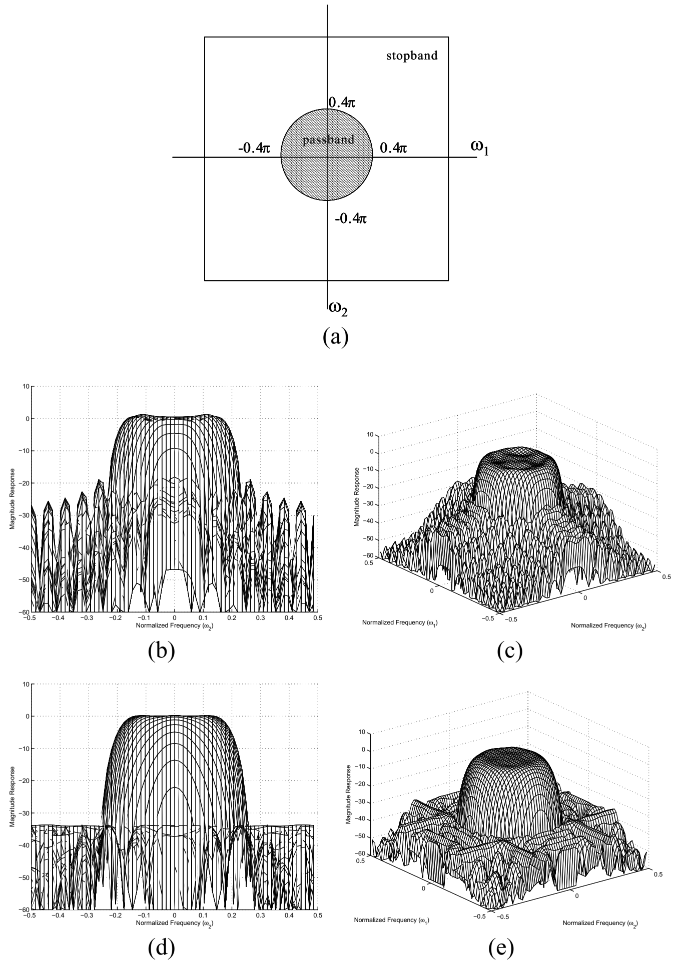

Fig. 2. (a) Design Specifications of Example 1. Results of Example 1. (b) L_2 Filter view from ω_2 axis. (c) L_2 Filter 3D View. (d) Peak constrained Filter view from ω_2 axis. (e) Peak constrained Filter 3D view.

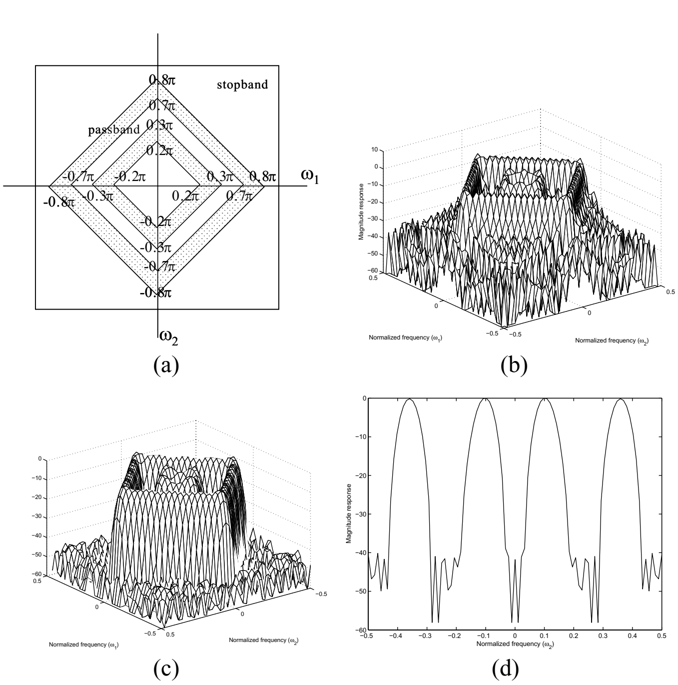

Fig. 3. (a) Design Specifications of Example 2. Results of Example 2. (b) L_2 Filter 3D View. (c) Peak constrained Filter 3D view. (d) Magnitude response at $\omega_1 = 0$.