DESIGN OF TWO-CHANNEL CAUSAL STABLE IIR PR FILTER BANKS AND WAVELET BASES USING CONSTRAINED MODEL REDUCTION

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ABSTRACT

This paper proposes a new method for designing two-channel causal stable IIR perfect reconstruction (PR) filter banks (FBs) with prescribed peak error and *K*-regularity constraints. It is based on the model reduction of the FIR functions in the structural PR filter banks of Phoong et al by a new model reduction technique, which was a modification of the technique previously proposed by Brandenstein et al. The proposed model reduction method retains the denominator of the conventional techniques and formulates the optimal design of the numerator as a semi-definite programming problem. Therefore, linear and convex quadratic inequalities such as peak error and *K*-regularity constraints for the IIR filters can be imposed and solved optimally. Design examples show that the proposed method gives better performance, more flexibility in incorporating a wide variety of constraints, and lower design complexity than conventional method.

I. INTRODUCTION

Perfect reconstruction (PR) multirate filter banks (FBs) have important applications in signal analysis, coding and the design of wavelet bases. An efficient structure of two-channel FIR/IIR FBs, which structurally satisfy the PR condition, was the structural PR FB proposed by Phoong et al [1]. The FBs of this structure are parameterized by two functions $\beta(z)$ and $\alpha(z)$ and some delay parameters. To meet different design specifications, these two functions can be chosen as arbitrary functions such as low-delay FIR [2] or IIR filters [3,4], while satisfying the PR condition. Because of these important results, the design of PR FBs can be simplified to general filter design problems. Moreover, wavelet bases can be constructed from these FBs by imposing additional *K*-regularity condition (which is equivalent to a certain number of zeros respectively at $\omega = \pi$ and $\omega = 0$ for the analysis lowpass and highpass filters) [1,2,5,6].

The design of causal stable IIR FBs using the structure in [1] was also studied by one of the author together with Mao et al [7] based on model reduction. In this approach, two FIR functions $\beta(z)$ and $\alpha(z)$ are first designed to meet the desired frequency characteristic. Model reduction [8] is then applied to these FIR functions to obtain an IIR FB having a similar characteristic as the original FIR FB. The advantages of this model reduction approach are its simple design procedure and the ability to preserve properties such as frequency characteristics, passband linear-phase, causality and stability. However, it does not allow precise control of the frequency response and other constraints, such as prescribed *K*-regularity or peak ripple constraints, to be imposed. One would also expect the performance of the model-reduced FB to be sub-optimal and it can be further improved. In addition, the problem of imposing a prescribed *K*-regularity to the IIR FB was not discussed.

In this paper, we propose a new design method for two-channel IIR structurally PR FBs using a new constrained model reduction technique, which is a modification of the model reduction method proposed in [9]. Important advantages of the method in [9] are that the numerator and denominator can be determined separately and the stability of the model-reduced filter is guaranteed. More precisely, the denominator is first determined, followed by the numerator. This property allows us to incorporate linear and convex quadratic constraints and shape the frequency response of the final IIR filter by designing the numerator using semi-definite programming (SDP), given the denominator at the first stage. For illustrative purpose, we mainly focus on the incorporation of peak stopband error and prescribed *K*-regularity constraints to the final IIR PR FBs. The former is useful to limit the undesirable sidelobe at the band edges and design results show that it yields considerable better performance compared to conventional model reduction in [7]. It should be note that given the denominator, the design of the numerator using SDP with linear and convex quadratic inequalities is a convex optimization problem. In other words, the solution, given the denominator, is guaranteed to be optimal. Owing to the improved frequency characteristics of the proposed design method, further optimization, as suggested in [7], is usually unnecessary. Hence the design complexity of our method, which basically can be regarded as a new model reduction technique with constraints, is considerably simpler. Since the prescribed *K*-regularity constraints are just linear equality constraint after the denominator has been determined, they can be incorporated easily under the SDP framework. Interested readers are referred to [10] for more details of SDP in filter design.

The paper is organized as follows: Section II is overview of the 2-channel structural PR FBs. The model reduction technique proposed in [9] and the principle of the proposed constrained model reduction are described in Section III. The details of the SDP formulation of the *K*-regularity and peak design error constraints for the IIR FBs will be given. Design examples are given in Section IV to demonstrate the effectiveness of the proposed approach, and finally, conclusion is drawn in Section V.

II. TWO-CHANNEL FIR STRUCTURALLY PR FBS

The structural PR FBs [1,2], as shown in Fig. 1, is parameterized by a sub-filter pairs, $\beta(z)$ and $\alpha(z)$, and two delay parameters N and M. In this structure, the z-transforms of the analysis and synthesis filters are given by:

$$H_0(z) = \tfrac{1}{2}(z^{-2N} + z^{-1}\beta(z^2)), \tag{2-1a}$$

$$H_1(z) = -\alpha(z^2)H_0(z) + z^{-2M-1}, \tag{2-1b}$$

$$F_0(z) = -H_1(-z) \text{ and } F_1(z) = H_0(-z) \tag{2-1c}$$

It can be shown [2] that the FB is PR for arbitrary choice of filter pairs $\beta(z)$ and $\alpha(z)$. Moreover, the desired responses of $\beta(e^{j\omega})$ and $\alpha(e^{j\omega})$ are given respectively by:

$$\beta_d(e^{j\omega}) = e^{j(-N+1/2)\omega} \text{ and } \alpha_d(e^{j\omega}) = \frac{e^{j(-M-1/2)\omega}}{H_0(e^{j\omega/2})}, \tag{2-2}$$

for $\omega \in [-\pi, \pi]$. According to (2-2), once $\beta(e^{j\omega})$ and hence $H_0(e^{j\omega})$ are designed, one can obtain the desired response of $\alpha(e^{j\omega})$. Consequently, the design of two-channel FIR PR FBs can be viewed as simple FIR filter design problems. A number of design methods are now available in literature [1,2,5,6].

III. DESIGN OF IIR PR FBS AND WAVELET BASES

A. — Model reduction

For the sack of presentation, a will be used to represent either β or α in the rest of this section, since the design procedures for both $\beta(e^{j\omega})$ and $\alpha(e^{j\omega})$ are very similar. To start with, suppose that we have designed the FIR filter $a(z)$ using any existing methods in the literature, the model reduction technique proposed in [9] is then applied to convert $a(z)$ to an IIR filter $\hat{a}(z)$ with the following form:

$$\hat{a}(z) = \frac{P_a(z)}{Q_a(z)} = \frac{\sum_{n=0}^{L_{pa}-1} p_a(n) z^{-n}}{\sum_{n=0}^{L_{qa}-1} q_a(n) z^{-n}}, \; q_a(0) = 1, \; L_{pa} \geq L_{qa}, \quad (3\text{-}1)$$

where L_{pa} and L_{qa} are respectively the length of numerator and denominator of $\hat{a}(z)$. As mentioned earlier, the advantage of this method is that $P_a(z)$ and $Q_a(z)$ can be determined separately. More precisely, $Q_a(z)$ can be found without the knowledge of $P_a(z)$. Therefore, unlike other model reduction technique, additional constraints can be readily incorporated during the determination of $P_a(z)$. In [9], a simple iterative design procedure was proposed to determine $Q_a(z)$. More importantly, the roots of the resulting $Q_a(z)$ are proved to strictly lie inside the unit circle, and thus $\hat{a}(z)$ is always stable. Details are omitted due to page limitation. Interested readers are referred to [9] for more details. Once $Q_a(z)$ is designed, we want to approximate the response of $a(z)$ by $P_a(z)$, given $Q_a(z)$, in the least square (LS) sense. That is:

$$E_{LS}(\boldsymbol{p}_a) = \int_{-\pi}^{\pi} |\{P_a(e^{j\omega}) / Q_a(e^{j\omega})\} - a(e^{j\omega})|^2 \, d\omega, \quad (3\text{-}2)$$

where $\boldsymbol{p}_a = [p_a(0), \dots, p_a(L_{pa}-1)]^T$. (3-2) can be written as the following matrix form:

$$\min_{p_a} \; \boldsymbol{p}_a^T \boldsymbol{U}_a \boldsymbol{p}_a + \boldsymbol{p}_a^T \boldsymbol{g}_a + v_a, \quad (3\text{-}3)$$

$$[\boldsymbol{U}_a]_{n1,n2} = \int_{-\pi}^{\pi} e^{j\omega(n2-n1)} / \left|Q_a(e^{j\omega})\right|^2 d\omega,$$

where $[\boldsymbol{g}_a]_n = \int_{-\pi}^{\pi} \mathrm{Re}\left\{a(e^{j\omega}) Q_a(e^{j\omega}) \cdot e^{j\omega n}\right\} / \left|Q_a(e^{j\omega})\right|^2 d\omega$ and

$$v_a = \int_{-\pi}^{\pi} \left|a(e^{j\omega})\right|^2 d\omega.$$

This is a standard quadratic programming problem, which can be solved readily. However, large sidelodes are usually encountered at the band-edge of the model-reduced filter. Therefore, additional constraints on the stopband ripple constraints should be imposed to improve the frequency characteristic. Here, we formulate (3-3) as a SDP problem. To start with, one can decompose $\boldsymbol{U}_a$ as $\boldsymbol{U}_a = \boldsymbol{G}_a^T \boldsymbol{G}_a$ so that it can be reformulated, by means of Schur complement [11], as the following linear matrix inequality (LMI):

$$\min_{x_a} \; \boldsymbol{c}_a^T \boldsymbol{x}_a$$

$$\text{subject to} \begin{bmatrix} \delta_a - \boldsymbol{p}_a^T \boldsymbol{g}_a - v_a & \boldsymbol{p}_a^T \boldsymbol{G}_a^T \\ \boldsymbol{G}_a \boldsymbol{p}_a & 1 \end{bmatrix} \succeq 0, \quad (3\text{-}4)$$

where $\boldsymbol{c}_a = [1, 0, \dots .0]^T$ and $\boldsymbol{x}_a = [\delta_a \;\; \boldsymbol{p}_a^T]^T$ .The advantage of formulating the objective function as LMI is that the resulting problem is convex and the optimal solution, if it exists, can be found. In addition, additional linear equalities and convex quadratic constraints can also be formulated as LMIs, as we shall illustrate in later sections. In order to approximate $a(z)$ with small enough errors using the technique in [9], we found that the length of the denominator of $\hat{a}(z)$ should satisfy the following condition:

$$L_{qa} \geq \lceil grd(a) \rceil + 1, \quad (3\text{-}5)$$

where $grd(y)$ is the passband group delay of the FIR function $y(z)$ and $\lceil w \rceil$ denotes the integer just larger than or equal to *w*. In other words, according to (2-2), we have:

$$L_{q\beta} \geq \lceil N - 0.5 \rceil + 1 \text{ and } L_{q\alpha} \geq \lceil M - N + 0.5 \rceil + 1. \quad (3\text{-}6)$$

(3-5) tells us that the savings of number of multiplications and additions would be more significant if model reduction is applied to FIR functions with lower system delay.

B. — **Peak stopband error constraint**

Denote δ_0 as the prescribed peak stopband ripple to be imposed on the analysis lowpass filter $H_0(z)$. These convex quadratic constraints are given by:

$$|H_0(e^{j\omega})|^2 \leq \delta_0, \; \omega \in [\pi - \omega_{\beta p}, \pi], \quad (3\text{-}7)$$

where $\omega_{\beta p}$ is the passband cutoff frequency of $\beta(z)$. Replacing $\beta(z)$ in (2-1a) with $\hat{\beta}(z)$ given $Q_\beta(z)$, (3-7) can be written as:

$$\delta_0 \geq \gamma_{R,0}^2(\omega) + \gamma_{I,0}^2(\omega), \quad (3\text{-}8)$$

$$\gamma_{R,0}(\omega) = \left|\boldsymbol{p}_\beta^T \cdot \mathrm{Re}\{\boldsymbol{e}_0(\omega)\} + \cos(2N\omega)\right|,$$

where $\gamma_{I,0}(\omega) = \left|\boldsymbol{p}_\beta^T \cdot \mathrm{Im}\{\boldsymbol{e}_0(\omega)\} - \sin(2N\omega)\right|$ and

$$\boldsymbol{e}_0(\omega) = [1, e^{-j2\omega}, \dots, e^{-j2(L_{p\beta}-1)\omega}]^T \cdot e^{-j\omega} / Q_\beta(e^{j2\omega}).$$

Using Schur complement [11], it can be shown that the constraints in (3-8) are equivalent to:

$$\Lambda(\boldsymbol{p}_\beta) = \begin{bmatrix} \delta_0 & \gamma_{R,0}(\omega) & \gamma_{I,0}(\omega) \\ \gamma_{R,0}(\omega) & 1 & 0 \\ \gamma_{I,0}(\omega) & 0 & 1 \end{bmatrix} \succeq 0, \quad (3\text{-}9)$$

Digitizing (3-9), these constraints on the peak ripples can be augmented to the existing LMI in (3-4) for determining $P_\beta(z)$. Similarly, the peak stopband error constraint of $H_1(z)$ can be written as:

$$|H_1(e^{j\omega})|^2 \leq \delta_1, \; \omega \in [0, \omega_{\alpha p}], \quad (3\text{-}10)$$

where δ_1 and $\omega_{\alpha p}$ are respectively the prescribed peak ripple of $H_1(z)$ and passband cutoff frequency of $\alpha(z)$. Similarly, it can be expressed as follows:

$$\Lambda(\boldsymbol{p}_\alpha) = \begin{bmatrix} \delta_1 & \gamma_{R,1}(\omega) & \gamma_{I,1}(\omega) \\ \gamma_{R,1}(\omega) & 1 & 0 \\ \gamma_{I,1}(\omega) & 0 & 1 \end{bmatrix} \succeq 0, \quad (3\text{-}11)$$

$$\gamma_{R,1}(\omega) = \left|\boldsymbol{p}_\alpha^T \cdot \mathrm{Re}\{\boldsymbol{e}_1(\omega)\} - \cos((2M+1)\omega)\right|,$$

where $\gamma_{I,1}(\omega) = \left|\boldsymbol{p}_\alpha^T \cdot \mathrm{Im}\{\boldsymbol{e}_1(\omega)\} + \sin((2M+1)\omega)\right|$ and

$$\boldsymbol{e}_1(\omega) = [1, e^{-j2\omega}, \dots, e^{-j2(L_{p\alpha}-1)\omega}]^T \cdot H_0(e^{j\omega}) / Q_\alpha(e^{j2\omega}).$$

Again, we digitizing (3-11) and augment these constraints on the peak ripples to the existing LMI in (3-4) for finding $P_\alpha(z)$.

C. — ***K*-regularity condition**

To construct a wavelet FB, the analysis filter pair $H_0(z)$ and $H_1(z)$ should posses at least one zero at $\omega = \pi$ and $\omega = 0$, respectively. Let K_0 and K_1 be the number of zeros to be imposed respectively at $\omega = \pi$ and $\omega = 0$ for $H_0(z)$ and $H_1(z)$ with $K_0 \geq K_1 \geq 1$. This is equivalent to:

$$\left[\frac{d^{k_0}}{d\omega^{k_0}} H_0(e^{j\omega})\right]_{\omega=\pi} = \left[\frac{d^{k_1}}{d\omega^{k_1}} H_1(e^{j\omega})\right]_{\omega=0} = 0 \quad (3\text{-}12)$$

for $k_0 = 0, \dots, K_0 - 1$ and $k_1 = 0, \dots, K_1 - 1$. In general, the number of zeros imposed for the analysis filters is closely related to the following halfband filters $H_a(z)$, $a = \alpha$ or β:

$$H_a(z) = [z^{-1} a(z^2) + z^{-2N_a}] / 2 \quad (3\text{-}13)$$

where $N_\beta = N$ and $N_\alpha = M - N + 1$. Obviously, if $H_\beta(z)$ has K_β zeros at $\omega = \pi$, then $H_0(z)$ also has $K_0 = K_\beta$ zeros at $\omega = \pi$. Similarly, $F_0(z) = -H_1(-z)$ can be written as:

$$F_0(z) = -\alpha(z^2)H_0(z) + H_\alpha(z)\cdot z^{-2N+1} \tag{3-14}$$

Again, it can be seen that if $H_\beta(z)$ and $H_\alpha(z)$ have respectively K_β and K_α zeros at $\omega = \pi$, then $F_0(z)$ has at least $K_1 = \min\{K_\beta, K_\alpha\}$ zeros at $\omega = \pi$. As defined in (3-13), the K-regularity condition can be obtained by considering the following form of the halfband filter:

$$H_a(z) = [\widetilde{H}_a(z)]/[2Q_a(z^2)] \tag{3-15}$$

where $\widetilde{H}_a(z) = z^{-1}\sum_{n=0}^{L_{pa}-1} p_a(n)z^{-2n} + z^{-2N_a}\sum_{n=0}^{L_{qa}-1} q_a(n)z^{-2n}$. The conditions for $H_a(z)$ to have K_a zeros at $\omega = \pi$ are equivalent to:

$$\left[\frac{d^{k_a}}{d\omega^{k_a}}\widetilde{H}_a(e^{j\omega})\right]_{\omega=\pi} = 0\ ,\ k_a = 0,\ldots,K_a - 1\ . \tag{3-16}$$

Expanding (3-16) and after slight manipulation, one gets a set of linear equality constraints as follows:

$$\begin{gathered}\sum_{n=0}^{L_{pa}-1}(2n+1)^{k_a} p_a(n) = \sum_{n=0}^{L_{qa}-1}(2n+2N_a)^{k_a} q_a(n)\ ,\\ k_a = 0,\ldots,K_a - 1\ .\end{gathered} \tag{3-17}$$

and its matrix form is given by:

$$\boldsymbol{A}_a \cdot \boldsymbol{p}_a = \boldsymbol{b}_a\ , \tag{3-18}$$

where $[\boldsymbol{b}_a]_{k_a} = \sum_{n=0}^{Lqa-1}(2n+2N_a)^{ka} q_a(n)$ and $[\boldsymbol{A}_a]_{k_a,n} = (2n+1)^{k_a}$. Here, $[\boldsymbol{A}]_{m,n}$ denotes the $(m,n)-th$ entry of matrix $\boldsymbol{A}$. Assume that the number of constraints is smaller than the number of variables, part of the variables, called the redundant variables, can be expressed in terms of the remaining variables, called the independent variables, when solving the SDP. First of all, rewrite (3-18) as follows:

$$\begin{bmatrix}\boldsymbol{A}_{a,L_{pa}-r} & \boldsymbol{A}_{a,r}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{p}_{a,L_{pa}-r}\\ \boldsymbol{p}_{a,r}\end{bmatrix} = \boldsymbol{b}_a\ , \tag{3-19}$$

where $\boldsymbol{A}_a = [\boldsymbol{A}_{a,L_{pa}-r}\quad \boldsymbol{A}_{a,r}]$; $\boldsymbol{p}_a = [\boldsymbol{p}_{a,L_{pa}-r}^T\quad \boldsymbol{p}_{a,r}^T]^T$; and r is the number of redundant variables in $P_a(z)$. Using (3-19), $\boldsymbol{p}_a$ can be written in terms of $\boldsymbol{p}_{a,L_{pa}-r}$ as:

$$\boldsymbol{p}_a = \begin{bmatrix}\boldsymbol{O}_{L_{pa}-r}\\ \boldsymbol{A}_{a,r}^{-1}\boldsymbol{b}_a\end{bmatrix} + \begin{bmatrix}\boldsymbol{I}_{L_{pa}-r}\\ -\boldsymbol{A}_{a,r}^{-1}\boldsymbol{A}_{L_{pa}-r}\end{bmatrix}\boldsymbol{p}_{a,L_{pa}-r}\ , \tag{3-20}$$

where $\boldsymbol{I}_D$ is an ($D \times D$) identity matrix; $\boldsymbol{O}_D$ is a D column zero vector. By substituting (3-20) into (3-3), $\boldsymbol{p}_{a,L_{pa}-r}$ can be found optimally by the SDP, while satisfying the prescribed constraints. Next, we shall summarize the design procedure.

D. — **Design Procedure**

1. Design FIR function $\alpha(z)$.
2. Model-reduce $\alpha(z)$ to an IIR function $\hat{\alpha}(z)$, possibly with constraints as described in section III-B and III-C.
3. Design FIR function $\beta(z)$, given $\hat{\alpha}(z)$.
4. Model-reduce $\beta(z)$ to an IIR function $\hat{\beta}(z)$, possibly with constraints as described in section III-B and III-C.

V. DESIGN EXAMPLES

Example 1: Low-delay two-channel structurally PR IIR FB

For comparison purpose, a two-channel structural PR IIR FB having the same specifications of example 3.1 in [7] is designed. The lengths of $\beta(z)$ and $\alpha(z)$ are 36 and 32, respectively. The delay parameters are $N = 8$ and $M = 23$. The passband cutoff frequencies of $H_0(e^{j\omega})$ and $H_1(e^{j\omega})$ are 0.45π and 0.55π, respectively. $\beta(z)$ and $\alpha(z)$ are FIR functions which are designed using the SDP method [6]. The stopband attenuation of $H_0(e^{j\omega})$ so obtained is 55.5 dB. In [7], model reduction is applied to $\beta(z)$ to obtain $\hat{\beta}(z)$ with $L_{p\beta} = L_{q\beta} = 11$ and $\alpha(z)$ is remained unchanged. It can be seen from the dash-dotted line in figure 2a that the worst-case stopband attenuation of the model-reduced $H_0(e^{j\omega})$ is about 53.5 dB, which is significantly worse than that in the FIR case. $\hat{\beta}(z)$ was then further optimized using the iterative SDP design method in [12] and the stopband attenuation of the resulting IIR analysis lowpass filter is improved to 55.5 dB. For the purposed method, comparable result for $H_0(e^{j\omega})$ is obtained by imposing the peak error constraint to limit the sidelode to 55.3 dB at the stopband and no further optimization is required, thus reducing considerably the design complexity. As for $H_1(e^{j\omega})$, our design gives a stopband attenuation of 55.5 dB, which is 2.7 dB better than that in [7]. This improvement is possibly due to the fact that the $\alpha(z)$ in the method proposed in [7] was designed with respect to the original FIR $\beta(z)$, but not the final IIR approximant $\hat{\beta}(z)$. Since model reduction changes $\beta(z)$, the response of the original FIR $\alpha(z)$ is no longer optimal. This explains why the frequency response of $H_1(e^{j\omega})$ in [7, figure 2b] is no longer equiripple and its stopband attenuation is degraded. Also, Model reducing this $\alpha(z)$ will further degrade the frequency characteristic of $H_1(e^{j\omega})$. Unlike the design procedure in [7], we design $\alpha(z)$ based on $\hat{\beta}(z)$, i.e. the model-reduced version of $\beta(z)$, as described in section III-D. This leads to a considerable better performance of $H_1(e^{j\omega})$ as shown in Table 1. In addition to the improved performance, no further optimizations are required in our approach, unlike the iterative SDP method in [7]. Hence the design complexity of our approach is considerably lower. Next, we shall illustrate how to incorporate prescribed K-regularity constraints into the model-reduced FBs. It should be noted that the possibility of imposing a prescribed K-regularity constraints was not addressed in [7].

Example 2: Low-delay two-channel PR IIR wavelet base

In this example, a two-channel structural PR IIR FB (i.e. $\beta(z)$ and $\alpha(z)$ are chosen as IIR filters) is designed. The design specifications are as follows: The cutoff frequencies of $H_0(z)$ and $H_1(z)$ are 0.48π and 0.52π, respectively. The filter lengths of $\beta(z)$ and $\alpha(z)$ are 60 and 73 respectively. The delay parameters are $N = 18$ and $M = 45$. The stopband attenuations of $H_0(e^{j\omega})$ and $H_1(e^{j\omega})$ of the two-channel PR FIR FB are found to be 43.81 dB and 43.74 dB, respectively. To obtain the IIR FB having similar characteristics as its FIR counterpart, model reduction is applied according to the design procedure described in section III-D. With $L_{p\beta} = L_{q\beta} = 20$ and $L_{p\alpha} = L_{q\alpha} = 30$, 35 multipliers and 35 adders are saved, compared to the FIR case. It can be seen from figure 3a and 3b that the LS solutions, which are showed in dash-dotted lines, exhibit significant ripples near the band edges, which significantly decrease the stopband attenuation. In order to illustrate the flexibility of the proposed method, peak error and K-regularity constraints are imposed during the model reduction of the FIR FB. Using the same specification as above, figure 3 shows the design results of the IIR FB so obtained. It can be seen from the solid line in figure 3a and 3b that the maximum stopband attenuation of the proposed IIR FB is now increased from 40.86 dB to 43 dB for $H_0(e^{j\omega})$, and from 42.56 dB to 44 dB for $H_1(e^{j\omega})$. Also, as depicted in figure 3c, $H_0(z)$ and $H_1(z)$ have two zeros at $\omega = \pi$ and $\omega = 0$, respectively. In exchange for slightly lower performance at the unconstrained frequency bands, both peak error and *K*-regularity constraints are satisfied. This demonstrates the effectiveness of the proposed method and its flexibility in incorporating linear and quadratic inequality constraints. The design results in this example are summarized in table 2.

V. CONCLUSION

A new method for designing two-channel causal stable IIR PR filter banks with prescribed peak error and *K*-regularity constraints is proposed. It is based on the model reduction of the FIR functions in the structural PR filter banks of Phoong et al by a new model reduction technique. The proposed model reduction method retains the denominator of the conventional techniques and formulates the optimal design of the numerator as a semi-definite programming problem. Linear and convex quadratic inequalities such as peak error and *K*-regularity constraints for the IIR filters can be imposed and solved optimally. Design examples show that the proposed method gives better performance, more flexibility in incorporating a wide variety of constraints, and lower design complexity than conventional method.

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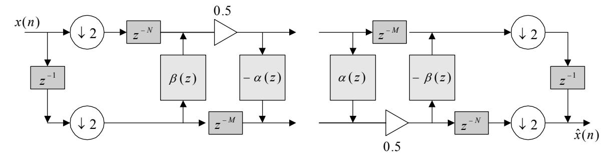

Figure 1: Structurally two-channel PR FB.

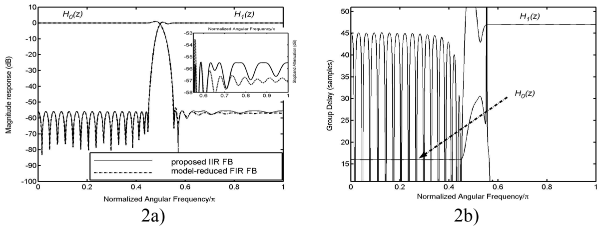

Figure 2. Design results of low-delay IIR FB in example 1 (peak stopband error constraint δ_0 = 55.3 dB). a) Frequency response of analysis filters (Stopband details of $H_0(z)$ in smaller figure): dash-dotted line – model-reduced FIR FB; solid line – proposed IIR FB. b) Group delay response of proposed IIR FB.

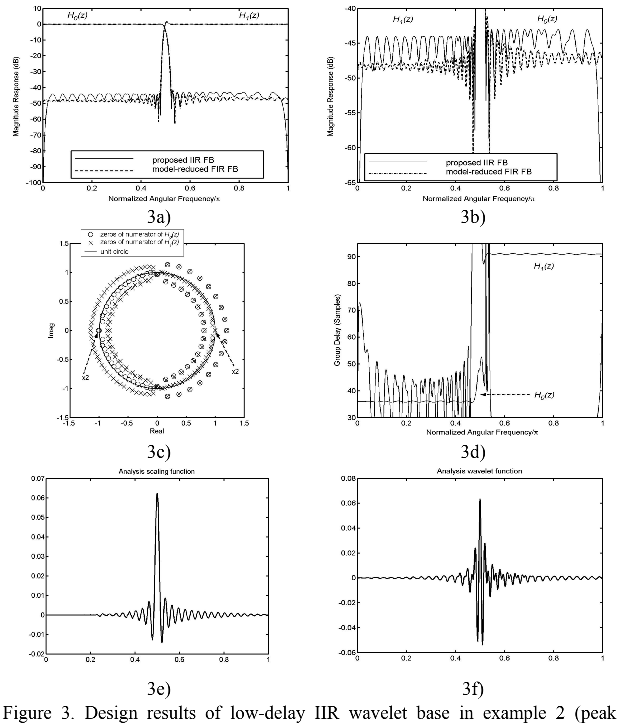

Figure 3. Design results of low-delay IIR wavelet base in example 2 (peak stopband error constraint: δ_0 = 53 dB and δ_1 = 54 dB; *K*-regularity constraint: K_0 = K_1 = 2): a) and b) Frequency response and stopband details of analysis filters: dash-dotted line – model-reduced FIR FB; solid line – proposed IIR FB. c) – f) Pole-zero plot, group delay response, analysis scaling function and analysis wavelet function of the proposed IIR FB..

	$\Delta\omega$	δ_{s0}	δ_{s1}	Mult.	Add.	Design Complexity
[7]	0.1π	55.5	52.8	53	51	model reduction (SDP-based)
Proposed approach	0.1π	55.3	55.5	53	51	model reduction, followed by iterative SDP

Table 1: Performance comparisons with [7] in example 1. $\Delta\omega$: transition bandwidth; $\delta_{s,m}$: stopband attenuation of $H_m(z)$, m = 0,1; Mult.: number of multipliers; Add.: number of adders.

	$\Delta\omega$	$\delta_{p,0}$	$\delta_{s,0}$	$\delta_{p,1}$	$\delta_{s,1}$	K_0, K_1	Mult.	Add.
FIR [6]	0.04π	0.055	43.81	0.055	43.74	0,0	133	131
Model reduction	0.04π	0.044	40.86	0.044	42.56	N/A	98	96
Proposed approach	0.04π	0.057	43.00	0.056	44.00	2,2	98	96

Table 2: Summary of design results in example 2. $\Delta\omega$: transition bandwidth; $\delta_{p,m}$: passband deviation of $H_m(z)$, m = 0,1; $\delta_{s,m}$: stopband attenuation of $H_m(z)$, m = 0,1; K_m: number of zeros of $H_m(z)$, m = 0,1; Mult.: number of multipliers; Add.: number of adders.