

# A ROBUST NARROWBAND ACTIVE NOISE CONTROL SYSTEM FOR ACCOMMODATING FREQUENCY MISMATCH

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## ABSTRACT

Narrowband active noise control (ANC) systems have many real-life applications where the noise signals generated by rotating machines are modeled as sinusoidal signals in additive noise. However, when the timing signal sensor, such as a tachometer that is used to extract the signal frequencies, and the cosine wave generator contain errors, the reference signal frequencies fed to each ANC channel will then be different from the noise signal true frequencies. This difference is referred to as frequency mismatch (FM). In this paper, through extensive simulations we demonstrate that the performance capabilities of a conventional narrowband ANC system using the filtered-X LMS (FXLMS) algorithm degrades significantly even for an FM as small as 1%. Next, we propose a new narrowband ANC system that will successfully compensate for the performance degradations due to FM. The amplitude/phase adjustment and the FM mitigations are performed simultaneously in a harmonic fashion such that the influence of the FM can be removed almost completely. Simulation results are provided to demonstrate the effectiveness of the proposed new system.

## 1. INTRODUCTION

In many real-life environments there may exist various unwanted sounds or noise signals that are harmful to the people working and/or living there. In many cases, the noise signals are generated by rotating machines, such as engines, cutting machines, fans, etc. and may be modeled as sinusoidal signals in additive noise. Usually, the frequencies of the noise signals are unknown, and their magnitudes are time-varying. Removing or reducing these noise signals, especially the lower frequency portion, is very important in various environmental and engineering systems. For example, large-scale cutting machines used in factories generate such noise signals which are harmful to their operators. As the environmental noise level regulations are made stricter, effective measures need to be developed to reduce the noise level. Active noise control (ANC) systems have been utilized in reducing these annoying noise signals, and research in this area has been carried out since the early 1970s [1]-[5].

In an ANC system, an unwanted noise signal in an acoustic region is suppressed by superimposing an equal-but-opposite phase source in the region. A sensor is placed to measure the noise source (reference signal) in real time, and a new sound signal made from the measurement and an adjustable filter/variable structure is sent to the region for the noise to be suppressed. An additional sensor is placed in the quieted region to measure an error feedback to control the filter properties. For most cases, finite-impulse-response (FIR) filters adapted using a filtered-X least mean square (FXLMS) algorithm are used as the variable structure [3]. Other techniques using recursive least squares (RLS) and Kalman filtering based algorithms have also been developed for many ANC systems [6, 3], that generally provide better noise reduction performance at the expense of more computational cost.

The conventional narrowband ANC systems are found to be effective in suppressing sinusoidal noise in many real-life applications [3] (see Fig. 1 for the block diagram of a typical ANC system considered in this work). However, in real applications, certain errors do occur with the timing signal sensor and/or the cosine wave generator. The timing signal sensor such as a tachometer is used to extract the rotational speed that is often linearly related to the signal frequencies. The so-called synchronization signal may be either the rotational speed or the signal frequencies. Invariably, in practice sensor errors do exist. A cosine wave generator produces the reference sinusoid signals for the ANC system using the synchronization signal or signal frequencies provided by the timing signal sensor. This generator is a piece of electronic hardware and thus inevitably is prone to some sort of error. If the above errors are negligible or sufficiently small, conventional narrowband ANC systems would work effectively. However, systems are not robust to these types of errors and may perform poorly in practice. Our extensive simulations have revealed that even with 1% of FM resulting from these errors one could render the system totally useless.

In this paper, a new robust narrowband ANC system is proposed that will effectively compensate for the performance degradations that are possible in conventional ANC system due to the FM. An amplitude/phase adjustment using a two-weight FIR filter and an AR model based FM mitigation filter are proposed simul-

taneously. Simulation results demonstrate the excellent performance capabilities of the proposed system in compensating FM of even as much as 10%.

## 2. THE LIMITATIONS OF A CONVENTIONAL NARROWBAND ANC SYSTEM

A typical conventional narrowband ANC system [3] is shown in Fig. 1. The noise signal is given by

$$d(n) = \sum_{i=1}^q \{a_{p,i} \cos(\omega_{p,i}n) + b_{p,i} \sin(\omega_{p,i}n)\} + v(n) \quad (1)$$

where  $q$  is the number of frequency components of the sinusoidal signal,  $\omega_{p,i}$  is the frequency of the  $i$ -th component,  $v(n)$  is a zero-mean additive white Gaussian noise with variance  $\sigma_v^2$ . The  $i$ -th reference sinusoid is given by

$$x_i(n) = a_i \cos(\omega_i n) + b_i \sin(\omega_i n) \quad (2)$$

The output of the  $i$ -th channel is expressed by

$$y_i(n) = h_{i,0}x_i(n) + h_{i,1}x_i(n-1) \quad (3)$$

The block  $S(z)$  corresponds to the secondary-path (or error-path) and is considered as an FIR lowpass filter with coefficients  $\{f_j\}_{j=0}^M$  which is assumed to be known *a priori*. The FXLMS algorithm is designed to update the two weights  $\{h_{i,0}, h_{i,1}\}$  corresponding to the Magnitude/Phase Adjuster (MPA) [5, 7] and is given by

$$h_{i,0}(n+1) = h_{i,0}(n) + \mu_i e(n) x_{i,s}(n), \quad (4)$$

$$h_{i,1}(n+1) = h_{i,1}(n) + \mu_i e(n) x_{i,s}(n-1) \quad (5)$$

where

$$x_{i,s}(n) = \sum_{j=0}^{M-1} \hat{f}_j x_i(n-j) \quad (6)$$

$$e(n) = \sum_{j=0}^{M-1} f_j \left( d(n-j) - \sum_{i=1}^q y_i(n-j) \right) \quad (7)$$

and  $\hat{f}_j$  is assumed to be known as an estimate of  $f_j$ .

If the FM is zero ( $\Delta\omega_i = \omega_i - \omega_{p,i} = 0, i = 1, \dots, q$ ), that is if the timing signal sensor (Sync signal) as well as the cosine wave generator have no or sufficiently small errors, then the conventional narrowband ANC system of Fig. 1 using the FXLMS algorithm provides a good performance. Unfortunately, these assumptions are hardly satisfied in real applications. A typical simulation result for an ideal case is depicted in Fig. 2 (b). It is easy to see that the system is effective in reducing the noise level. The simulation result for a case with 1% FM ( $:= |\Delta\omega_i|/\omega_{p,i} \times 100\%$ ) is given in Fig. 2 (c). Clearly, the ANC system is completely ineffective, which implies the sensitivity of the system to presence of FM or sensor and generator errors. Corrective measures have to be taken in order to prevent such a drastic degradations in the performance. In all the simulation results presented in this work, an FIR lowpass filter of  $M^{th}$  order and a cutoff frequency  $0.4\pi$  is generated by the MATLAB function and is used as the error-path.

## 3. THE NEW ROBUST NARROWBAND ANC SYSTEM

To mitigate the influence of the FM, one needs to provide the MPA cells with reference sinusoids whose frequencies are as close as possible to those of the primary noise signal  $d(n)$ . Towards this end, in this paper we propose to use a software-based sinusoid signal generator to replace the conventional hardware-based cosine wave generator. Of course, the information from the timing signal sensor such as the rotation speed has to be mapped to the signal frequencies in a certain way. The regressive relationship, usually linear, between the rotation speed and the signal frequencies may be used for this purpose.

Recall that the  $i$ -th reference sinusoid obeys the following AR relation [8]

$$x_i(n) = -c_i(n)x_i(n-1) - x_i(n-2), \quad n \geq 2 \quad (8)$$

$$x_i(0) = a_i, \quad x_i(1) = a_i \cos(\omega_i(0)) + b_i \sin(\omega_i(0))$$

where  $c_i(n)$  is a frequency-related coefficient whose initial value is defined as  $-2\cos(\omega_i(0))$  and  $\omega_i(0)$  is the signal frequency obtained from the timing signal sensor. When the distance between the frequency derived from  $c_i(n)$ , i.e.,  $\arccos(-c_i(n)/2)$ , and that ( $\omega_i(n)$ ) obtained from the sensor is above certain threshold,  $c_i(n)$  may be refreshed to  $-2\cos(\omega_i(n))$ . Practical consideration is needed to determine a proper threshold for the above refreshment of  $c_i(n)$  for any given application.

The new narrowband ANC system is depicted in Fig. 3. The update of the MPA weights and the coefficient  $c_i(n)$  may be done simultaneously by minimizing the ANC output error  $e(n)$ . The recursions for the MPA weights are the same as those given in (4) and (5). The frequency-related coefficient may be updated using the following expression

$$\begin{aligned} \frac{\partial e^2(n)}{\partial c_i(n)} &= -2e(n) \frac{\partial y_i(n)}{\partial c_i(n)} \\ &\rightarrow 2e(n) \hat{h}_{i,0}(n) x_{i,s}(n-1) \end{aligned} \quad (9)$$

as

$$c_i(n+1) = c_i(n) - \mu_{c_i} e(n) \hat{h}_{i,0}(n) x_{i,s}(n-1) \quad (10)$$

The simulation results given for the new ANC system is shown in Fig. 4, where the simulation conditions are identical to those in Fig. 2, except that the step size parameters for the frequency related coefficients are newly set. The mean convergence of  $c_1(n)$  is shown in Fig. 5. It is clear that the new ANC system can effectively compensate for the performance deteriorations that are due to the FM, resulting in more robustness that is required in real-life applications.

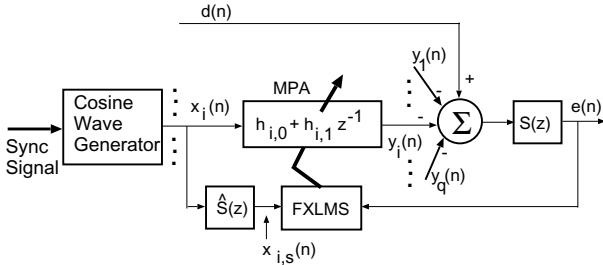
Moreover, to illustrate the strength of the proposed scheme simulation results for an FM of 10% are also provided in Fig. 6. It can be observed clearly that the new ANC system is performing excellently in suppressing the FM. The same claim can be made for cases with severe additive noise and/or much longer error-path (to as long as  $M = 64 \sim 100$ ). This implies that the new system is sufficiently robust for potential implementation in real applications.

#### 4. CONCLUSIONS

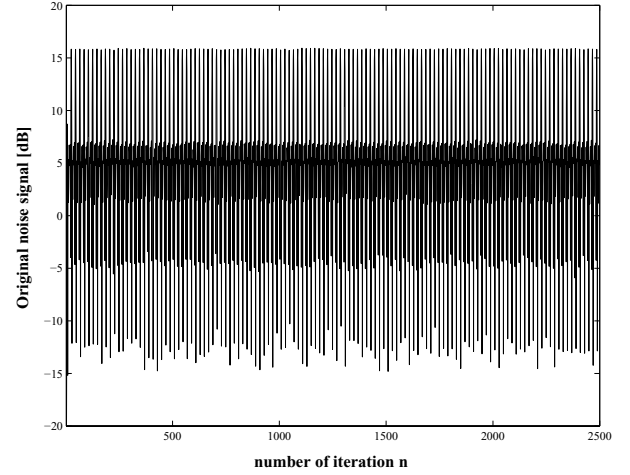
In this work a new robust narrowband ANC system is proposed. An AR-based filter is introduced in the proposed ANC system to generate reference sinusoids to be fed to the MPA cells, thus providing an opportunity for reducing the hostile FM that embodies the error of the timing signal sensor. The FM is reduced by an LMS-like algorithm, which preserves the cost-efficiency of the conventional ANC system. Furthermore, the proposed ANC system no longer needs the cosine wave generator used in the conventional system. Nevertheless, the new ANC system provides excellent robustness against the sensor error. Extensive simulations are conducted to demonstrate that the proposed system is powerful enough to suppress an FM as much as 10%. The only sacrifice that one needs to make is the added delay in the convergence of the adaptive process. DSP-based implementation and performance analysis of the proposed system are topics for further research.

#### REFERENCES

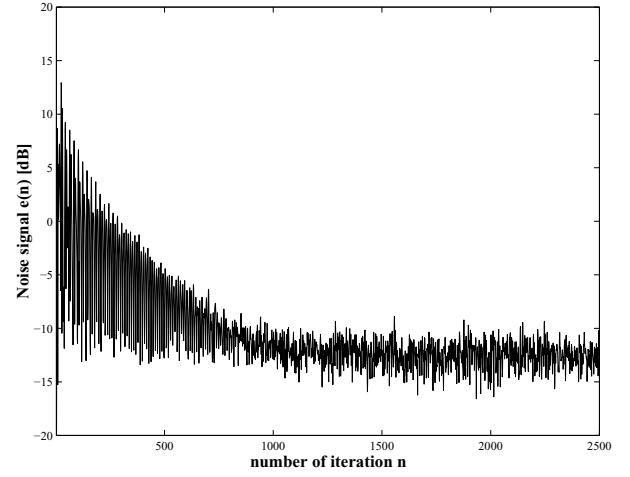
- [1] B. Widrow et al., "Adaptive noise canceling: Principles and applications," Proc. IEEE, vol. 63, pp. 1692-1716, Dec. 1975.
- [2] P. A. Nelson and S. J. Elliott, *Active Control of Sound*, Academic Press INC., 3rd ed., 1995.
- [3] S. M. Kuo and D. R. Morgan, *Active Noise Control Systems, Algorithms and DSP Implementations*, John Wiley & Sons, INC., 1996.
- [4] L. J. Eriksson, "Development of the filtered-U algorithm for active noise control," J. Acoust. Soc. Am., 89(1), pp. 257-265, Jan. 1991.
- [5] D. R. Hush, N. Ahmed, R. David, and A. D. Stearns, "An adaptive IIR structure for sinusoidal enhancement, frequency estimation, and detection," IEEE Trans. Acoust., Speech, Signal Processing, vol. 34, no. 6, pp. 1380-1390, Dec. 1986.
- [6] P. A. C. Lopes and M. S. Piedade, "A Kalman filter approach to active noise control," in Proc. EUSIPCO 2000, Finland, 2000.
- [7] Y. Xiao, N. Tani, A. Ikuta, "Performance analysis of an LMS magnitude/phase adjuster for an active noise controller," Proc. of 8th Intl. Congress on Sound & Vibration, pp. 245-252, Sept 2001.
- [8] Y. Xiao, R. Ward, L. Ma, L. Xu, "A new LMS-based Fourier analyzer in the presence of frequency mismatch," Proc. of IEEE ISCAS2003, vol. IV, pp. 369-372, May 2003.



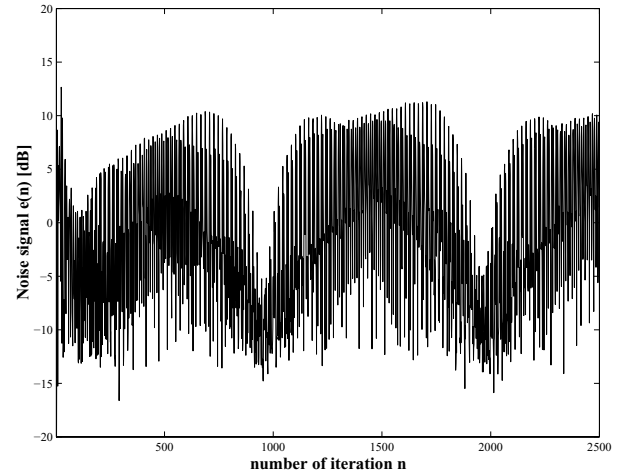
**Fig. 1** The conventional narrowband ANC system ( $i$ -th channel).



(a) The original noise signal level

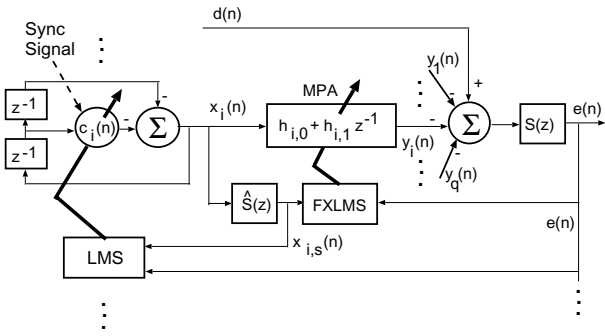


(b) The output signal without FM

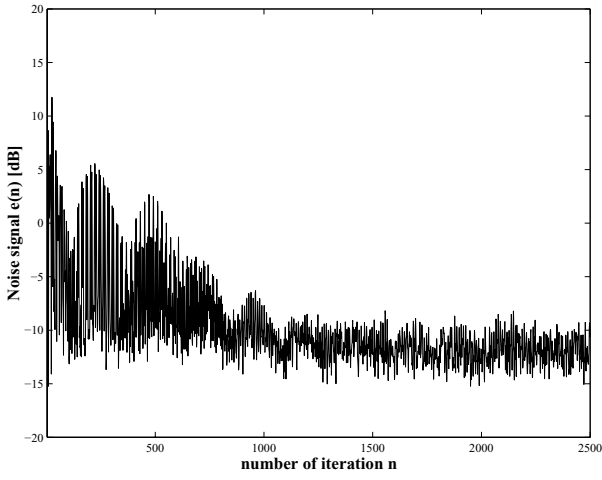


(c) The output signal with 1% FM

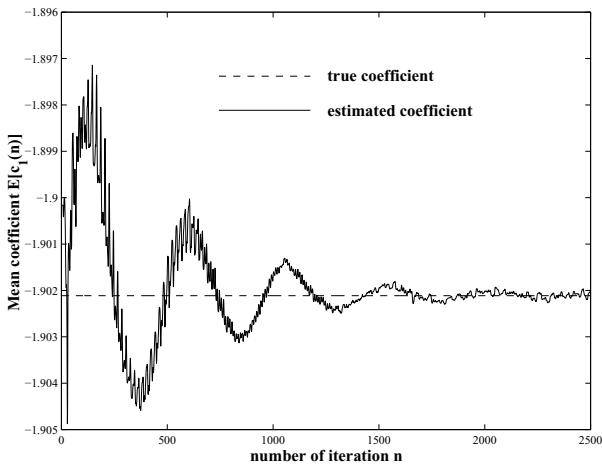
**Fig. 2** Levels of original noise signal and output noise signals of the conventional ANC system (true signal frequency:  $\Omega_0 = [0.1\pi, 0.2\pi, 0.3\pi]^T$ , reference signal frequency:  $\Omega(1) = [0.101\pi, 0.202\pi, 0.303\pi]^T$ ,  $a_{p,1} = 3, b_{p,1} = 1.0, a_{p,2} = 2, b_{p,2} = 1.0, a_{p,3} = b_{p,3} = 1.0, \mu_1 = \mu_2 = \mu_3 = 0.025$ , error-path order  $M = 11, \hat{f}_j = f_j$  ( $j = 0, \dots, M-1$ ),  $\sigma_v = 0.33$ , 40 runs).



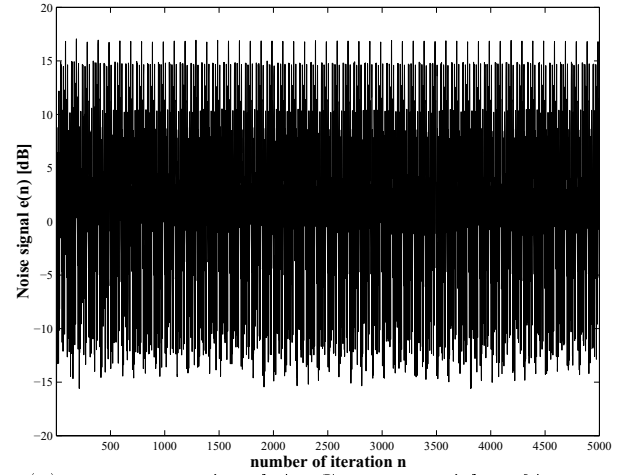
**Fig. 3** The proposed robust narrowband ANC system (  $i$ -th channel).



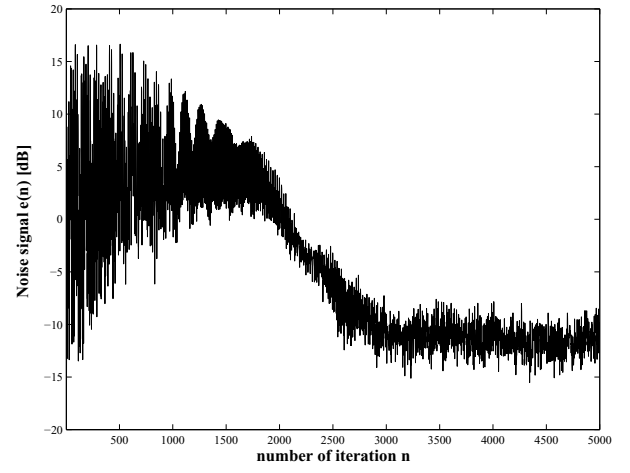
**Fig. 4** Output noise (error) signal produced by the proposed robust narrowband ANC, where  $\mu_{c_i} = 0.0005$  ( $i = 1, 2, 3$ ), with other conditions identical as in Fig. 2.



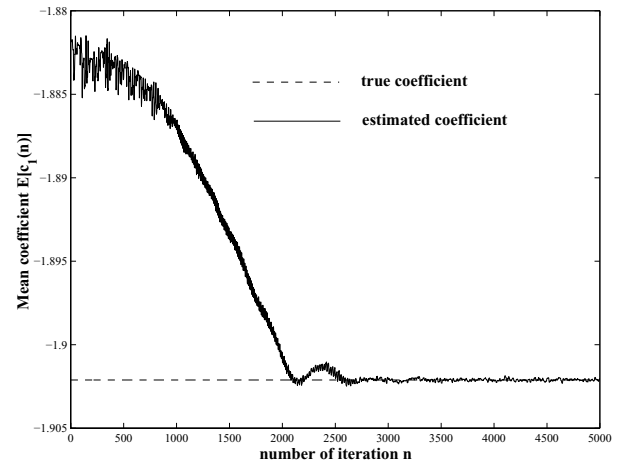
**Fig. 5** Mean convergence of  $c_1(n)$  of the proposed robust narrowband ANC for the simulations in Fig. 4.



(a) The conventional ANC system with 10% FM



(b) The proposed ANC system with 10% FM



(c) The mean of frequency-related coefficient  $c_1(n)$

**Fig. 6** Levels of output noise signals for the conventional and the proposed ANC systems, and the mean convergence of a frequency related coefficient (true signal frequency:  $\Omega_0 = [0.1\pi, 0.2\pi, 0.3\pi]^T$ , reference signal frequency:  $\Omega(1) = [0.11\pi, 0.22\pi, 0.33\pi]^T$ ,  $a_{p,1} = 3, b_{p,1} = 1.0$ ,  $a_{p,2} = 2, b_{p,2} = 1.0$ ,  $a_{p,3} = b_{p,3} = 1.0$ ,  $\mu_1 = \mu_2 = \mu_3 = 0.05$ , error-path order  $M = 11$  ( $f_j = f_j$  ( $j = 0, \dots, M-1$ )),  $\mu_{c_1} = 0.0001$ ,  $\mu_{c_2} = 0.00025$ ,  $\mu_{c_3} = 0.0005$ ,  $\sigma_v = 0.33$ , 40 runs).