EFFICIENT MULTISTAGE COMB-MODIFIED ROTATED SINC (RS) DECIMATOR

Gordana Jovanovic-Dolecek[1] and *Sanjit K. Mitra*[2]

[1] INAOE, P. O. Box 51&216, Puebla, Pue., Mexico

[2] University of California, Santa Barbara, CA 93106-950

Tel. & Fax: +52 222 2470517, (805) 893 8312, E-mail: gordana@inaoep.mx, mitra@ece.ucsb.edu

ABSTRACT

This paper presents a new multistage comb-rotated sinc (RS) decimator. The proposed structure consists of different cascaded comb sections, each down-sampled by a specific down-sampling factor. The number of sections depends on the decimation factor of the original comb decimator. The first section is realized in a non-recursive form. Using the polyphase decomposition, the sub-filters of the first section can be operated at lower rate which depends on the down-sampling factor of the first section. Additionally, the rotated sinc (RS) filter is cascaded in the second section, thus permitting both multipliers of the RS filter to work at the lower rate. The magnitude response of the proposed structure is better than that of the original comb decimator.

1. INTRODUCTION

In many communication and signal processing systems, it is necessary to isolate a very narrowband signal from a very wideband signal, referred to as sub-band tuning [1]. The decimation filter is the key component required to provide an efficient all-digital sub-band tuner systems [1].

A commonly used decimation filter is the cascaded integrator comb (CIC) filter [2], consisting of an integrator section (first stage) and a differentiator section (second stage) separated by a down-sampler with a down-sampling factor M. Each of the main sections is a cascade of K identical filters as shown in Figure 1. The transfer function of the resulting decimation filter is given by

$$H(z)=\left[\frac{1}{M}\left(\frac{1-z^{-M}}{1-z^{-1}}\right)\right]^{K}. \qquad (1)$$

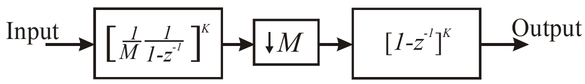


Figure 1: CIC decimation filter

The above decimation filter is attractive in many applications because of its very low complexity. It should be noted that while the differentiator section operates at the lower data rate, the integrator section works at the higher input data rate resulting in a larger chip area and a higher power consumption especially when the decimation factor and the filter order are high [3]. The use of a non-recursive equivalent to Eq. (1) reduces power consumption and increases the circuit speed [3-5]. More details on a comparison of the performances of the recursive and non-recursive implementation are given in [3]. In this paper we propose a new multistage structure in which the first stage is implemented non-recursively while all other stages are implemented recursively. The magnitude response of this structure is improved over that of the original comb filter by using a modified rotated sinc (RS) filter introduced in [6]. Unlike the structure advanced in [6], where one multiplier works at the high input rate, in the structure proposed in this paper, both multipliers work at the lower rate.

The paper is organized as follows. In Section 2 we first define the modified multistage comb filter, and in Section 3 we propose modified RS filter. An efficient multistage decimation structure is described in Section 4.

2. THE MULTISTAGE COMB FILTER

Considering the case when the down-sampling factor can be expressed as

$$M = M_1 M_2 M_3 ... M_N \qquad (2)$$

we can rewrite Eq. (1) as

$$H(z) = \left[\prod_{i=0}^{N-1} H_i(z) \right]^K \tag{3}$$

where

$$H_i(z) = \frac{1}{M_{N-i}} \left(\frac{1 - z^{-\prod_{j=0}^{N-i} M_j}}{1 - z^{-\prod_{j=0}^{N-(i+1)} M_j}} \right); \; M_0 = 1 . \tag{4}$$

For example, for $M = 32$ and $N = 3$, we can select

$$M_1 = 4, M_2 = 4, M_3 = 2, \tag{5}$$

yielding

$$H_0(z) = \frac{1}{2}\left(\frac{1 - z^{-32}}{1 - z^{-16}} \right), \; H_1(z) = \frac{1}{4}\left(\frac{1 - z^{-16}}{1 - z^{-4}} \right)$$

$$H_2(z) = \frac{1}{4}\left(\frac{1 - z^{-4}}{1 - z^{-1}} \right). \tag{6}$$

Using Eqs. (3) and (4) we express the modified comb filter $H_m(z)$ as,

$$H_m(z) = H_0^{K_0}(z) H_1^{K_1}(z) \cdots H_{N-1}^{K_{N-1}}(z). \tag{7}$$

For

$$H_k(z) = \left[\frac{1}{L_1} \left(\frac{1 - z^{L_2}}{1 - z^{L_3}} \right) \right]^{L_4}, \; k = 0, \ldots, N-1, \tag{8}$$

the corresponding magnitude response is

$$\left| H_k(e^{j\omega}) \right| = \left| \left[\frac{1}{L_1} \left(\frac{\sin(\omega L_2 / 2)}{\sin(\omega L_2 / 2)} \right) \right]^{L_4} \right|, \tag{9}$$

where $L_i, i = 1, \ldots, 4$, are integers. For $M = 32$ from Eq. (1) using Eq. (9) we arrive at

$$\left| H(e^{j\omega}) \right| = \left| \left[\frac{1}{32} \left(\frac{\sin(16\omega)}{\sin(\omega/2)} \right) \right]^K \right|. \tag{10}$$

We get from Eq. (7) using Eq. (5)

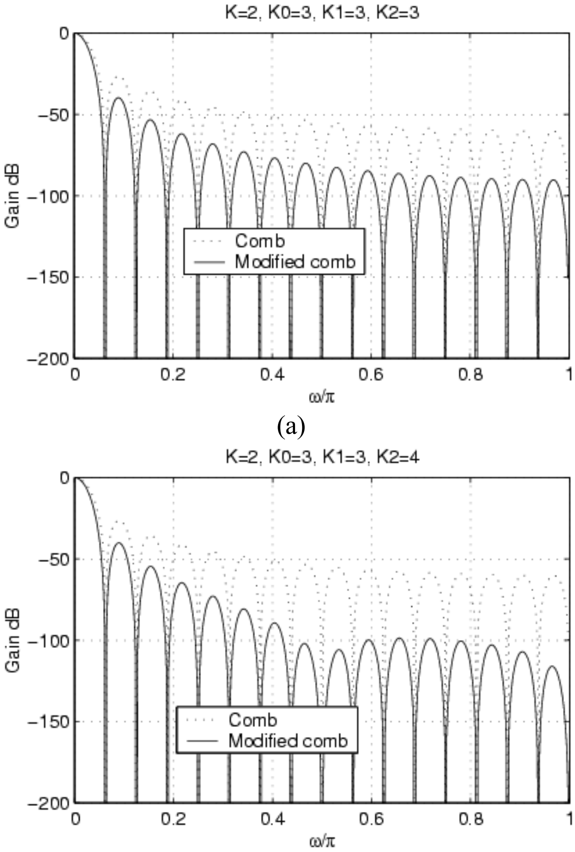
$$\left| H_m(e^{j\omega}) \right| =$$

$$\left| \left[\frac{1}{2} \left(\frac{\sin(16\omega)}{\sin(8\omega)} \right) \right]^{K_0} \left[\frac{1}{4} \left(\frac{\sin(8\omega)}{\sin(2\omega)} \right) \right]^{K_1} \left[\frac{1}{4} \left(\frac{\sin(2\omega}{\sin(\omega/2)} \right) \right]^{K_2} \right| \tag{11}$$

Figure 2 shows the magnitude responses given in Eqs. (10) and (11) for different values of K_0, K_1, and K_2. It should be noted that by using different values for the number of factors K_i in each stage, the magnitude characteristic of the new filter can be improved over that of the original comb decimator. We exploit this fact in the structure proposed in the next two sections.

3. THE MODIFIED RS FILTER

The RS filter is proposed in [6] increases the stopband attenuation of the comb filter without increasing too much the computational complexity. By applying a clockwise rotation of α radians to any zero of Eq. (1), we obtain the following transfer function [6]



(a)

(b)

Figure 2: Gain responses of the comb and the modified comb filter.

$$H_u(z) = \frac{1}{M} \frac{1 - z^{-M} e^{j\alpha M}}{1 - z^{-1} e^{j\alpha}} . \tag{12}$$

An equivalent expression is obtained by applying the opposite rotation

$$H_d(z) = \frac{1}{M} \left(\frac{1 - z^{-M} e^{-j\alpha M}}{1 - z^{-1} e^{-j\alpha}} \right). \tag{13}$$

These two filters have complex coefficients, but they can be cascaded, thus obtaining a filter $H_r(z)$ with real coefficients

$$H_r(z) = H_u(z) H_d(z)$$

$$= \frac{1}{M^2} \left(\frac{1 - 2\cos(\alpha M) z^{-M} + z^{-2M}}{1 - 2\cos(\alpha) z^{-1} + z^{-2}} \right). \tag{14}$$

The cascade of three filters given in Eqs. (1) and (14) is referred in [6] as the RS filter $H_R(z)$:

$$H_R(z) = H(z) H_r(z). \tag{15}$$

The magnitude response of this filter is as follows

$$\left|H_R(e^{j\omega})\right| = \frac{1}{M^3}\left|\frac{\sin(\omega M/2)}{\sin(\omega/2)}\right| \cdot$$
$$\left|\frac{\sin((\omega+\alpha)M/2)}{\sin((\omega+\alpha)/2)}\frac{\sin((\omega-\alpha)M/2)}{\sin((\omega-\alpha)/2)}\right|. \qquad (16)$$

We modify Eq. (14) as indicated in Eq. (17):

$$H_{rm}(z) = H_{um}(z)H_{dm}(z)$$
$$= \frac{1}{(M/M_1)^2}\left(\frac{1-2\cos(\alpha M)z^{-M}+z^{-2M}}{1-2\cos(\alpha M_1)z^{-M_1}+z^{-2M_1}}\right), \qquad (17)$$

where M_1 is a factor of M as indicated in Eq. (2). As a result, $H_{rm}(z)$ can be moved to the lower rate which is M_1 times smaller than the high input rate.

The corresponding modified RS filter is obtained as the cascade of the modified comb filter Eq. (7) and the modified rotated filter Eq. (17).

$$H_{Rm}(z) = H_m(z)H_{rm}(z)\,. \qquad (18)$$

The corresponding magnitude response is

$$\left|H_{Rm}(e^{j\omega})\right| = \left|H_m(e^{j\omega})\right|$$
$$\left(\frac{M_1^2}{M^2}\frac{\sin((\omega+\alpha)M/2)}{\sin((\omega+\alpha)M_1/2)}\frac{\sin((\omega-\alpha)M/2)}{\sin((\omega-\alpha)M_1/2)}\right). \qquad (19)$$

We illustrate the procedure for $M = 32$. Using Eqs. (5), (11), and (19), we have

$$\left|H_{Rm}(e^{j\omega})\right| = \left|\frac{1}{2}\left(\frac{\sin(16\omega)}{\sin(8\omega)}\right)\right|^{K_0} \cdot$$
$$\left|\left[\frac{1}{4}\left(\frac{\sin(8\omega)}{\sin(2\omega)}\right)\right]^{K_1}\left[\frac{1}{4}\left(\frac{\sin(2\omega)}{\sin(\omega/2)}\right)\right]^{K_2}\right| \cdot$$
$$\frac{1}{8^2}\left|\left(\frac{\sin((\omega+\alpha)16)}{\sin((\omega+\alpha)2)}\right)\left(\frac{\sin((\omega-\alpha)16)}{\sin((\omega-\alpha)2)}\right)\right|. \qquad (20)$$

The plots $H_R(\omega)$ and $H_{Rm}(\omega)$ for α = 0.0184 and $K = K_1 = K_2 = 1$, $K_0 = 2$ are shown in Figure 3(a), whereas the corresponding plots for $K = 1$, $K_1 = K_2 = 2$, $K_0 = 4$ are shown in Figure 3(b). It should be noted that any desired stopband attenuation can be attained by choosing appropriate values of K_i and α in Eq. (20).

4. EFFICIENT REALIZATIONS

Using Eq. (7) and the cascade equivalence [7] we construct the multistage structure of the modified comb filter as shown in Figure 4(a). The structure is composed of cascaded comb filters separated by their corresponding down-samplers. As shown in [4], the maximum decimation factor in the first stage can considerably improve the power consumption area. We use an approach similar to that given in [4]. The cascade of K_{N-1} comb filters of length M_1 has a transfer function given by

$$H_{N-1}^{K_{N-1}}(z) = \left(\frac{1}{M_1}\right)^{K_{N-1}} H'_{N-1}(z) \qquad (21)$$

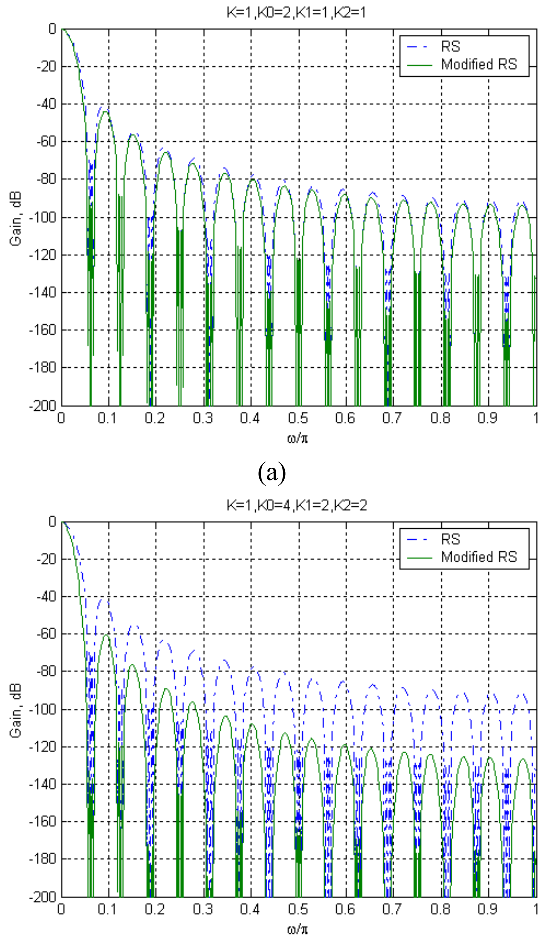
where

$$H'_{N-1}(z) = \sum_{n=0}^{K_{N-1}(M_1-1)} h(n)z^{-n}\,. \qquad (22)$$

The coefficients of this filter are integers and are symmetric [4]. A polyphase decomposition of the transfer function of Eq. (22) is given by

$$H'_{N-1}(z) = E_0(z^{M_1}) + \ldots + z^{-(M_1-1)}E_{M_1-1}(z^{M_1}) \qquad (23)$$

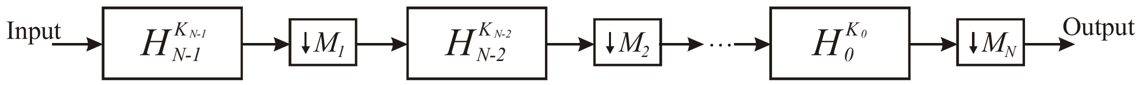
where $E_s(z_1^{M_1}), s = 0,\ldots,M_1-1$, denote the M_1 polyphase components. Using the cascade equivalence [7], the down-sampler can be placed before filtering. As a result, the polyphase filters in the first section are moved to a lower rate, which is M_1 times lower than the input rate.



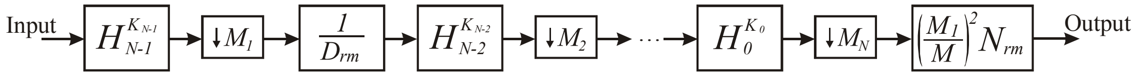
(a)

(b)

Figure 3: Gain responses of the RS and modified RS filters.



(a) Modified comb filter structure



(b) Modified comb-RS structure

Figure 4: Efficient realizations.

From (17) we have

$$H_{rm}(z) = \frac{1}{(M/M_1)^2} N_{rm}(z^M) / D_{rm}(z^{M_1}) \quad (24)$$

where

$$N_{rm}(z^M) = \frac{1}{(M/M_1)^2}(1 - 2\cos(\alpha M) z^{-M} + z^{-2M}. \quad (25)$$

$$D_{rm}(z^{M_1}) = \frac{1}{1 - 2\cos(\alpha M_1) z^{-M_1} + z^{-2M_1}}. \quad (26)$$

As a result, the denominator $D_{rm}(z^{M_1})$ of $H_{rm}(z)$ can be moved to the second stage, i.e. after the factor-of-M_1 downsampler. Similarly, the numerator $N_{rm}(z^M)$ can be moved after the factor-of-M_N downsampler. A more efficient realization of the comb-modified RS filter is thus as shown in Figure 4(b).

5. CONCLUDING REMARKS

This paper has presented a new efficient multistage structure for comb-RS decimation filter. First stage is realized in a nonrecursive form. Applying the polyphase decomposition the polyphase filters in the first section are moved to a lower rate, which is M_1 times lower than the high input rate. The magnitude response of this structure is improved over that of the original comb filter using a modified rotated sinc (RS) filter. Unlike the structure proposed in [6], where one multiplier works at the high input rate, in the structure proposed in this paper, both multipliers work at lower rate. Besides, as the examples included have illustrated, we obtain an improved magnitude characteristic compared to that of the original RS filter [6] saving a low complexity of the comb filter.

REFERENCES

[1] A. Kwentus, Z. Jiang, and A. Willson, Jr., "Application of filter sharpening to cascaded integrator-comb decimation filters," *IEEE Trans. on Signal Processing*, vol. 45, pp. 457-467, February 1997.

[2] E. B. Hogenauer, "An economical class of digital filters for decimation and interpolation," *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. ASSP-29, No.2, pp.155-162, April 1981.

[3] Y. Gao, L. Jia, J. Isoaho and H. Tenhunen, "A comparison design of comb decimators for sigma-delta analog-to-digital converters," *Analog Integrated Circuits and Signal Processing*, vol. 22, pp. 51-60, 1999.

[4] H. Aboushady, Y. Dumonteix, M. M. Loerat, and H. Mehrezz, "Efficient polyphase decomposition of comb decimation filters in Σ-Δ analog-to-digital converters," *IEEE Trans. on Circuits & Systems – II: Analog and Digital Signal Processing*, vol. 48, pp. 898-903, October 2001.

[5] F. Daneshgaran and M. Laddomada, "A novel class of decimation filters for ΣΔ A/D converters," *Wireless Communications and Mobile Computing* , vol. 2, No. 8, pp.867-882, December 2002.

[6] L. Lo Presti, "Efficient modified-sinc filters for sigma-delta A/D converters," *IEEE Trans. on Circuits and Systems-II: Analog and Digital Signal Processing*, vol.47, N0.11, pp. 1204-1213, November 2000.

[7] S. K. Mitra, *Digital Signal Processing: A Computer-Based Approach*, Second edition, McGraw-Hill, 2001.