NONLINEAR ACTIVE NOISE CONTROL

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ABSTRACT

In this paper a brief review of recent contributions on nonlinear methods for active noise control is presented. Models based on polynomial filters are considered in some detail and an account on novel techniques used to adapt multichannel nonlinear controllers is given. Possible future research lines are prospected, too.

1. INTRODUCTION

Methods for active noise control (ANC) are nowadays intensively studied and have already provided promising applications in vibration and acoustic noise control tasks. The initial activities originated in the field of control engineering [1] while in recent years a signal processing approach has been succesfully applied [2, 3]. This approach strongly benefited of the advances in electroacoustic transducers, flexible digital signal processors and efficient adaption algorithms. The technique used for ANC is based on the destructive interference in a given location of the noise produced by a primary source and the interefering signal generated by a secondary source. Successful implementations of this principle can be found in systems such as air conditioning ducts, to attenuate the low frequency noise due to the fans, or in transport systems, to reduce the noise generated by the engines inside propeller aircraft, automobiles and helicopters. Physical limitations generally restrict the frequency range of these active control systems to below a few hundred hertz. The main limitation is related to the wavelength of the acoustic waves in connection to the extension of the silenced area. At higher frequencies (up to 500 Hz), it is still possible to cancel the sound in a limited zone around, for example, a listener's ear, and suitable systems have been successfully implemented for headsets and zonal control. For disturbance frequencies above 1 kHz passive systems, based on the absorption and/or reflection properties of materials, are still the better choice.

Most of the studies presented in the literature refer to linear models while it is often recognized that nonlinearities can affect actual applications. In fact, nonlinear effects may be present according to the behavior of the noise source and the paths modeling the acoustic systems. Therefore, nonlinear modeling techniques may bring new insights and suggest new developments in the design of active noise controllers.

In this paper the active control of acoustic noises is considered in the framework of a signal processing approach, with particular reference to a nonlinear environment. From this point of view, the main problems to discuss are the derivation of efficient adaptation algorithms for the nonlinear controller and their fast implementation for real-time applications.

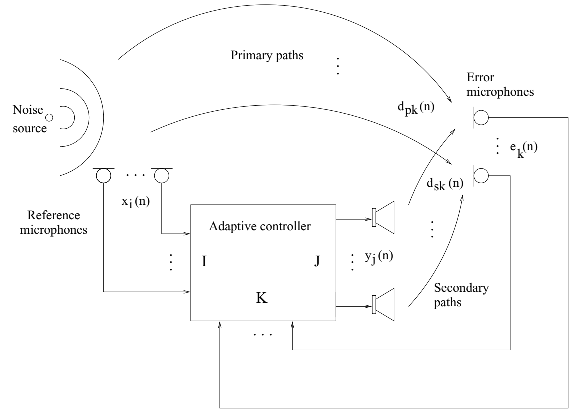

Figure 1: Multichannel active noise control.

The scenario describing the problem of ANC is briefly depicted in Section 2. The nonlinearities involved will be reviewed in Section 3, together with some recent contributions on nonlinear approaches. Models based on polynomial filters will be considered in some detail and an account on novel techniques used to adapt the nonlinear controller will be given in Section 4. A few simulation results will be reported in Section 5. Finally, possible future research lines will be prospected.

2. THE ANC SCENARIO

As mentioned in the Introduction, the principle of ANC is the cancellation of an acoustic disturbance based on the destructive interference with another noise produced by the controller with the same amplitude but opposite phase. The specific ANC strategies are usually divided in two classes, namely *feed-forward* methods, where reference signals measured in proximity of the noise source are available, and *feedback* methods, where the reference signals are not available. The ANC systems can then be further subdivided in *single-channel* and *multichannel* systems, according to the number of reference sensors, error sensors and actuator sources used. A multichannel ANC feed-forward scheme is shown in Figure 1. The multichannel scheme is used to spatially expand the silenced region with respect to the single-channel approach at the expenses of an increased computational complexity.

As shown in Figure 1, I microphones are used to collect I input signals $x_i(n)$ generated by the noise source and fed to the adaptive controller. The original noise propagates up to the region to be silenced through the so-called primary paths. The adaptive controller generates J noises $y_j(n)$ that are propagated through the so-called secondary paths. The undesired sound $d_{pk}(n)$ destructively interfer with the generated sounds $d_{sk}(n)$. The error microphones thus collect K errors $e_k(n) = d_{pk}(n) + d_{sk}(n)$ and send them back to the controller, where they are used together with the input signals, to adapt it. Commonly, the controller is described by means of $I \times J$ FIR filters connecting any input i to any output j. The secondary paths are still modeled as FIR filters and preliminary and independent evaluations of their impulse responses $s_{jk}(n)$ are needed. Then, according to the linearity assumptions, it results

$$d_{pk}(n) + \sum_{j=1}^{J} s_{jk}(n) * y_j(n) =$$

$$d_{pk}(n) + \sum_{j=1}^{J} s_{jk}(n) * \left\{ \sum_{i=1}^{I} \mathbf{h}_{ij}^T(n)\mathbf{x}_i(n) \right\} =$$

$$d_{pk}(n) + \sum_{j=1}^{J}\sum_{i=1}^{I} \mathbf{h}_{ij}^T(n) \left\{ s_{jk}(n) * \mathbf{x}_i(n) \right\}, \qquad (1)$$

where $\mathbf{x}_i(n)$ is the vector of the last N_i input samples collected at the reference microphone i

$$\mathbf{x}_i(n) = [x_i(n)\ x_i(n-1) \cdots x_i(n - N_i + 1)]^T \qquad (2)$$

and $\mathbf{h}_{ij}(n)$ is the vector of the N_i coefficients of the FIR filter connecting the input i to the output j of the controller

$$\mathbf{h}_{ij}(n) = [h_{ij}(0)\ h_{ij}(1) \cdots h_{ij}(N_i - 1)]^T. \qquad (3)$$

The symbol $*$ indicates the operation of linear convolution. Due to the presence of these convolutions, the adaptation algorithms derived on the basis of this model take the attribute of *Filtered-X*.

As shown in [4], the main drawbacks of the multichannel approach are the complexity of the coefficient updates, the data storage requirements and the slow convergence of the adaptive algorithms, even though the continuing evolution in the DSPs permits us to be optimistic about real-time implementations. On the other hand, single-channel schemes offer a reduced complexity but with a limited extension of the cancellation zone. In consideration of the additional complexities introduced by nonlinearities, research papers dealing with nonlinear controllers are presently limited to the single-channel case.

3. NONLINEAR EFFECTS IN ANC

In this section we review the main nonlinear effects that may influence the behavior of an active control system and thus motivate the use of nonlinear active noise controllers.

3.1 Nonlinear reference noise

It has been recently noted [5, 6] that the noise generated by a dinamic system can be often modeled as a nonlinear and deterministic process of chaotic rather than stochastic nature. In particular, it has been shown in [7] that the noise of the fan in a duct is well modeled by a chaotic process. Three kinds of chaotic noises, *i. e.* logistic, Lorenz and Duffing noises, have been applied in [5] to test the nonlinear single-channel controller proposed there. Among these noises, the logistic chaotic noise offers a simple and useful test signal since it is a second-order white and predictable nonlinear process generated by the following expression

$$\xi(n+1) = \lambda\xi(n)(1 - \xi(n)), \qquad (4)$$

where $\lambda = 4$ and $\xi(0)$ is a real number in the open interval $0 - 1$ (with the exclusion of the values 0.25, 0.5 and 0.75). The impact of this kind of noise on the controller performance becomes evident when the secondary path is modeled as a non-minimum phase FIR filter. In fact, the transfer function of the single-channel controller that minimizes the square of the error signal needs the approximation of the inverse of the transfer function of the secondary path with a noncausal linear filter. It has been shown in [5] that, while a zero-lag linear inverse does not exist, a zero-lag nonlinear inverse exists if the input signal is stochastic non-Gaussian or deterministic.

3.2 Nonlinear primary path

Another situation in which a nonlinear model is required is when the primary path exhibits some nonlinear distortions. Evidence of this fact can be found, for example, in ducts where the noise is propagating with high sound pressure [6] and the nonlinearity of the air is taken into account.

A practical model commonly used to describe mild nonlinearities is based on the well-known Volterra series [8]. This kind of model demands a nonlinear controller described by a Volterra filter. It is worth noting that for these filters it is possible to derive efficient updating algorithms by exploiting the linearity of their output with respect to the kernel coefficients. A nonlinear Filtered-X LMS algorithm based on a Volterra model has been first proposed in [6], where a multichannel implementation for the nonlinear controller is analyzed, by exploiting the so-called diagonal coordinate system proposed in [9]. This representation allows a truncated Volterra system to be described by a "diagonal" arrangement of the entries of each one of its kernels. A similar representation has been used in [10] to derive the so-called Filtered-X Affine Projections (AP) algorithm.

3.3 Nonlinear secondary path

The effects on nonlinearities in the secondary path have been also studied in the literature. ANC systems may use, in fact, in the secondary paths A/D and D/A converters, power amplifiers, loudspeakers and transducers. Overdriving the electronics gives rise to nonlinear effects. An accurate study of the impact of a nonlinearity in the secondary path has been presented in [11] for the Filtered-X LMS algorithm. The analysis is limited, however, to the presence of a memoryless saturation nonlinearity. The analytical and simulation results show that even a small nonlinearity may significantly affect the controller behavior.

Nonlinear schemes and nonlinear distortions have been also considered in the framework of the feedback schemes. In particular, in [15] a frequency selective feedback structure has been described. This structure works essentiallly in the frequency domain and is considered as nonlinear since signal amplitudes and phases are adapted in place of the controller coefficients. It is also confirmed that in active headphone sets, which still contain microphones, amplifiers and loudspeakers in the secondary path, nonlinear behaviors have been noticed [15].

3.4 Nonlinear models for the controller

The presence of nonlinear effects in the source noise, the primary and secondary paths calls for a nonlinear controller. The models used in recent contributions can be grouped in the following categories:

- Neural networks [7, 12, 13, 14].
- Radial basis functions [5].
- Volterra filters [5, 6, 10].
- Frequency selective filters [15].
- Fuzzy systems [16].

4. FILTERED-X ALGORITHMS FOR MULTICHANNEL ACTIVE NOISE CONTROLLERS

In this Section we specifically consider truncated Volterra models and show how the Filtered-X algorithms for single-channel feedforward schemes can be extended to multichannel systems.

According to the stochastic gradient approximation, the classical Filtered-X LMS algorithm for a linear multichannel system is derived by minimizing the sum of the squared values of the signals measured by the error microphones. The final updating relationship for the FIR filter connecting the input i to the output j of the controller is given by

$$\mathbf{h}_{ij}(n+1) = \mathbf{h}_{ij}(n) - \sum_{k=1}^{K} \mu_k e_k(n) \left\{ s_{jk}(n) * \mathbf{x}_i(n) \right\} \quad (5)$$

where μ_k, $k = 1, \ldots, K$, are the step sizes, or adaptation constants, controlling the convergence properties of the algorithm. In the nonlinear case, the linearity of the output of a Volterra filter with respect to the kernel coefficients makes it possible to derive an updating relation which is formally equal to Eq. (5). According to the diagonal representation [9], the Volterra filter connecting the input i to the output j is represented as a multichannel filter bank. The input to each channel is formed with appropriate products of input samples arranged in a vector in order to actually derive the final updating relation.

The multichannel interpretation of a Volterra filter has been exploited in [10] to derive the Filtered-X AP algorithm for a single-channel quadratic controller and to indicate how this algorithm can be extended to higher-order kernels. The main difference with respect to the Filtered-X LMS algorithm is that, in addition to the present one, $L-1$ previous errors are simultaneously considered. In the multichannel case, a set of k vectors $(1 \le k \le K)$ of the *a priori* errors $\mathbf{e}_k(n) = [e_{k1}(n)\ e_{k2}(n) \cdots e_{kL}(n)]^T$ needs to be exploited. A first extension of the single channel Filtered-X AP algorithm to a multichannel quadratic controller can be obtained in the case of a single reference microphone ($i = 1$). In this case, in fact, the updating terms can be derived by considering the $N_T \times L$ matrices

$$\mathbf{G}_{jk}(n) = [s_{jk}(n) * \mathbf{x}(n)\ \ s_{jk}(n-1) * \mathbf{x}(n-1) \cdots$$
$$s_{jk}(n-L+1) * \mathbf{x}(n-L+1)], \quad (6)$$

where $N_T = \sum_{q=1}^{M}(N-q+1)$ and N is assumed to be the memory length of the quadratic filters connecting the single input $x(n)$ to each output j of the controller. Within some approximations, the final updating expression for each channel m, $1 \le m \le M \le N$, of the quadratic filters becomes, for $1 \le j \le J$,

$$\mathbf{h}_{m,j}(n+1) = \mathbf{h}_{m,j}(n) -$$
$$\sum_{k=1}^{K} \mu_k \mathbf{G}_{m,jk}(n) \left[\mathbf{G}_{jk}^T(n) \mathbf{G}_{jk}(n) \right]^{-1} \mathbf{e}_k(n), \quad (7)$$

where $\mathbf{G}_{m,jk}(n)$ are submatrices of $\mathbf{G}_{jk}(n)$ of congruent dimensions. The updating rule requires the inversion of a set of $L \times L$ matrices at any time step. It is worth remarking that this operation is an affordable task for low orders of affine projections L. Since there is no space here to report the specific derivations, they are deferred to another paper presently in preparation, where the case of a number of reference microphones greater than one will be considered, too.

5. SOME SIMULATION RESULTS

As an example, we report in this Section the result of an experiment involving a multichannel controller with one input ($I = 1$), two actuators ($J = 2$) and two error microphones ($K = 2$). The source noise is a logistic chaotic noise as in Eq. (4) with $\xi(0) = 0.9$. The nonlinear process is then normalized in order to have a unit signal power $x(i) = \xi(i)/\sigma_\xi$. The primary paths are modeled with two FIR filters and the secondary paths are described by four non-minimum phase FIR filters. The system is identified with a second order Volterra filter with linear and quadratic parts of memory length 10. The quadratic kernel is formed with only two channels ($M = 2$), corresponding to the two main diagonals ($m = 1, 2$) in the diagonal representation. Figure 2 plots the ensamble average of the mean attenuation at the error microphones for 50 runs of the simulation system using the updating rule of Eq. (7). The four curves refer to different values of the affine projection order L. The order $L = 1$ corresponds to a normalized LMS adaptation algorithm. In the experiment all the adaptation constants have been fixed equal to 0.001. For higher orders of affine projections it is evident the improvement in the convergence behavior of the algorithm.

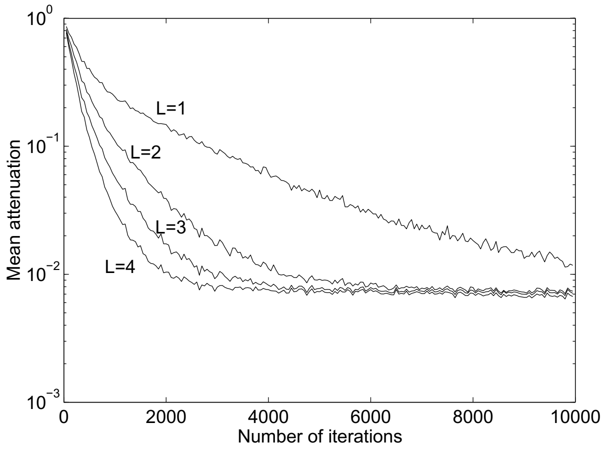

Figure 2: Mean attenuation in a multichannel active noise controller.

6. FUTURE PERSPECTIVES

In this paper, we have first presented the principles of single- and multichannel active noise control. Then, we have motivated the use of nonlinear controllers and studied in particular the realizations based on the Volterra filters used in feed-forward schemes. Filtered-X LMS algorithms for linear and Volterra filters have been reviewed and extended with particular reference to multichannel implementations. Filtered-X AP algorithms have been considered, too. To the knowledge of the Authors, adaptation rules for nonlinear multichannel controllers have not yet been demostrated in the literature.

One of the main problems that requires further investigations is the complexity of the implementations. A fast implementation of the multichannel Filtered-X LMS has been actually proposed in [4] for linear controllers. For nonlinear controllers, the single-channel Filtered-X AP algorithm has been shown [10] to be not too expensive in term of complexity in comparison with the Filtered-X LMS algorithm. The updating rule requires, in fact, about L times the number of operations necessary when using the Filtered-X LMS algorithm. On the other hand, the Filtered-X AP algorithm offers better convergence and tracking behaviors. This fact depends on the correlations which are present in the signals (products of input samples) entering the multichannel filter bank implementing the Volterra filter even in presence of not correlated input signals. An open problem remains the investigation of efficient implementations for the nonlinear multichannel Filtered-X LMS and AP algorithms. When using the Volterra models, there are in fact two multichannel representations, one relative to the structure of the controller and one relative to the realization of the Volterra filters, that require to be efficiently nested together.

Another open problem is the study of ANC methods when the secondary paths include nonlinearities with memory. The computation of nonlinear inverse or pseudoinverse systems for polynomial filters is thus required. A recent account on this topic can be found in [8].

REFERENCES

[1] P. A. Nelson and S. J. Elliott, *Active Control of Sound*, 3rd ed. London: Academic Press, 1995.

[2] S. M. Kuo and D. R. Morgan, *Active Noise Control systems - Algorithms and DSP Implementations.* Nwe York: Wiley, 1996.

[3] S. J. Elliott. *Signal Processing for Active Control.* London: Academic Press, 2000.

[4] S. C. Douglas, "Fast implementations of the Filtered-X LMS and LMS algorithms for multichannel active noise control," *IEEE Trans. Speech Audio Processing*, vol. 7, pp. 454-465, July 1999.

[5] P. Strauch and B. Mulgrew, "Active control of nonlinear noise processes in a linear duct," *IEEE Trans. Signal Processing*, vol. 46, No. 9, pp. 2404-2412, Sept. 1998.

[6] L. Tan and J. Jiang, "Adaptive Volterra filters for active noise control of nonlinear processes," *IEEE Trans. Signal Processing*, vol. 49, pp. 1667-1676, Aug. 2001.

[7] T. Matsuura, T. Hiei, H. Itoh and K. Torikoshi, "Active noise control by using prediction of time series data with a neural network," in *Proc. IEEE SMC Conf.*, vol. 3, pp. 2070-2075, 1995.

[8] V. J. Mathews and G.L.Sicuranza, *Polynomial Signal Processing*, New York: John Wiley, 2000.

[9] G. V. Raz and B. D. Van Veen, "Baseband Volterra filters for implementing carrier based nonlinearities," *IEEE Trans. Signal Processing*, vol. 46, pp. 103-114, Jan. 1998.

[10] A. Carini and G. L. Sicuranza, "Filtered-X Affine Projection algorithm for nonlinear active noise controllers," in *Proc. NSIP-03*, Grado, Italy, 2003.

[11] M. H. Costa, J. C. Bermudez and N. J. Bershad, "Stochastic analysis of the Filtered-X LMS algorithm in systems with nonlinear secondary path," *IEEE Trans. Signal Processing*, vol. 50, No. 6, pp. 1327-1342, June 2002.

[12] S.D.Snyder and N. Tanaka, "Active control of vibration using a neural network," *IEEE Trans. Neural Networks*, vol. 6, pp.819-828, July 1995.

[13] D. Pavisic, L. Blondel and J.P. Draye, "Active noise control with dynamic recurrent neural networks," in *Proc. ESANN*, Brussels, Belgium, pp. 45-50, Apr. 1995.

[14] M. Bouchard, B. Pailard and C.T.L. Dinh, "Improved training of neural networks for nonlinear active control of sound and vibration," *IEEE Trans. Neural Networks*, vol. 10, pp. 391-401, Mar. 1999.

[15] T. Meurers, S. M. Veres and S. J. Elliott, "Frequency selective feedback for active noise control," *IEEE Control Syst. Mag.*, Vol. 22, No. 4, pp. 32-41, Aug. 2002

[16] C.A. Silva, J.M. Sousa and J.M.G. Sá da Costa, "Active noise control based on fuzzy models," in *4th European Conf. on Noise Control, EURONOISE 2001*, Patras, Greece, 14-17 Jan. 2001.