BER-BASED VS. GAME-THEORETIC POWER ALLOCATION STRATEGIES FOR MULTIUSER MISO SYSTEMS

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ABSTRACT

Motivated by the extensive use of game-theoretic strategies for uplink power control in CDMA, we compare in this paper a strategy based on the widespread utility function used in the literature with other traditional schemes based on the BER. Here, we focus on the downlink of a communication system. Basically, that utility function is a ratio between the frame success rate and the used power. It is shown in this paper that the strategy maximizing the utility implies a higher error rate than for other classical schemes, which was not shown in the literature to the best of our knowledge. Finally, we briefly discuss the usefulness of pricing mechanisms in a game-theoretic formulation of the power control.

1. INTRODUCTION

Game-theoretic power control has been widely studied in the literature not only in the context of Code Division Multiple Access (CDMA) since late nineties, see e.g. [1], [2], and [3], but also for digital subscriber lines [4]. Concentrating on CDMA, the authors model the uplink power control problem as a game. Provided that the users are selfish and rational, game theory provides an elegant mathematical tool to obtain a distributed solution to the problem. However, complete information is needed at the terminals, which means that they shall know the channel from the other users in the cell. Therefore, the term distributed refers to the computation of the solution. Some advantages of the game-theoretic formulation for the uplink are that it is easy scalable and that it provides fairness among the users since they are granted their maximum satisfaction.

This degree of satisfaction is expressed mathematically in terms of a convenient utility function, which is a key issue. If data is transmitted, the utility should be increasing with respect to the Signal to Interference Ratio (SIR) if the transmit power is fixed, or it should be decreasing with power if the SIR is kept constant, among other properties [5]. Therefore, it is sensible to use a ratio between the Frame Success Rate (FSR), that is, the probability that the frame is correct, and the transmitted power, as the authors suggest [2].

Related to this issue, we address the power control in the downlink of a communication system, where the Access Point (AP) or Base Station (BS) is equipped with multiple antennas, whereas the terminals have a single one, as it happens nowadays for most communication standards. Essentially, we wish to compare the widespread utility-based strategy used in e.g. [1], [2], or [3] with other schemes based on the Bit Error Rate (BER). We show that although the utility-based optimization maximizes the utility within the cell, while minimizing the power, the FSR is penalized, or equivalently, the BER is higher than for other methods. This result reflects the difficulty in choosing convenient utility functions.

Within our context, the first issue is the transmit beamforming. As in [6], we assume a Zero Forcing (ZF) beamforming criterion, which eliminates the inner-cell interference among the users that are being served by the multi-antenna AP. This scheme is especially well-suited for Space Division Multiple Access (SDMA) systems, since the resources granted for the users do not overlap. Note that a similar idea holds for Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA). ZF creates parallel and orthogonal equivalent channels for the users without inner-cell interference with a simple and closed-form solution [7].

We deal in this paper mainly with the allocation of the limited total instantaneous power among the users, where fairness considerations come to the scene [8]. Then, the figure of merit should not be an aggregate magnitude, thus a trade-off among the performance of the active users should be taken into account [9]. We concentrate on techniques based on the BER, and either minimize the sum of BER for the users, or all of them are given the same BER. The former is the Minimum Sum BER (MSB) strategy whereas the latter Minimizes the Maximum BER (MMB). These schemes are compared to the well-known Uniform Power Allocation (UPA), and to a utility-based framework, particularly the Maximization of the Sum of Utilities (MSU). The MSU reflects a situation where the AP wishes to maximize the global perceived satisfaction. To the best of our knowledge, a similar study has not been conducted in the literature. The final remark is that all the considered problems have been solved using convex optimization [10].

In Section 2 we give an overview of the problem, just before the power allocation criteria exposed in Section 3. Simulation results are shown in Section 4, and then we discuss the usefulness of pricing schemes and conclude.

2. PROBLEM STATEMENT

In the following, boldface capital (lowercase) letters refer to matrices (vectors). The conjugate transpose of $\mathbf{a}$ is $\mathbf{a}^H$ and the element at row ith and column jth of $\mathbf{A}$ is denoted by $[\mathbf{A}]_{i,j}$. The square matrix with diagonal elements given by $a_1, a_2, \ldots, a_n$ is expressed as $diag(a_1, a_2, \ldots, a_n)$, and $a^+ = \max(0, a)$. The cardinality of the set $\mathscr{K}$ is given by $|\mathscr{K}|$, $\exp(x)$ is the exponential function of x, and $tr(\mathbf{A})$ denoted the trace of the matrix $\mathbf{A}$.

We focus on the downlink, where a Q-antenna AP communicates simultaneously with K single-antenna terminals, which are gathered in the set $\mathscr{K} = \{1, \ldots, K\}$ and we assume that $K \leq Q$. At any time instant, the received signal vector for this model is

$$\mathbf{y} = \mathbf{HBs} + \mathbf{w} \in \mathbb{C}^{K \times 1}, \tag{1}$$

where the kth position of vector $\mathbf{y}$ $(\mathbf{s})$ is the received (transmitted) signal for user k. $\mathbf{H}$ is the $K \times Q$ complex flat-fading channel matrix, the ith row of which contains the $1 \times Q$ vector of the channel gains for the ith user, i.e. $\mathbf{h}_i^T$, and we assume that the components of the channel matrix are independent and identically distributed Gaussian random variables with zero mean and unit variance. In a TDMA/TDD system at pedestrian speed, it might not be far from reality the assumption that the channel matrix is known at the AP, whereas the receivers are only aware of their own channel response. The noise vector is complex Gaussian, i.e. $\mathbf{w} \sim \mathscr{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_Q\right)$,

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and the transmit beamvectors for the K users are gathered in the matrix $\mathbf{B} = [\mathbf{b}_1 \mathbf{b}_2 \ldots \mathbf{b}_K] \in \mathbb{C}^{Q \times K}$.

With a ZF beamforming criterion, the K channels become parallel and orthogonal, thus the users receive their transmitted symbol corrupted only by additive white Gaussian noise, without inner-cell interference. In this problem, it is meaningful to separate the effect of the channel and the power allocation. Therefore, we normalize the beamvector, so that the effect of the equivalent channel is captured by α_k. The modified ZF criterion becomes $\mathbf{H}\widetilde{\mathbf{b}}_k = \alpha_k \mathbf{1}_k, \forall k$, where the vector $\mathbf{1}_k$ has zeros at all positions but the kth. The normalized beamvector for the kth user is obtained as

$$\widetilde{\mathbf{b}}_k = \alpha_k \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathbf{1}_k, \tag{2}$$

where the $\alpha_k = 1/\sqrt{\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]_{k,k}}$ are real and positive by construction. The K normalized beamvectors are gathered in the matrix $\widetilde{\mathbf{B}} = \left[\widetilde{\mathbf{b}}_1 \widetilde{\mathbf{b}}_2 \ldots \widetilde{\mathbf{b}}_K\right]$, thus $\mathbf{H}\widetilde{\mathbf{B}} = \mathbf{D}_\alpha = diag(\alpha_1, \alpha_2, \ldots, \alpha_K)$. The beamforming matrix contains also the power factors β_k, i.e. $\mathbf{B} = \widetilde{\mathbf{B}}\mathbf{D}_\beta$, where $\mathbf{D}_\beta = diag(\beta_1, \beta_2, \ldots, \beta_K)$. The signal model in (1) finally reduces to

$$\mathbf{y} = \mathbf{D}_\alpha \mathbf{D}_\beta \mathbf{s} + \mathbf{w} \Rightarrow y_k = \alpha_k \beta_k s_k + w_k, \tag{3}$$

in which the equivalent gain α_k depends on the channels from the rest of the users. With this model, the Signal to Noise Ratio (SNR) for the kth user is given by

$$\gamma_k = \frac{\alpha_k^2 \beta_k^2}{\sigma^2}, \tag{4}$$

where we have assumed that the symbols have unitary mean energy, particularly, normalized Quadrature Amplitude Modulation (QAM) symbols are considered. For the sake of simplicity, which is an important feature for cross-layer designs, we use the easy-differentiable BER expression given in [11] for QAM signals, i.e.

$$\mathrm{BER}(\gamma) \approx c_1 \exp(-c_2 \gamma), \tag{5}$$

where c_1 and c_2 depend on the concrete signal mapping. Since this paper assumes that there is no channel coding at the transmitter, the channel is time-invariant, and the noise is Gaussian, the Frame Error Rate (FER) can be expressed as a function of the BER and the frame length L in bits as $\mathrm{FER} = 1 - (1 - \mathrm{BER})^L$, thus the Frame Success Rate (FSR) can be obtained as $\mathrm{FSR} = 1 - \mathrm{FER}$.

3. POWER ALLOCATION STRATEGIES

In this section, we propose several alternatives for allocating the total available instantaneous power P_T among the users. We assume that they are homogeneous, i.e. their data traffic requirements are the same, and we focus on optimizing a function of the BER while imposing a constraint on the instantaneous power, in contrast to the approach taken e.g. in [6]. Besides, we evaluate the utility-based cost function proposed e.g. in [2] among other papers. However, it is already pointed out in [12] that further understanding of the utility functions is needed. In this sense, there is a number of cost functions that could be used. Since in this paper a centralized system is considered, we assume that the AP would like to maximize the sum of utilities for all the users, see Section 3.3, which reflects the situation where the AP obtains the highest global satisfaction.

Without any channel knowledge, the best option would be the well-known Uniform Power Allocation (UPA), in which the whole power is divided equally among the active users in the cell, so that we do not care about their actual channel gain nor how we can improve the performance. The power allocated to the kth user is $\beta_k^2 = \frac{P_T}{K}$, thus the SNR for the kth user is given by $\gamma_k = \frac{\alpha_k^2}{\sigma^2} \frac{P_T}{K}$, which leads to a lower BER for the users having a better channel. Since this paper assumes that the AP has perfect channel knowledge, more efficient power allocation criteria could be applied, as we can see next. We point out that the presented techniques are considered to deliver a best-effort service, since the AP optimizes a function of the BER regardless of the individual Quality of Service (QoS) achieved by the users. For further details, see e.g. [9].

3.1 Minimum Maximum BER (MMB)

A possible optimization criterion consists of minimizing the maximum BER among the users. We will see in this subsection that it finally reduces to assigning the same BER to all users, regardless of their channel quality. The cost function is expressed as

$$\min_{\beta_k^2} \max_k \mathrm{BER}_k \tag{6}$$

$$s.t. \sum_{k \in \mathscr{K}} \beta_k^2 \leq P_T, \tag{7}$$

where we have implicitly assumed that the β_k^2 are non-negative, since they are power allocation factors. According to convex optimization theory [10], the previous problem is convex because the BER approximation is an exponential and the constraints are linear. In order to properly solve this problem, one should express it according to the convex formulation. Recalling (5),

$$\min_{\beta_k^2} \; t \tag{8}$$

$$s.t. \sum_{k \in \mathscr{K}} \beta_k^2 - P_T \leq 0, \tag{9}$$

$$c_1 \exp(-c_2 \frac{\alpha_k^2 \beta_k^2}{\sigma^2}) - t \leq 0, \forall k \in \mathscr{K}, \tag{10}$$

$$-\beta_k^2 \leq 0, \forall k \in \mathscr{K}, \tag{11}$$

to which we can apply the Karush-Kuhn-Tucker (KKT) conditions [10]. Using these,we can find the following solution[1]

$$\alpha_k^2 \beta_k^2 = \frac{P_T}{tr\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]}, \tag{12}$$

which implies that all the users attain the same SNR, and thus the same BER. However, in this case the AP is using some resources to increase the performance of the worse users.

3.2 Minimum Sum BER (MSB)

Another possibility is to minimize the total BER, regardless of the users with poorer channels. In one sentence, we would like to minimize the sum BER of all the users in the cell subject to the power constraint, which means

$$\min_{\beta_k^2} \sum_{k \in \mathscr{K}} \mathrm{BER}_k \tag{13}$$

$$s.t. \sum_{k \in \mathscr{K}} \beta_k^2 - P_T \leq 0, \tag{14}$$

$$-\beta_k^2 \leq 0, \forall k \in \mathscr{K}. \tag{15}$$

We can apply the KKT conditions [10] because the problem is convex, and one can see that the solution is similar to a waterfilling:

$$\beta_k^2 = \frac{\sigma^2}{c_2 \alpha_k^2} \left[log\left(\frac{c_1 c_2 \alpha_k^2}{\sigma^2}\right) - log\lambda \right]^+, \forall k \in \mathscr{K}, \tag{16}$$

where $log\lambda$ is obtained in order to fulfill (14) with equality. A remark about implementation is that since the α_k^2 change (increase)

[1] Note that $\sum_{k \in \mathscr{K}} 1/\alpha_k^2 = tr\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]$.

when the number of users is reduced, they shall be recomputed if there is any user j for which $\beta_j^2 = 0$. Then, user j is removed from the active set $\mathscr{K}$, thus the jth row is eliminated from $\mathbf{H}$. Therefore, the solution in (16) shall be computed again. By construction, it is clear that this scheme will provide a lower BER than the MMB, but the drawback is that for the sake of the collective revenue, some users might not even be allocated for transmission.

3.3 Maximum Sum of Utilities (MSU)

Differently to other papers, e.g. [1], [2], or [3], we solve a utility-based downlink power control using convex optimization [10]. Although the game is generally competitive [4], the MSU here is analogous to a refereed game in which a cooperative strategy is sought, see [2] and references therein. Whereas in the refereed game the AP would tell the terminals the uplink power, in this case the AP allocates a certain power to the users for downlink transmission[2].

With minor modifications to the utility function given e.g. in [2], the utility perceived by the kth user can be expressed as

$$u_k = \frac{\left(1 - \frac{\text{BER}}{c_1}\right)^L}{\beta_k^2} = \frac{\left(1 - \exp(-c_2 \frac{\alpha_k^2 \beta_k^2}{\sigma^2})\right)^L}{\beta_k^2}, \tag{17}$$

in which, in agreement with e.g. [5], the FSR in the numerator has been slightly modified. By dividing the BER by the constant c_1 we guarantee that if $\beta_k^2 = 0 \Rightarrow u_k = 0$, and that the utility tends to 0 as the power goes to infinity, i.e. $\lim_{\beta_k^2 \to \infty} u_k = 0$. If we had not proceeded so, at null power, $\beta_k^2 = 0$, the utility would be infinity, and the terminal would choose not to transmit. This modification does not have a deep impact in the trend of the FSR [1].

In the literature for the uplink power control, each user maximizes its own utility. Then, if we derive the utility in (17), the solution is the β_k^2 whose equilibrium SNR γ_k^* satisfies

$$\exp(-c_2\gamma_k^*)(1 + Lc_2\gamma_k^*) - 1 = 0, \tag{18}$$

so that the power allocation factors can be obtained using (4) as

$$\beta_k^2 = \frac{\sigma^2}{\alpha_k^2}\gamma^* = c. \tag{19}$$

This point c is a maximum of the utility function in (17). Therefore, since there exists a point c such that u_k is non-decreasing for $t \leq c$, and non-increasing for $t > c$, the function in (17) is quasi-concave [10]. Moreover, this point constitutes a Nash Equilibrium (NE) for the uplink power control game, which is taken as a benchmark in existing literature, e.g. [5] and [2]. Therefore, we denote the equilibrium SNR as $\gamma^{NE} = \gamma_k^*$. A NE is a point where no user can increase its own utility function by changing its own transmitted power, given the transmitted power from the other users [13].

Since we focus on the downlink, the AP shall distribute the limited instantaneous power among the users in the cell. This constitutes a difference with respect to existing literature, see e.g. [12] and references therein. For this multiuser communication, the AP has several alternatives involving fairness issues [9]. In this paper, the AP wishes to maximize the sum of utilities of all the users in the cell, which means that the total perceived satisfaction would be maximum. Since the objective function (sum of utilities) is quasi-concave because it is obtained by a sum of quasi-concave functions [10], minus a sum of quasi-concave functions is quasi-convex. Therefore, we can formulate the optimization in convex form as

$$\min_{\beta_k^2} - \sum_{k \in \mathscr{K}} u_k \tag{20}$$

$$s.t. \sum_{k \in \mathscr{K}} \beta_k^2 - P_T \leq 0, \tag{21}$$

$$-\beta_k^2 \leq 0, \forall k \in \mathscr{K}. \tag{22}$$

[2]Note that if the system is TDMA/TDD and the channel is quasi-static, the same power could be used for the uplink

Table 1: Maximization of the Sum of Utilities (MSU) Algorithm

1. Set $\mathscr{K} = \{1, \ldots, K\}$.
2. Build matrix $\mathbf{H}$ with the users in the set $\mathscr{K}$, and compute $\alpha_k^2 = 1/\left[\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\right]_{k,k}, \forall k \in \mathscr{K}$.
3. If the condition in (24) is satisfied, go to step 5.
4. Otherwise, select the user $k^s : \min_k \alpha_k^2$, and remove it from the active set, $\mathscr{K} = \mathscr{K} - k^s$. Go to step 2.
5. Compute the power for the users in $\mathscr{K}$ according to (19). For the users not in $\mathscr{K}$, set $\beta_k^2 = 0$.

Applying the KKT conditions [10], we obtain that the solution β_k^2 might be in the set

$$\beta_k^2 \in \left\{0, \frac{\sigma^2}{\alpha_k^2}\gamma^{NE}\right\}, \forall k \in \mathscr{K}. \tag{23}$$

If the power were unbounded, the utility maximization would yield the same performance as the MMB, since all the users would get the same equilibrium SNR given by γ^{NE}. However, since the power is limited, either the user is allocated at a point such that its own utility is maximized or it is not scheduled. The question is which users will not be allocated for transmission.

If we add the power factors β_k^2 obtained in (19) as if all the users were active, the total power is $\sigma^2\gamma^{NE} tr\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]$. Therefore, the MSU problem serves all the users with the SNR of the NE, γ^{NE}, if[3]

$$tr\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right] \leq \frac{P_T/\sigma^2}{\gamma^{NE}}. \tag{24}$$

In any other case, we should decide which users are allocated null power. It can be easily seen that if we substitute the equilibrium β_k^2 obtained in (19) in the utility function in (17), the utility for the kth user at the NE is $u_k^{NE} = k_{NE}\alpha_k^2$, where k_{NE} is a constant. Therefore, the user with lower α_k^2 (worst channel) is the selected candidate to be allocated null power, since it is the user that penalizes the performance of the rest of the users.

With these issues, we summarize in Table 1 the algorithm that yields the highest sum of utilities. First, it tries to allocate all the users, but if the problem is not feasible, the best strategy is to remove the user with worst channel, see step 4 in Table 1. Step 5 reflects (23). Note that if a user is allocated null power, the α_k^2 shall be recomputed because they increase when less users are served. In most cases, $\sum_{k \in \mathscr{K}} \beta_k^2 < P_T$ because the power is determined by (19). The idea behind is that using more or less power than the NE for any user would imply a lower utility, which is not the objective.

4. SIMULATIONS

We have an AP provided with $Q = 6$ antennas, which tries to serve $K = 6$ active users in the cell. The SNR in the figures refers to the ratio P_T/σ^2, and the range is from 4 dB to 28 dB in steps of 4 dB. We assume 2 bits of symbol without loss of generality, thus the constants in (5) are $c_1 = 0.2$ and $c_2 = 1.6/3$. We have conducted simulations to evaluate the utility, the BER, and the power.

First, we plot in Figure 1 the sum of utilities for all users in the cell with respect to the SNR. It is clear that the technique based on the maximization of the utility yields the best results compared to the UPA, the MMB, and the MSB. Moreover, these techniques always use the total available power, whereas the MSU only uses the following percentages of total transmitted power $[89.24, 78.10, 73.41, 68.17, 55.59, 42.27, 30.83]$ for the simulated SNR. Therefore, we can conclude that the utility is maximized while the used power is the lowest among the studied methods.

[3]In this case, we say that the problem is feasible.

We plot in Figure 2 the sum of BER vs. the SNR for the proposed methods. The BER is set to 0.5 if the users do not transmit. The MSB yields the optimum performance since it is designed for that purpose. It is important to see that the maximization of the sum of utilities does not yield a good performance in terms of BER. Finally, this SDMA system based on ZF fully exploits the multiplexing gain because it is serving the maximum number of users, i.e. $K = Q$, but the diversity gain is penalized [14]. The authors show in [9] that even when $K = Q-1$, the BER decreases in more than one magnitude order for moderate SNR.

As final comments, note that the alternatives based on game theory, e.g. pricing [5] or repeated games [12], would increase the utility while reducing even more the power. These options are studied in order to overcome the Pareto deficiency of the NE. Briefly, a Pareto optimum point means that no user can increase its own utility without decreasing the utility obtained by other users [13]. To the best of our knowledge, it is not shown in existing papers how the BER (or FSR) performance degrades, see e.g. [2]. Therefore, constraints on the SNR or on the BER should be added to the problem in order to fulfill the real traffic requirements from the users.

5. DISCUSSION AND CONCLUSIONS

In this paper, we have compared the utility-based power control with some schemes based on the BER, namely the minimum sum BER and the minimum maximum BER. We have solved the problems using convex optimization, and results have shown that the maximization of utility does not yield a good performance in terms of BER, even compared to the classical uniform power allocation. To the best of our knowledge, BER performance was not shown in previous papers developing a game-theoretic formulation of the power control in CDMA. In any case, game theory provides an attractive mathematical framework, and concepts such as pricing can be useful for future communication systems.

The pricing factor can be set by the AP in order to force the terminal to transmit at a certain power level in the uplink. For instance, we can assume that each selfish terminal wishes to maximize the following modified utility function $\tilde{u}_k = u_k - c_k\beta_k^2$, where c_k is a different pricing factor for each user. The pricing c_k would be chosen by the AP in a way such that when the terminal optimizes individually $\tilde{u}_k$ with respect to β_k^2, the selected power would be the one previously computed by the AP in order to optimize a certain cost function. For instance, the AP could select among the UPA, the MSB, or the MMB. Since the AP has all the necessary information and computational capabilities, it can communicate the pricing value to the terminals, so that the power allocation is computed in a distributed manner. However, note that some information is needed at the terminals, which shall be provided by the AP.

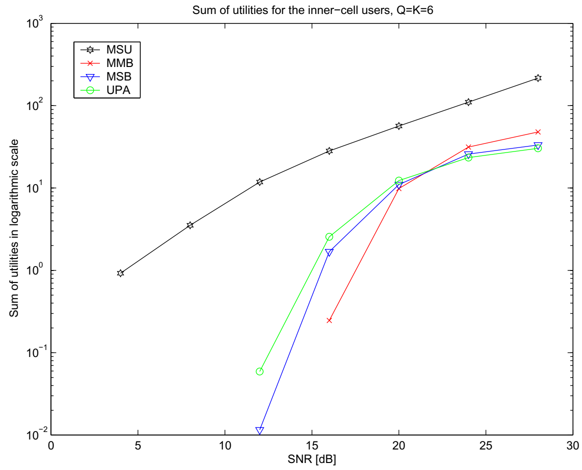

Figure 1: Sum of utilities for the inner-cell users.

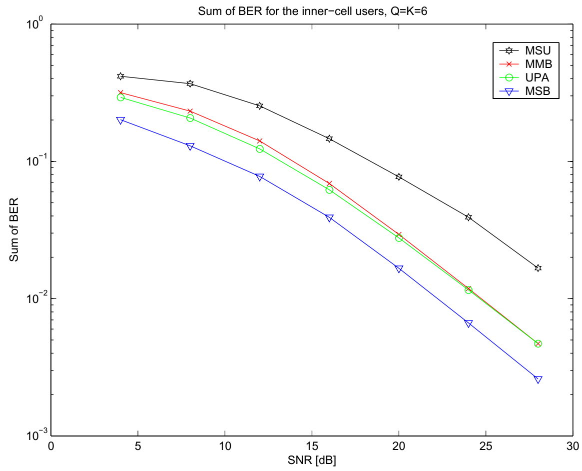

Figure 2: Sum of BER for the inner-cell users.

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