ON BLIND FREQUENCY-OFFSET SYNCHRONIZATION FOR OFDM COMMUNICATIONS

Mounir Ghogho

University of Leeds - School of EEE
LS29JT, United Kingdom
m.ghogho@ieee.org

ABSTRACT

We address the problem of carrier frequency offset (CFO) synchronization in Orthogonal Frequency Division Multiplexing (OFDM) communications systems in the context of frequency-selective fading channels. We consider the case where the transmitted symbols are drawn from Phase-Shift-Keying (PSK) constellations. We propose and compare two algorithms to blindly estimate the CFO. The first method exploits the constant modulus of the PSK constellations. The second method exploits the finite alphabet property of these constellations. For both methods, using adequate parameterization, the estimation of the CFO is decoupled from the estimation of the other signal parameters. The first method is shown to outperform the second technique.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) modulation is attractive for high data rate wireless networks. It has been adopted for a number of applications including Digital Audio Broadcasting and local area networks such as IEEE 802.11a. The main advantage of OFDM systems is the reduced complexity of the equalizer at the receiver, which allows for inexpensive hardware implementation. OFDM systems are however more sensitive to Carrier Frequency Offset (CFO) than single carrier systems [1]. The presence of a CFO causes loss of orthogonality between the sub-carriers, and leads to increased bit-error-rate. Consequently, there has been considerable work in the area of CFO estimation. A number of pilot-assisted CFO synchronization techniques are available in the literature. Blind CFO synchronization is attractive because it saves bandwidth, i.e., no training pilots are required. A blind CFO estimator was recently proposed in [2] (see also [3] for more details); this estimator, designed to work with dispersive channels, exploits the fact that practical OFDM systems are not fully loaded, i.e., the number of information-bearing sub-carriers is smaller than the size of the FFT block. In [4], we extended this approach by inserting a few extra nullsubcarriers in order to improve performance and ensure identifiability.

Here, we consider the case where the transmitted symbols are drawn form PSK constellations. We propose two algorithms to blindly estimate the CFO. The first method exploits the constant modulus of the PSK constellations. The second method exploits the finite alphabet property of these constellations. For both methods, using adequate parameterization, the estimation of the CFO is decoupled from the estimation of the other signal parameters.

This work was supported by the UK Royal Academy of Engineering.

2. SIGNAL MODEL

OFDM modulation consists of N (usually a power of 2) sub-carriers, equi-spaced at a separation of $\Delta f = B/N$, where B is the total system bandwidth. All sub-carriers are mutually orthogonal over a time interval of length $T = 1/\Delta f$. Each sub-carrier is modulated independently with symbols drawn from PSK constellations. Each OFDM block is preceded by a cyclic prefix whose duration is longer than the delay spread of the propagation channel, so that inter-block interference can be eliminated at the receiver, without affecting the orthogonality of the sub-carriers. Practical OFDM systems are in general not fully loaded in order to avoid aliasing. In this case, some of the sub-carriers at the edges of the OFDM block are not modulated; these sub-carriers are referred to as virtual sub-carriers. Their number is dictated by system design requirements and is, in general, about 10% of N. Let $\mathcal{N} = \{-N/2+1, ..., N/2\}$ denote the entire set of sub-carriers; also let $\mathcal{N}_a = \{-(N_a-1)/2, ..., (N_a-1)/2\}$ denote the N_a-element set of active or modulated sub-carriers ($N_a < N$ and N_a is odd), and let $\mathcal{N}_v = \mathcal{N} - \mathcal{N}_a$ denote the set of $N_v = N - N_a$ virtual sub-carriers[1].

At the receiver, the output of the matched filter is sampled with period $T_s = T/N$. After discarding the cyclic prefix, the complex envelope of the baseband received sig-

[1] The assumptions that N is even and N_a is odd are not critical.

nal in an OFDM block can be described as

$$x(k) = e^{j2\pi k\xi_o/N} \sum_{n\in\mathcal{N}_a} H_n s_n e^{j2\pi kn/N} + v(k) \qquad (1)$$

with $k = 0, ..., N-1$, where $\{s_n\}$ are randomly drawn from M-PSK constellations, i.e. $\{\exp(j\pi(2i+1)/M),\ i = 0, ..., M-1\}$; ξ_o (a real number, $|\xi_o| < N/2$) is the CFO normalized to $1/T$, H_n is the complex channel response at the nth subcarrier frequency

$$H_n = \sum_{l=0}^{L} h_l e^{-j2\pi ln/N} \ . \qquad (2)$$

Here, $\{h_l\}$ are the coefficients of the $L+1$-tap channel, and $v(k)$ is additive noise which is assumed to be zero-mean, uncorrelated, circularly symmetric and Gaussian with variance $\sigma_v^2 = E\left\{|v(k)|^2\right\}$.

3. CFO ESTIMATION EXPLOITING THE CONSTANT MODULUS PROPERTY

Since $\{s_n\}$ is a constant-modulus sequence, $H_n s_n$ in eq. (1) may be rewritten as $H_n s_n = |H_n|e^{j\theta_n}$ where θ_n is the angle of $H_n s_n$, $|\theta_n| \le \pi$. It is worth noting that the N_a unknown parameters, $\{|H_n|\}$ in (1), are parameterized by only $(L+1)$ complex parameters which are the channel coefficients $\{h_l\}$. This property is the core of the following algorithm.

We consider the parameters $\{|H_n|\}$ and $\{\theta_n\}$ as unknown deterministic parameters. Since the additive noise is white, circularly symmetric and Gaussian, the ML estimates of ξ_o, $|\mathbf{H}| = \{|H_n|,\ n \in \mathcal{N}_a\}$ and $\boldsymbol{\theta} = \{\theta_n,\ n \in \mathcal{N}_a\}$ are obtained by minimizing the L_2 norm:

$$J(\xi, |\mathbf{H}|, \boldsymbol{\theta}) = \sum_{k=0}^{N-1} \left| x(k) - e^{j2\pi k\xi/N} \sum_{n\in\mathcal{N}_a} |H_n| e^{j\theta_n} e^{j2\pi kn/N} \right|^2 \qquad (3)$$

The criterion in eq. (3) can be rewritten as[2]

$$J(\xi, |\mathbf{H}|, \boldsymbol{\theta}) = \sum_{k=0}^{N-1} |x(k)|^2 + N \sum_{n\in\mathcal{N}_a} |H_n|^2 - 2N\mathcal{R}\left[\sum_{n\in\mathcal{N}_a} |H_n| X(n+\xi) e^{-j\theta_n}\right] \qquad (4)$$

where $\mathcal{R}[z]$ denotes the real part of a complex variable z ($\mathcal{I}[z]$ denotes the imaginary part), and $X(f)$ is the discrete-Fourier transform of $\{x(k)\}$ at frequency f/N

$$X(f) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{-j2\pi kf/N} \ .$$

[2]If m is an integer, we have that $\sum_{k=0}^{N-1} e^{j2\pi km/N} = N\delta(m)$, where $\delta(m) = 1$ if $m = 0$ mod N, and $\delta(m) = 0$ otherwise.

Setting $\partial J/\partial\theta_n = 0$, and assuming that $|H_n| \neq 0$[3], the ML estimate $\hat{\theta}_n$, satisfies

$$\mathcal{I}\left[X(n+\xi)e^{-j\hat{\theta}_n}\right] = 0, \qquad n \in \mathcal{N}_a \qquad (5)$$

which implies

$$\hat{\theta}_n = \arg\{X(n+\xi)\} \qquad (6)$$

where $\arg\{z\}$ denotes the argument of a complex variable z. Substituting $\hat{\theta}_n$ into eq. (4), the criterion to minimize becomes

$$\begin{aligned} J(\xi, |\mathbf{H}|) &= \frac{1}{N}\sum_{k=0}^{N-1} |x(k)|^2 + \sum_{n\in\mathcal{N}_a} |H_n|^2 \\ &\quad -2\sum_{n\in\mathcal{N}_a} |H_n||X(n+\xi)| \\ &= J_{VSC}(\xi) + J_A(\xi, |\mathbf{H}|) \qquad (7) \\ J_{VSC}(\xi) &= \sum_{n\in\mathcal{N}_v} |X(n+\xi)|^2 \qquad (8) \\ J_A(\xi, |\mathbf{H}|) &= \sum_{n\in\mathcal{N}_a} \left(|X(n+\xi)| - |H_n|\right)^2 \qquad (9) \end{aligned}$$

where we have used Parseval's theorem,

$$(1/N)\sum_{k=0}^{N-1} |x(k)|^2 = \sum_{n\in\mathcal{N}} |X(n+\xi)|^2 \ , \quad \xi \in \mathcal{R} \ , \ (10)$$

and the fact that $H_n s_n = 0$ for $n \notin \mathcal{N}_a$. The linear relations in eq. (2) should be taken into account in the minimization of the above criterion.

Notice that J_{VSC} is equivalent to the cost function used in an existing blind CFO estimator [2], which exploits the virtual sub-carriers ($\mathcal{N}_v$). The extra term J_A exploits the constant-modulus property of the transmitted symbols. As for J_{VSC}, minimizing J_A alone provides a consistent estimate of the CFO. Of course, J_{VSC} (respectively J_A) is a valid criterion only if the system is not fully loaded (respectively the constant-modulus property is satisfied). There are however some identifiability conditions associated with each criterion [5].

Note that J_{VSC} is not a function of $|\mathbf{H}|$. Therefore, only J_A needs to be considered in the minimization of J with respect to $|\mathbf{H}|$. This should be carried out under the constraint

$$|H_n|^2 = \sum_{l,p=0}^{L} h_l h_p^* e^{-j2\pi(l-p)n/N} \qquad (11)$$

which follows from (2). Constraint (11) states that $|H_n|^2$, $n \in \mathcal{N}_a$, are quadratic forms of the $(L+1)$ channel coefficients.

[3]if $|H_n| = 0$, θ_n becomes non-identifiable. A channel with $L+1$ taps can have at most L zeros that coincide with the sub-carriers; clearly $N_a > L$ ensures that $H_n \not\equiv 0, \forall n \in \mathcal{N}_a$. With $N_a > L$, all the H_n's cannot be zero; hence having $H_n = 0$ for some $n \in \mathcal{N}_a$ will not affect the final CFO estimator.

We modify J_A as follows

$$J'_A(\xi, |\mathbf{H}|) = \sum_{n\in\mathcal{N}_a} \left(|X(n+\xi)|^2 - |H_n|^2\right)^2 \ . \quad (12)$$

The motivation for using this modified criterion is given next. Since our goal is to estimate the CFO by using a one-dimensional optimization procedure, we need to eliminate the $|H_n|$'s from the criterion in eq. (9). Towards this objective, we use $|H_n|^2$ instead of $|H_n|$ since the former can be re-parameterized as follows:

$$|H_n|^2 = \boldsymbol{c}_n^T \boldsymbol{\lambda}$$

where

$$\begin{aligned}
\boldsymbol{c}_n &= [1, \sqrt{2}\cos(2\pi n/N), \cdots, \sqrt{2}\cos(2\pi nL/N), \\
&\quad \sqrt{2}\sin(2\pi n/N), \cdots, \sqrt{2}\sin(2\pi nL/N)]^T \\
\boldsymbol{\lambda} &= [g_0, \sqrt{2}\mathcal{R}[g_1], \cdots, \sqrt{2}\mathcal{R}[g_L], \\
&\quad \sqrt{2}\mathcal{I}[g_1], \cdots, \sqrt{2}\mathcal{I}[g_L]]^T \\
g_i &= \sum_{l=0}^{L-i} h_l^* h_{l+i} \ .
\end{aligned}$$

The sequence $\{|H_n|^2\}$ is then described by linear combinations of the $(2L+1)$ elements of the parameter vector $\boldsymbol{\lambda}$. Note that both $\boldsymbol{c}_n$ and $\boldsymbol{\lambda}$ are real-valued vectors. The cost function J'_A is no longer the ML cost, but it does lead to a consistent estimate of ξ_o

By assumption we have that $\boldsymbol{c}_n^T\boldsymbol{\lambda} \not\equiv 0, \ \forall n \in \mathcal{N}_a$; further, from the definition of $\boldsymbol{c}$, we have that

$$\mathbf{C}_2 := \sum_{n\in\mathcal{N}_a} \boldsymbol{c}_n \boldsymbol{c}_n^T \ .$$

Setting $\partial J'_A/\partial\boldsymbol{\lambda} = 0$ yields the following closed-form estimator

$$\hat{\boldsymbol{\lambda}} = \mathbf{C}_2^\dagger \sum_{n\in\mathcal{N}_a} |X(n+\xi)|^2 \boldsymbol{c}_n \ , \quad (13)$$

where † denotes the pseudo-inverse. Substituting $\widehat{|H_n|^2} = \boldsymbol{c}_n^T\hat{\boldsymbol{\lambda}}$ into eqs. (7)-(9), we obtain the following criterion

$$\begin{aligned}
J(\xi) &= J_{VSC}(\xi) + J_{CM}(\xi) && (14)\\
J_{VSC}(\xi) &= \sum_{n\in\mathcal{N}_v} |X(n+\xi)|^2 && (15)\\
J_{CM}(\xi) &= \sum_{n\in\mathcal{N}_a} \left(|X(n+\xi)| - \sqrt{Y(n;\xi)}\right)^2 && (16)
\end{aligned}$$

where

$$Y(n;\xi) = \boldsymbol{c}_n^T \mathbf{C}_2^\dagger \sum_{n\in\mathcal{N}_a} |X(n+\xi)|^2 \boldsymbol{c}_n \ . \quad (17)$$

In the above criterion, J_{CM} exploits the constant modulus property of the symbol constellations. Using Parseval's theorem, eqn. (10), the two criteria can be merged together; after dropping constant terms, the pseudo-ML estimator of the CFO is obtained as

$$\hat{\xi}_o = \arg\min_{\xi} \sum_{n\in\mathcal{N}_a} \left(Y(n;\xi) - 2|X(n+\xi)|\sqrt{Y(n;\xi}\right) \quad (18)$$

where recall that $X(f)$ is the discrete Fourier transform of $x(k)$ in (1) at frequency f/N, and $Y(n;\xi)$ is defined in (17). We refer to the estimator minimizing J_{VSC} as the VSC-based estimator, which is equivalent to the estimator in [2] for a specific choice of $\mathcal{N}_v$. We refer to our estimator in eq. (18) as the VSC&CM-based estimator.

In [5], the performance improvement of the VSC&CM-based estimator over the VSC-based estimator was shown to be more than one order of magnitude (i.e., more than 10dB!) provided $N_a \gg L$ (typically $N_a > 2L$), which is usually the case in practice.

4. CFO ESTIMATION EXPLOITING THE FINITE ALPHABET PROPERTY

PSK constellations of size M satisfy the following property:

$$s_n^M = 1.$$

Therefore, in the noiseless case and for the true CFO, we obtain

$$[X(n+\xi_o)]^M = H_n^M = \left[\sum_{l=0}^{L} h_l e^{-j2\pi ln/N}\right]^M$$

which can also be rewritten as

$$[X(n+\xi_o)]^M = \sum_{l=0}^{ML} u_l e^{-j2\pi ln/N} = \boldsymbol{\gamma}_n^H \boldsymbol{u}$$

where $\boldsymbol{\gamma}_n = [1, e^{j2\pi n/N}, ..., e^{j2\pi MLn/N}]^T$ and $\boldsymbol{u}$ is an $(ML+1)\times 1$ vector whose elements are functions of the channel coefficients.

We now propose the following estimator

$$\begin{aligned}
J(\xi) &= J_{VSC}(\xi) + w\bar{J}_{FA}(\xi, \boldsymbol{u}) && (19)\\
\bar{J}_{FA}(\xi, \boldsymbol{u}) &= \sum_{n\in\mathcal{N}_a} \left|[X(n+\xi]^M - \boldsymbol{\gamma}_n^H \boldsymbol{u}\right|^2 && (20)
\end{aligned}$$

where w is a weight parameter to control the contribution of the finite-alphabet-based criterion with respect to that of the VSC-based criterion. If $ML+1 < N_a$, the parameter vector $\boldsymbol{u}$ can be uniquely determined as

$$\hat{\boldsymbol{\lambda}} = \boldsymbol{\Gamma}^\dagger \sum_{n\in\mathcal{N}_a} [X(n+\xi)]^M \boldsymbol{\gamma}_n \ , \quad (21)$$

where

$$\boldsymbol{\Gamma} := \sum_{n\in\mathcal{N}_a} \boldsymbol{\gamma}_n \boldsymbol{\gamma}_n^H \ .$$

Substituting this estimate in eq. (22), the finite alphabet-based criterion becomes

$$J_{FA}(\xi) = \sum_{n \in \mathcal{N}_a} \left| [X(n+\xi]^M - Z(n;\xi) \right|^2 \qquad (22)$$

where

$$Z(n;\xi) = \boldsymbol{\gamma}_n^H \boldsymbol{\Gamma}^\dagger \sum_{n \in \mathcal{N}_a} [X(n+\xi)]^M \boldsymbol{\gamma}_n$$

The final VSC&FA-based estimator is thus obtained as

$$\hat{\xi}_o = \arg\min_{\xi} [J_{VSC}(\xi) + w J_{FA}(\xi)] \qquad (23)$$

We have, therefore, shown that the constant-modulus and the finite alphabet properties can be exploited in blindly estimating ξ_o without increasing the dimension of the optimization procedure. Indeed, the VSC&CM-based and the VSC&FA-based estimators are obtained by a one dimensional search, the same as with the existing VSC-based estimator. The computations of the criteria in eqs. (18) and eqs. (23) are more demanding than that of J_{VSC}.

If the system is fully loaded, then $N_a = N$, and the VSC-based estimator fails. Our estimators, which in this case should be referred to as the CM-based and FA-based estimators, continue to perform well. Furthermore, in this case, $\mathbf{C}_2 = \boldsymbol{\Gamma} = N\mathbf{I}$ where $\mathbf{I}$ is the $N \times N$ identity matrix, and hence no matrix inversion is required. However, in this case, the CFO can be unambiguously estimated only in the interval $[-0.5, 0.5)$.

5. PERFORMANCE ANALYSIS

Here we only consider fully loaded systems so that the comparison of the algorithms proposed in this paper would not be affected by the VSC-based criterion. We compare the two estimators using Monte-Carlo simulations. Only one OFDM block is used. We consider a total of $N = Na = 64$ sub-carriers. The transmitted symbols are drawn from equiprobable MPSK constellations with M=2,4 or 8. The channel coefficients are generated using an uncorrelated Rayleigh scattering model with exponential power delay profile, i.e., $E\{h_i^* h_j\} = \exp(-\beta i)\delta(i-j)$, with $\beta = 1/5$. The CFO is generated randomly between -0.5 and 0.5. Both the channel coefficients and the CFO are generated randomly for each Monte-Carlo run. The signal-to-noise ratio (SNR) is defined as

$$SNR = 10 \log_{10} \frac{N_a \sum_{i=0}^{L} E\{|h_i|^2\}}{\sigma_v^2}.$$

where σ_v^2 is the variance of the additive noise. Mean-square error (MSE) was estimated empirically from 200 Monte-Carlo runs.

For BPSK symbols, the performance of the two estimators were found to be nearly the same (see fig. 1). For higher size constellations, the CM-based estimator outperformed the FA-based estimator.

6. CONCLUSIONS

In this paper, we proposed two blind frequency offset estimators for PSK-OFDM systems in the context of frequency-selective fading channels. We showed that exploiting the constant property of PSK symbols was more beneficial than exploiting the finite alphabet property of these constellations.

7. REFERENCES

[1] T. Pollet, M.V. Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise," *IEEE Trans. Commun.*, 43(2/3/4), pp. 191-193, 1995.

[2] H. Liu and U. Tureli, " A high-efficiency carrier estimator for OFDM communication, *IEEE Commun. Letts.*, 2(4), pp. 104-106, 1998.

[3] U. Tureli, H. Liu, M.D. Zoltowski, " OFDM blind carrier offset estimation: ESPRIT," *IEEE Trans. Commun.*, 48(9), pp. 1459-1461, 2000.

[4] M. Ghogho, A. Swami and G. Giannakis, "Opimized null-subcarrier selection for CFO estimation in OFDM over frequency-selective fading channels," *Proc. Globecom*-2001, San Antonio, USA, Nov. 2001.

[5] M. Ghogho and A. Swami, "A highly accurate blind frequency-offset synchronization fo OFDM communications," European Wireless Conference, Florence, Italy, Feb. 2002.

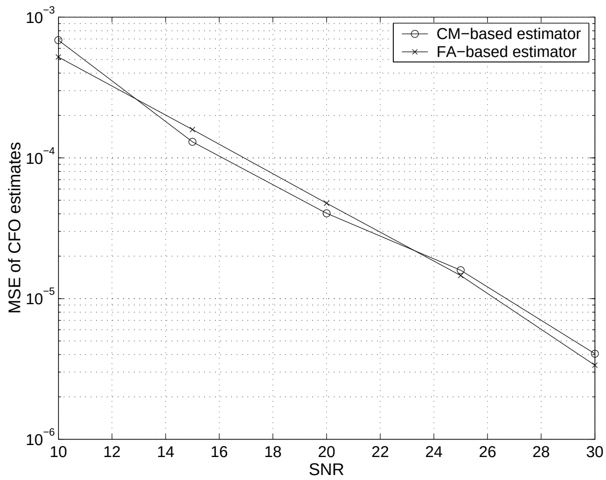

Fig. 1. MSE of CFO estimators vs. SNR; BPSK symbols; $L = 6$.