

# Interference Rejection in Systems Employing Transmit Diversity

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## ABSTRACT

Space diversity methods provide excellent means for increasing the robustness against interference and noise in communication systems. This is well known for systems using multiple receive antennas and it holds for systems applying transmit diversity as well. In fact, the use of transmit diversity provide two independent means for reducing the impact of interference. Via the diversity gain and via the code rate, both related to the underlying space-time code. In this paper, we are considering the use of space diversity methods in conjunction with interference rejection combining (IRC) to mitigate the impact of interference and noise for interference limited scenarios. Transmit diversity schemes based on space-time block codes (STBC) are assumed. We elucidate the importance of taking into account the spatio-temporal correlation induced by cochannel users employing transmit diversity. A novel space-time IRC (STIRC) scheme is proposed to handle the problem. The STIRC scheme combines the simplicity of conventional IRC while taking the structure of interfering users into account. Simulation examples demonstrate significant gains compared to the use of conventional IRC.

## 1 Introduction

The fact that most communication systems are interference limited has called for an emerging interest in finding algorithms to mitigate the impact of interference in systems employing space diversity. In practice, to suppress interference seen at the receiver two main tracks exist. Either the interference is modeled as a deterministic but unknown signal or it is modeled as a stochastic signal with some suitable statistics. Taking the former approach, interference cancellation and/or subspace filtering methods arise naturally whereas the later approach lends itself directly to noise whitening methods. In for example [1, 2], the space-time code structure is utilized to derive multiuser maximum likelihood (ML) decoding algorithms based on both spatio-temporal filtering as well as on interference cancellation techniques. In [3], decoding of STBC using the stochastic model approach is considered. Based on the assumption that the interference experienced by the detector is well modeled as a Gaussian unstructured noise signal i.e., as a temporally white but potentially spatially colored stochastic process, ML detectors using conventional in-

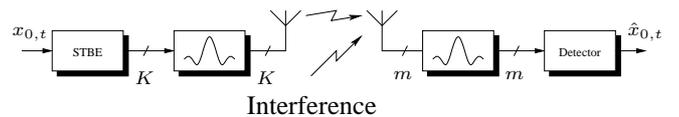


Figure 1: System model.

terference rejection combining (IRC) techniques are derived for both known and unknown channels.

Adopting the notion of modeling interference and background noise as a stochastic Gaussian distortion, in this paper the implications of using space diversity in conjunction with noise whitening methods are discussed. We introduce a novel space-time IRC (STIRC) scheme to handle the presence of cochannel interference (CCI) due to users employing transmit diversity based on space time block codes (STBC). The proposed scheme is an extension of the conventional IRC algorithm in the sense that it take advantage of the space-time structure imposed by the interfering users STBC. The outline of the paper is as follows. In Section 2 we introduce our system and data models. In Section 3, we give a brief introduction to the basic properties of STBC in terms of providing robustness against interference. Next we highlight the importance of taking the space-time structure of transmit diversity users into account in order not to deteriorate the performance of the receiver and we introduce the STIRC scheme. To illustrate the discussions in Section 3 some numerical examples are presented in Section 4. Finally, in Section 5 we summarize the paper and state our conclusions.

## 2 Data Models

Consider the system depicted in Fig. 1. Blocks of independent data symbols,  $\mathbf{x}_{0,p} = [x_{0,pk+1}, \dots, x_{0,pk+k}]$ , where each symbol  $x_{0,t}$  belongs to the same unitary finite complex alphabet  $\mathcal{X}$ , are mapped using a space-time block encoder (STBE) into sets of  $K$  codewords of length  $n$ . Transformed into parallel symbol sequences and pulse shaped, the waveforms representing the codewords are transmitted from different antenna elements. At the receiver, after passing an assumed dispersive time-varying propagation environment, the superposition of wavefields received from the desired and potentially a number of interfering users is sampled using an

array of  $m$  antenna elements. Down converted and processed by parallel noise reduction filters, the antenna output signals are sampled and fed to a detector producing estimates of the sent data.

A space-time block code is a mapping between two vector spaces defined by the code book,

$$\mathcal{S}(\mathcal{X}^{1 \times k}) = \{\mathbf{Y} \in \mathbb{C}^{K \times n} : \mathbf{Y} = \mathbf{S}(x), x \in \mathcal{X}^{1 \times k}\}.$$

Clearly, many different mappings  $\mathbf{S} : \mathcal{X}^{1 \times k} \mapsto \mathbb{C}^{K \times n}$  can be defined and represented in a number of different ways. In this paper, with  $\bar{x}$  and  $\tilde{x}$  denoting the real and imaginary part of  $x$  respectively, we will confine ourself to codes that may be written,

$$\mathbf{S}(x; \{\mathbf{A}_\kappa\}_{\kappa=1}^k, \{\mathbf{B}_\kappa\}_{\kappa=1}^k) = \sum_{\kappa=1}^k \mathbf{A}_\kappa \bar{x}_\kappa + j \mathbf{B}_\kappa \tilde{x}_\kappa, \quad (1)$$

where the mapping matrices  $\mathbf{A}_\kappa \in \mathbb{R}^{K \times n}$  and  $\mathbf{B}_\kappa \in \mathbb{R}^{K \times n}$ . The issue of finding good mapping matrices has received considerable attention in the literature. Based on both coding theoretical as well as signal-to-noise ratio (SNR) considerations it has been shown that optimal matrices must satisfy the following orthogonality constraints [4, 5],

$$\begin{aligned} \mathbf{A}_\kappa \mathbf{A}_\kappa^T &= \mathbf{I}_K, \quad \mathbf{B}_\kappa \mathbf{B}_\kappa^T = \mathbf{I}_K \\ \mathbf{A}_\kappa \mathbf{A}_\rho^T &= -\mathbf{A}_\rho \mathbf{A}_\kappa^T, \quad \mathbf{B}_\kappa \mathbf{B}_\rho^T = -\mathbf{B}_\rho \mathbf{B}_\kappa^T, \quad \kappa \neq \rho \\ \mathbf{A}_\kappa \mathbf{B}_\rho^T &= \mathbf{B}_\rho \mathbf{A}_\kappa^T, \end{aligned} \quad (2)$$

where  $\mathbf{I}_K$  denotes an identity matrix of dimension  $K \times K$  and  $(\cdot)^T$  the standard transpose operation.

Throughout this paper we will assume that the joint impulse response between any pair of transmit and receive filters are identical, real and satisfies the Nyquist criterion i.e.,  $g(t) \in \mathbb{R}$  and  $g(nT) = 0$ ,  $|n| > 0$ . For notational convenience, it will also be assumed that  $g(0) = 1 \geq |g(t)|$ . Justified by the assumptions that sufficiently spaced transmit and receive antennas are used, the time variations of the medium are slow and that the dispersion is small compared to the symbol period of the data, the propagation channels between the transmit and receive antennas will be modeled as mutually independent, frequency non-selective and block-wise fading.

### 3 Space Diversity Methods in Conjunction with IRC

Although the use of space diversity methods provides robustness against interference by their own right, it is natural to combine these methods with other forms interference suppressing measures. In this section we will consider IRC.

#### 3.1 Basic properties of STBC

From the space-time block code definition (1) it is clear that for a receiver to take advantage of the structure imposed by the code, the receiver has to process the input data stream in blocks of at least  $n$  samples. Assuming that the receiver knows the synchronization state of the codewords, the minimum decision statistics to retrieve the source data of the  $p$ th

sent space-time codeword is thus  $\mathbf{R}_p = [\mathbf{r}_{pn+1}, \dots, \mathbf{r}_{pn+n}]$ , where  $\mathbf{R}_p \in \mathbb{C}^{m \times n}$ .

If no cochannel interference is present the decision statistics can be related to the sent data as

$$\mathbf{R}_p = \mathbf{H}_0 \mathbf{S}_0(\mathbf{x}_{0,p}) + \mathbf{E}_p, \quad (3)$$

where  $\mathbf{x}_{0,p} \in \mathcal{X}^{1 \times k}$ ,  $\mathbf{S}_0(\mathbf{x}_{0,p}) = [\mathbf{s}_{0,pn+1}, \dots, \mathbf{s}_{0,pn+n}]$  is defined according to (1) and for notational convenience we have omitted the parameters of the STBC included in the definition.  $\mathbf{E}_p$  is the noise contribution matrix defined similar to  $\mathbf{R}_p$ . To handle the fact that (1) is not, in general, a linear function in  $x$  but rather in  $\bar{x}$  and  $\tilde{x}$ , it is advantageous to cast (3) into its equivalent real representation. If we define,  $\boldsymbol{\rho}_p = [\text{vec}(\bar{\mathbf{R}}_p)^T, \text{vec}(\tilde{\mathbf{R}}_p)^T]^T$ , the relationship between the decision statistics and the sent data can be formulated as,

$$\begin{aligned} \boldsymbol{\rho}_p &= \begin{bmatrix} [\mathbf{I}_n \otimes \bar{\mathbf{H}}_0] \mathbf{A}_0 & -[\mathbf{I}_n \otimes \tilde{\mathbf{H}}_0] \mathbf{B}_0 \\ [\mathbf{I}_n \otimes \tilde{\mathbf{H}}_0] \mathbf{A}_0 & [\mathbf{I}_n \otimes \bar{\mathbf{H}}_0] \mathbf{B}_0 \end{bmatrix} \begin{bmatrix} \bar{\mathbf{x}}_{0,p}^T \\ \tilde{\mathbf{x}}_{0,p}^T \end{bmatrix} + \boldsymbol{\epsilon}_p \\ &= [\mathbf{P}_{0,1}, \mathbf{P}_{0,2}] \begin{bmatrix} \bar{\mathbf{x}}_{0,p}^T \\ \tilde{\mathbf{x}}_{0,p}^T \end{bmatrix} + \boldsymbol{\epsilon}_p, \end{aligned} \quad (4)$$

where  $\mathbf{A}_0 = [\text{vec}(\mathbf{A}_{0,1}), \dots, \text{vec}(\mathbf{A}_{0,k})]$ ,  $\mathbf{B}_0$  is defined similar to  $\mathbf{A}_0$  and  $\boldsymbol{\epsilon}_p \sim \mathcal{N}(\mathbf{0}_{2mn \times 1}, \frac{\sigma^2}{2} \mathbf{I}_{2mn})$  is the corresponding noise term defined similar to  $\boldsymbol{\rho}_p$ . Using the optimality criterion defined by (2) it follows directly that the equivalent channel matrices  $\mathbf{P}_{0,1}$  and  $\mathbf{P}_{0,2}$  satisfy the following properties:

$$\mathbf{P}_{0,i}^T \mathbf{P}_{0,j} = \delta_{ij} \text{tr}(\bar{\mathbf{H}}_0^T \bar{\mathbf{H}}_0 + \tilde{\mathbf{H}}_0^T \tilde{\mathbf{H}}_0) \mathbf{I}_k, \quad (5)$$

where  $\delta_{ij}$  is the Kronecker delta function and  $\text{tr}(\cdot)$  denotes the standard trace operation. From (4) and (5), two important and general observations can be made. First, from (5), the celebrated capability of STBC to provide a diversity gain of  $mK$  is readily verified. Secondly, by observing that  $\mathbf{P}_i \in \mathbb{R}^{2mn \times k}$  it directly follows that the signal space of  $\boldsymbol{\rho}_p$  occupies  $2k$  dimensions in the  $2mn$  dimensional observation space. Hence, leaving  $2mn - 2k$  degrees of spatio-temporal freedom available for interference suppression means. To sum up, transmit diversity methods based on STBC provide robustness against interference in two ways; via the diversity gain controlled by the parameter  $K$ , and via the code rate controlled by the ratio of the  $k$  and  $n$  parameters.

#### 3.2 Interference rejection combining

Interference rejection combining amounts to suppressing the joint impact of interference and noise in a detector by modeling it as a colored noise distortion with suitable properties and distribution [6, 7]. Due to the noise model assumption, the ML detection rule is often trivially derived. For example, in our considered case where, if we adopt a Gaussian model to describe the impact of cochannel interference and background noise, the ML estimates are found as

$$\begin{aligned} \hat{\mathbf{x}}_{0,t} &= \arg \max_{\mathbf{x} \in \mathcal{X}} \boldsymbol{\rho}_t^T \mathbf{Q}^{-1} [\mathbf{P}_{0,1}, \mathbf{P}_{0,2}] \begin{bmatrix} \bar{\mathbf{x}}^T \\ \tilde{\mathbf{x}}^T \end{bmatrix} \\ &= \arg \max_{\mathbf{x} \in \mathcal{X}} \sum_{\kappa=1}^k \mathbf{P}_{0,1,\kappa}^T \mathbf{Q}^{-1} \boldsymbol{\rho}_t \bar{x}_\kappa + \mathbf{P}_{0,2,\kappa}^T \mathbf{Q}^{-1} \boldsymbol{\rho}_t \tilde{x}_\kappa, \end{aligned} \quad (6)$$

where  $\mathbf{Q}$  denotes the covariance matrix of the error term  $\epsilon$  which now accounts for the impact of both cochannel interference as well as background noise.  $\mathbf{P}_{0,j,\kappa}$  denotes the  $\kappa$ th column of  $\mathbf{P}_{0,j}$ . Note that due to the fact that the columns of  $\mathbf{P}_{0,1}$  and  $\mathbf{P}_{0,2}$  are orthogonal, the joint ML criterion decouples into  $k$  scalar detection problems.

From (6) it is clear that the performance of the detector will be closely related to how well the assumed second order statistics match the true characteristics of the interference and noise. To study this problem more in detail, let us assume that the desired user is interfered by a small number of synchronous cochannel users. In addition, to model the interference as a Gaussian distortion, let us further assume that the data samples of the interfering users,  $[\tilde{x}_{d,t}, \tilde{x}_{d,t}]^T$  for  $d > 0$ , are independent and Gaussian distributed with zero mean and covariance matrix given by  $\frac{1}{2}\mathbf{I}_2$ .

### 3.2.1 Conventional IRC

In [3], transmit diversity signaling in combination with conventional IRC is considered. Under the assumption that the interference and noise seen at the input port of the detector may be modeled as temporally white, an unstructured ML estimator for the covariance matrix of the noise is derived. Since the derived estimator is consistent it will, cast into the framework of this paper, converge to

$$\mathbf{Q}^{\text{us}} = \frac{1}{2} \sum_{d=1}^D \begin{bmatrix} \mathbf{Q}_{d,1}^{\text{us}} & \mathbf{Q}_{d,2}^{\text{us}} \\ \mathbf{Q}_{d,2}^{\text{us}T} & \mathbf{Q}_{d,1}^{\text{us}} \end{bmatrix} + \frac{\sigma^2}{2} \mathbf{I}_{2mn}, \quad (7)$$

$$\mathbf{Q}_{d,1}^{\text{us}} = \mathbf{I}_n \otimes (\bar{\mathbf{H}}_d \bar{\mathbf{H}}_d^T + \tilde{\mathbf{H}}_d \tilde{\mathbf{H}}_d^T),$$

$$\mathbf{Q}_{d,2}^{\text{us}} = \mathbf{I}_n \otimes (\bar{\mathbf{H}}_d \tilde{\mathbf{H}}_d^T - \tilde{\mathbf{H}}_d \bar{\mathbf{H}}_d^T).$$

### 3.2.2 Space-Time IRC

For interference limited scenarios where cochannel users potentially employing transmit diversity schemes based on STBC may also be present, the temporal whiteness assumption made in [3] is not appropriate. The reason for this is due to the fact that the algebraic structures imposed by the transmit diversity schemes of the interfering users are not considered. Taking these structures into account it is easily verified that a consistent estimator for  $\mathbf{Q}$  will converge to<sup>1</sup>

$$\mathbf{Q}^{\text{st}} = \frac{1}{2} \sum_{d=1}^D \begin{bmatrix} \mathbf{Q}_{d,1}^{\text{st}} & \mathbf{Q}_{d,2}^{\text{st}} \\ \mathbf{Q}_{d,2}^{\text{st}T} & \mathbf{Q}_{d,1}^{\text{st}} \end{bmatrix} + \frac{\sigma^2}{2} \mathbf{I}_{2mn}, \quad (8)$$

$$\mathbf{Q}_{d,1}^{\text{st}} = [\mathbf{I}_n \otimes \bar{\mathbf{H}}_d] \mathbf{A} \mathbf{A}^T [\mathbf{I}_n \otimes \bar{\mathbf{H}}_d^T]$$

$$+ [\mathbf{I}_n \otimes \tilde{\mathbf{H}}_d] \mathbf{B} \mathbf{B}^T [\mathbf{I}_n \otimes \tilde{\mathbf{H}}_d^T],$$

$$\mathbf{Q}_{d,2}^{\text{st}} = [\mathbf{I}_n \otimes \bar{\mathbf{H}}_d] \mathbf{A} \mathbf{A}^T [\mathbf{I}_n \otimes \tilde{\mathbf{H}}_d^T]$$

$$- [\mathbf{I}_n \otimes \tilde{\mathbf{H}}_d] \mathbf{B} \mathbf{B}^T [\mathbf{I}_n \otimes \bar{\mathbf{H}}_d^T].$$

<sup>1</sup>Note: To arrive at the result in (8) we have assumed that the mapping matrices of all cochannel users satisfy the properties defined by (2). This assumption is clearly satisfied if all cochannel users employ the same STBC as the desired user. It however also holds for scenarios where non-transmit diversity and transmit diversity users are mixed as long as all transmit diversity users use the same STBC.

Clearly, for the two estimators to be identical,  $\mathbf{A} \mathbf{A}^T = \mathbf{I}_{Kn}$  and  $\mathbf{B} \mathbf{B}^T = \mathbf{I}_{Kn}$ , must hold. This is however a property that is obviously not true for space-time block codes in general. The implication of this is that if the unstructured estimator is used in scenarios where the receiver has to deal with strong interference due to transmit diversity users, a substantial performance degradation can be expected since the noise whitening operation performed by the detector is based on an estimate that does not accurately describe the second order statistics of the interference. For noise limited cases the difference between the two estimators is expected to be negligible.

## 4 Numerical Examples

To examine the performance and study the behavior of a space diversity system operating under interference limited conditions, simulations have been conducted for several different cases. For all examined cases, the setup shown in Fig. 1 was considered assuming synchronous cochannel users, perfect knowledge regarding all users and their channel state information as well as known properties of the background noise. Throughout the tests, the channels were assumed constant during the transmission of one frame of data, comprising a total of 116 QPSK source symbols. The fading was assumed independent from frame to frame.

The definitions of the considered signal-to-noise and signal-to-interference ratios are  $\text{SNR} = \frac{E\{\|\mathbf{H}_0 \mathbf{s}_{0,t}\|_F^2\}}{mK\sigma^2}$  and  $\text{SIR} = \frac{E\{\|\mathbf{H}_0 \mathbf{s}_{0,t}\|_F^2\}}{E\{\|\sum_{d=1}^D \mathbf{H}_{d,q}(q,q^{-1};\tau_d) \mathbf{s}_{d,t}\|_F^2\}}$  respectively where  $\|\cdot\|_F$  denotes the Frobenius norm. For each presented result,  $10^4$  frames of data were sent through the system.

### 4.1 Example 1: Robustness provided by STBC

In Section (3.1) it was shown that STBC can provide robustness against interference in two ways. Via the diversity gain and via the code rate. To illustrate these properties and in addition highlight the importance of taking the structure of the interference into account, three different setups have been studied for the case of no receive diversity ( $m = 1$ ). The setups are, no transmit diversity and transmit diversity with  $(k, n, K) = \{(2, 4, 2), (2, 2, 2)\}$  respectively. For the  $k/n = 1/2$  rate case we have considered a complex orthogonal design, straightforwardly constructed from a real (2,2,2) orthogonal design, c.f. [4] for details, in our framework defined as

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}, \quad (9)$$

whereas in the full rate case the well known space-time block code proposed by Alamouti[8] was considered i.e.,

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (10)$$

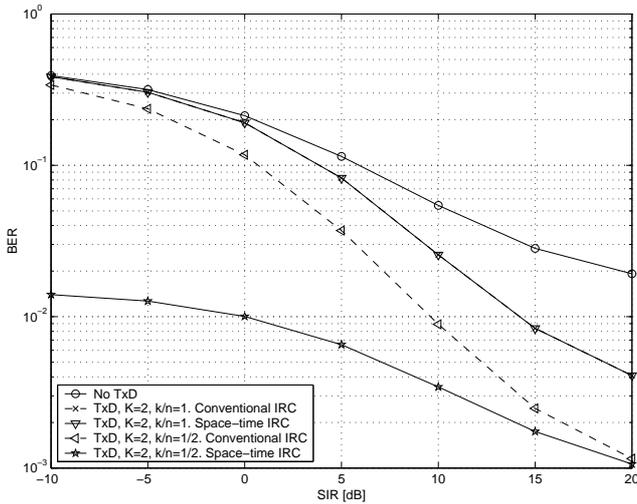


Figure 2: BER versus SIR. 1 CCI user. SNR = 15 dB.  $m = 1$ .

From Fig. 2 the impact of the diversity and code gains are clearly observed. In the full-rate case a distinct and increasingly growing gain as the SIR increases and the performance of the detector converge towards its error floor due to the background noise can be seen. Also, note that the performance due to the conventional and the space-time IRC are identical as expected. For the 1/2-rate case we see the first example of the dramatic performance difference experienced by neglecting the structure of the interference. For the receiver using the conventional IRC method a non-insignificant gain as compared to the full-rate case can be noted. However, compared to the performance attainable using the STIRC method, the gain of the former method is small. Especially for low SIR although the performance difference between the two methods decreases as the SIR level increases.

#### 4.2 Example 2: Robustness utilizing space diversity

In this subsection we highlight the impact of combining transmit and receive diversity conditioned that the structure of the interference is/is not considered. We restrict the study to the case of full rate STBC, c.f. (10).

In Fig. 3, the simulation results obtained for the case of one dominating interfering user is presented. For the receive diversity case using  $m = 2$  antennas, we again see a substantial performance drop for low to moderate SIR values when not taking the structure of the interference into account. Compared to the 1/2 rate case discussed earlier, we can note that an even larger relative performance degradation is experienced for this case.

## 5 Conclusions

In this paper we have introduced a novel space-time IRC (STIRC) scheme which exploits the structure of interferers using space-time block codes (STBC). Furthermore, we discuss the implications of using conventional interference rejection combining (IRC) in system where cochannel users

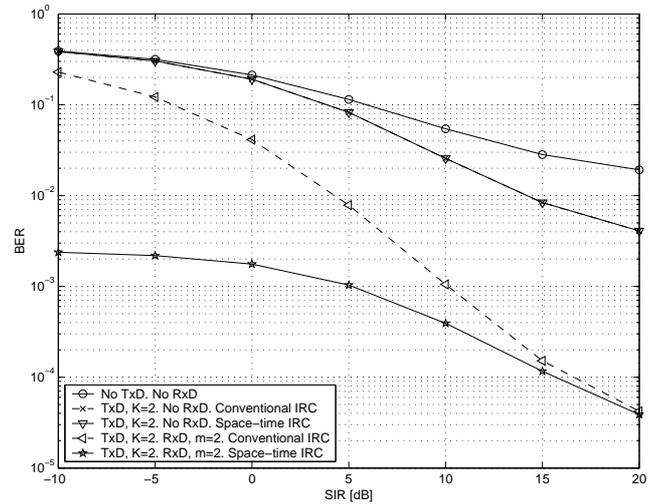


Figure 3: BER versus SIR. 1 CCI user. SNR = 15 dB.

employing transmit diversity may be present. Via simulation examples we demonstrate both the effectiveness of the proposed STIRC scheme as well as the importance of taking the structure of transmit diversity users into account in order not to substantially degrade the performance of an interfered receiver.

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