ON THE GENERATION OF SEQUENCES SIMULATING HIGHER ORDER WHITE NOISE FOR SYSTEM IDENTIFICATION

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ABSTRACT

In this paper, we study the generation of binary sequences that exhibit the characteristics of higher order white noise signals. Three classes of binary sequences are examined: Gold sequences, dual–BCH sequences, and sequences generated by the term–by–term modulo 2 addition of two maximal length sequences whose least periods are relatively prime. It is shown that in all of the cases the autocorrelation and the higher order moment spectra are determined by the crosscorrelation function of the component sequences used in each construction. The number of peaks appearing in higher order moments is significantly reduced or vanished in all classes. The quality and efficiency of sequences from each class in simulating higher order white noise signals is demonstrated by simulations in the context of bilinear input–output system identification.

1 INTRODUCTION

White noise signals have been widely used in the area of system identification because they have a rich spectrum and will therefore affect a large number of system modes, making them particularly suitable for linear [10] and especially nonlinear system identification [7], [8]. Binary maximal length sequences (m–sequences) can approximate very well second order white noise signals and their properties have been studied analytically in the context of linear [10] and nonlinear [7] system identification.

However, maximal length sequences have the disadvantage that they cannot be used to approximate higher order white noise signals due to the existence of peaks in their higher order statistics (moments and cumulants). In many important applications like blind identification of linear systems [4], identification of finite Volterra series [6], [7], [8] and bilinear input–output models [11], e.t.c., the identification procedure requires that the unknown system is excited with a higher order white noise input signal. In this paper, we study the generation of binary sequences that exhibit the characteristics of higher order white noise signals and are obtained from appropriately selected pairs of m–sequences of either the same or different least periods.

We show that the autocorrelation function and the higher order moments of a binary sequence constructed by the modulo 2 addition of two m–sequences of arbitrary least periods, depend on their crosscorrelation almost everywhere. When the two m–sequences have the same least period then under certain conditions ([2], [9]) their crosscorrelation function takes three specific values, and the number of higher order moment peaks of the resulting binary sequence is significantly reduced or vanished. We also show that if a sequence belongs to the dual code of a binary t–error correcting BCH code then all its higher order moments up to order $2t$ are free of peaks. When the two m–sequences have relatively prime least periods their crosscorrelation function is constant everywhere, and thus the values of the statistics of the resulting binary sequence and the specific positions where these values appear are determined a priori by the choice of the particular m–sequences. Moreover, although peaks appear in the higher order moments almost as often as in the case of m–sequences, these peaks are usually positioned quite far from the origin, thus making these sequences almost ideal for simulating higher order white noise signals in some identification problems. The quality of the above binary sequences in simulating higher order white noise signals is demonstrated by simulations in the identification of bilinear input–output models, where m–sequences fail to provide unbiased estimates of the system parameters. All the results presented in this paper are stated without proof which can be found in [5].

2 MAXIMAL LENGTH SEQUENCES

In this section we consider the case of binary m–sequences. We also introduce the notion of trinomial pairs and show that such pairs characterize m–sequences as inappropriate for simulating higher order white noise signals. In what follows $\mathbb{F}_2$ is the prime field $\{0, 1\}$ of 2 elements, $\oplus$ denotes modulo 2 addition and $\mathbb{Z}_N = \{0, 1, \ldots, N-1\}$ is the set of integers modulo N.

Let x be an infinite periodic binary sequence of least period N generated by a linear feedback shift register (LFSR) with characteristic polynomial $g(z) = 1 \oplus g_1 z \oplus \cdots \oplus g_{n-1} z^{n-1} \oplus z^n \in \mathbb{F}_2[z]$, and let $\boldsymbol{x} = (x_0, x_1, \ldots, x_{N-1})$ be the first N elements of x, where $x_t \in \mathbb{F}_2$ for all $t \geq 0$. Then sequence x satisfies the *linear recurrence relation* [3]

$$x_t = g_{n-1} x_{t-1} \oplus \cdots \oplus g_1 x_{t-n+1} \oplus x_{t-n}, \quad t \geq n \qquad (1)$$

which is associated to the polynomial $g^*(z) = 1 \oplus g_{n-1} z \oplus \cdots \oplus g_1 z^{n-1} \oplus z^n = z^n g(1/z)$, called the *reciprocal* of $g(z)$. For the rest of the paper, $g(z)$ will be referred to as the *minimal polynomial* of sequence x, i.e. the lowest degree polynomial whose corresponding LFSR generates the sequence, and for a given sequence x it is unique. Moreover, periodic binary sequences with the property that the numbers of ones and zeros in one period differ by at most 1 will be called *balanced sequences*, and the statistics of sequence x will be defined in terms of the $\{+1, -1\}$ version $\widetilde{x}$ of x, given by

$\widetilde{x}_t = (-1)^{x_t}$, since this is the one that normally arises in systems analysis [9].

Sequence x is an m–sequence, i.e. its least period N has the maximum possible value which is equal to $2^n - 1$, if and only if $g(z)$ is a primitive polynomial. If D denotes the delay operator which shifts vectors cyclically to the right by one position, then clearly $D^t\boldsymbol{x} = (x_{N-t}, \ldots, x_{N-1}, x_0, \ldots, x_{N-t-1})$ and obviously it holds that $D^{-t}\boldsymbol{x} = D^{N-t}\boldsymbol{x}$. The most important properties of m–sequences are described next ([3], [4]):

(i) m–sequences have the well known *shift–and–add* property, that is for any $t_1, t_2 \in \mathbb{Z}_N$, $t_1 \neq t_2$, there exists a unique $t_3 \in \mathbb{Z}_N$ distinct from both t_1 and t_2 such that $D^{t_1}\boldsymbol{x} \oplus D^{t_2}\boldsymbol{x} = D^{t_3}\boldsymbol{x}$.

(ii) The number of ones in $\boldsymbol{x}$, denoted by $\mathrm{wt}(\boldsymbol{x})$, is equal to 2^{n-1} and hence m–sequences are balanced.

The periodic autocorrelation function R_x of sequence x is the real–valued function given by the formula

$$R_x(\tau) = \frac{1}{N}\sum_{t=0}^{N-1}(-1)^{x_t \oplus x_{t-\tau}} = 1 - \frac{2}{N}\,\mathrm{wt}(\boldsymbol{x} \oplus D^{\tau}\boldsymbol{x}) \quad (2)$$

where $\tau \in \mathbb{Z}_N$. By properties (i) and (ii) we infer that m–sequences have the *ideal autocorrelation* property

$$R_x(\tau) = \begin{cases} 1 & \text{if } \tau \equiv 0 \pmod{N}, \\ -1/N & \text{otherwise.} \end{cases}$$

The autocorrelation of an m–sequence with least period 1023 and minimal polynomial $g(z) = 1 \oplus z^3 \oplus z^{10}$ is shown in Fig. 1 where we have used the obvious identity $R_x(-\tau) = R_x(N - \tau)$. The periodic binary sequence x forms a white noise signal of order k if its cumulants up to order k are multidimensional impulse functions, that is: $\mathrm{cum}[\boldsymbol{x}, D^{\tau_1}\boldsymbol{x}, \ldots, D^{\tau_{s-1}}\boldsymbol{x}] = \gamma_s \delta(\tau_1)\cdots\delta(\tau_{s-1})$ for all $s = 2, \ldots, k$, where "cum" denotes the cumulant of order s, $\delta(\cdot)$ is the usual delta function and γ_s is the sth order intensity of x. Cumulants and moments are closely related and the previous definition is equivalent to higher order independence of x ([4]). If the expected value of x is zero, as it is considered to be the case of balanced sequences, the cumulants up to order three are equal to the corresponding moments of equal order. As a result, m–sequences are well suited for simulating second order white noise signals as long as their least period N is sufficiently long. The third order moment sequence of x is defined in accordance to (2), as follows

$$\begin{aligned} R_x(\tau_1, \tau_2) &= \frac{1}{N}\sum_{t=0}^{N-1}(-1)^{x_t \oplus x_{t-\tau_1} \oplus x_{t-\tau_2}} \\ &= 1 - \frac{2}{N}\,\mathrm{wt}(\boldsymbol{x} \oplus D^{\tau_1}\boldsymbol{x} \oplus D^{\tau_2}\boldsymbol{x}) \end{aligned}$$

where $\tau_i \in \mathbb{Z}_N$, and is symmetric. Properties (i) and (ii) imply that the third order moment of m–sequences takes the value $-1/N$ everywhere in $\mathbb{Z}_N^2$ except at pairs $(\tau_1, \tau_2) \in \mathbb{Z}_N^2$ such that $\boldsymbol{x} = D^{\tau_1}\boldsymbol{x} \oplus D^{\tau_2}\boldsymbol{x}$. These pairs (τ_1, τ_2) are called *trinomial pairs* of sequence x and they correspond to maximal peaks in the third order moment sequence, since then we have $R_x(\tau_1, \tau_2) = 1$. Since m–sequences are balanced it is clear that trinomial pairs cause obstruction of higher order white noise randomness. Moreover, in the case of m–sequences for each $\tau_1 = 1, \ldots, N-1$ there exists exactly one trinomial pair (τ_1, τ_2) due to property (i). In the sequel, the set of all trinomial pairs of the sequence x with minimal polynomial $g(z)$ is denoted by T_g and its order by $|T_g|$.

The third order moments of the m–sequence x with minimal polynomial $g(z) = 1 \oplus z^3 \oplus z^{10}$ is shown in Fig. 1.

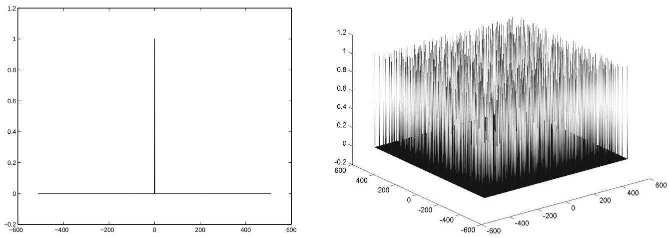

Figure 1: The autocorrelation and the third order moments of the m–sequence x with minimal polynomial $g(z) = 1 \oplus z^3 \oplus z^{10}$

As expected $|T_g| = 1022$. Hence, m–sequences cannot be employed in applications where a higher order white noise signal is required. We also note that all of the above and subsequent results can easily be generalized to the case of moment sequences of order greater than three.

3 GOLD AND DUAL–BCH SEQUENCES

In this section we study the case of binary sequences obtained from pairs of m–sequences of the same least period, as well as the case of dual–BCH sequences. The number of peaks appearing in the higher order moments of these sequences is significantly reduced or vanished.

If the minimal polynomials of any two binary sequences x and y are $g(z)$ and $f(z)$ respectively, then the minimal polynomial of sequence $w = x \oplus y$ is the least common multiple (lcm) of $g(z)$ and $f(z)$ ([3]). The interest in such sequences is motivated by the following Theorem which implies that the number of peaks appearing in the third order moments of w is reduced or vanished, depending on the particular choice of x and y.

Theorem 1. *Let x and y be two distinct binary sequences of the same least period N, and let their minimal polynomials be $g(z)$ and $f(z)$ respectively. Then, the set of all trinomial pairs of sequence $w = x \oplus y$ is given by: $T_{\mathrm{lcm}(g,f)} = T_g \cap T_f$.*

Theorem 1 is still valid if x and y have different least periods. For example, if sequence x has least period M, where M divides N, then Theorem 1 can be applied by replacing T_g by the set of all trinomial pairs T_g' of x in $\mathbb{Z}_N^2$ which is given as follows:

$$T_g' = \{(\tau_1, \tau_2) \in \mathbb{Z}_N^2 : (\tau_1 \bmod M, \tau_2 \bmod M) \in T_g\}.$$

The periodic crosscorrelation function $R_{x,y}$ of sequences x and y with least period N is the real–valued function defined similarly to (2) as follows

$$R_{x,y}(\tau) = \frac{1}{N}\sum_{t=0}^{N-1}(-1)^{x_t \oplus y_{t-\tau}} = 1 - \frac{2}{N}\,\mathrm{wt}(\boldsymbol{x} \oplus D^{\tau}\boldsymbol{y}) \quad (3)$$

where $\tau \in \mathbb{Z}_N$. If x and y have different least periods, say N_1 and N_2 respectively, then (3) is still valid but now N equals the $\mathrm{lcm}(N_1, N_2)$, and is the least period of sequence $w = x \oplus y$. The following two Theorems imply that the statistics of sequence w can be fully determined by the crosscorrelation spectra of x and y when both sequences are maximal length.

Theorem 2. *Let x and y be m–sequences, of least periods N_1 and N_2, and with minimal polynomials $g(z)$ and $f(z)$ respectively. Then, the autocorrelation function R_w of sequence $w = x \oplus y$, of least period $N = \mathrm{lcm}(N_1, N_2)$, is given by*[1]

[1]The notation $\tau \equiv 0 \pmod{N_i}$ $i = 1, 2$, implies that τ is a nonzero multiple of N_i but not of N.

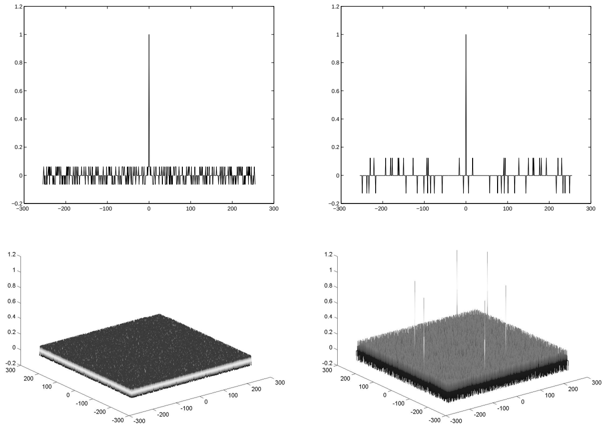

Figure 2: The autocorrelation and the third order moments of sequence $w = x \oplus y$, where y is obtained by decimating the m–sequence x with minimal polynomial $g(z) = 1 \oplus z^4 \oplus z^9$, by 5 (left) and by 9 (right)

$$R_w(\tau) = \begin{cases} 1 & \text{if } \tau \equiv 0 \pmod{N}, \\ -1/N_1 & \text{if } \tau \equiv 0 \pmod{N_2}, \\ -1/N_2 & \text{if } \tau \equiv 0 \pmod{N_1}, \\ R_{x,y}(\tau') & \text{otherwise,} \end{cases}$$

for some integer $\tau' \in \mathbb{Z}_N$.

Theorem 3. *Let x and y be m–sequences as defined above. Then, the third order moments $R_w(\tau_1, \tau_2)$ of sequence $w = x \oplus y$ are given by*

$$R_w(\tau_1, \tau_2) = \begin{cases} 1 & \text{if } (\tau_1, \tau_2) \in T'_g \cap T'_f, \\ -1/N_1 & \text{if } (\tau_1, \tau_2) \in T'_f \setminus (T'_g \cap T'_f), \\ -1/N_2 & \text{if } (\tau_1, \tau_2) \in T'_g \setminus (T'_g \cap T'_f), \\ R_{x,y}(\tau) & \text{otherwise,} \end{cases}$$

for some integer $\tau \in \mathbb{Z}_N$.

T'_g, T'_f denote the sets of all trinomial pairs of sequences x, y respectively in $\mathbb{Z}_N^2$, and the notation $A \setminus B$ represents the subset of all elements of A which do not belong in B, where B is a subset of A. The moments of order greater than three of sequence w can still be determined by Theorem 3.

If x and y have the same least period and the roots of their minimal polynomials are related to each other as described below, then their crosscorrelation function takes a fixed set of values ([2], [9]), and the resulting class of binary sequences $w = x \oplus y$ are called *Gold sequences.*

Theorem 4 (Gold). *Let us consider the m–sequences x and y, of least period $N = 2^n - 1$, with minimal polynomials $g(z)$ and $f(z)$ respectively. Let the roots of $f(z)$ be the dth powers of the roots of $g(z)$, where either $d = 2^k + 1$ or $d = 2^{2k} - 2^k + 1$, and $e = \gcd(k, n)$ is such that n/e is odd. Then, the spectrum of $R_{x,y}$ is three–valued and*

$(-1 + 2^{(n+e)/2})/N$	occurs	$2^{n-e-1} + 2^{(n-e-2)/2}$	times,
$(-1)/N$	occurs	$2^n - 2^{n-e} - 1$	times,
$(-1 - 2^{(n+e)/2})/N$	occurs	$2^{n-e-1} - 2^{(n-e-2)/2}$	times,

per period.

Obviously, the value of e affects the magnitude of $R_{x,y}$. In particular, there is a tradeoff between the number of times the values $(-1 \pm 2^{(n+e)/2})/N$ appear, and their magnitude. Hence, the value of e should be chosen according to the context of the specific application. We also note that the assumption of Theorem 4 concerning the roots of $f(z)$ and $g(z)$ can be satisfied if y is obtained by decimating x by d ([3]).

As a consequence, Theorems 2 and 3 (for $N_1 = N_2 = N$) together with Theorem 4 can be used to obtain sequences whose autocorrelation and higher order moments can be controlled by the choice of certain parameters. Moreover, the number of higher order moment peaks is significantly reduced or even vanished. This is illustrated in Fig. 2, where y is obtained by decimating the m–sequence x with minimal polynomial $g(z) = 1 \oplus z^4 \oplus z^9$ and least period 511 ($n = 9$), by 5 ($e = 1$, $k = 2$) and by 9 ($e = 3$, $k = 3$). The sequences w used in this example were balanced. This can be easily achieved if we notice that since $\sum_{t=0}^{N-1}(-1)^{w_t} = NR_{x,y}(0)$, w is balanced if and only if $R_{x,y}(0) = -1/N$. This example also illustrates the tradeoff between the magnitude of the values of $R_{x,y}$ and the number of times they occur in one period.

The class of dual–BCH sequences is also of great importance since their higher order moments are free of peaks up to an order determined by the designed distance of the BCH code:

Theorem 5. *Let $g^*(z)$ be the generating polynomial of a binary t–error correcting BCH code ([1], [4]). Then, the binary sequence w with minimal polynomial $g(z)$ has no peaks in its sth order moment, for all integers $s \leq 2t$.*

We note that when the integer n in Theorem 4 is odd and $e = 1$, $k = 1$ and $d = 3$, the resulting Gold sequence $w = x \oplus y$ is a double–error correcting dual–BCH sequence with both sequences x and y being maximal length.

4 M–SEQUENCES WITH RELATIVELY PRIME LEAST PERIODS

In this section we study the case of binary sequences obtained from pairs of m–sequences with relatively prime least periods. The crosscorrelation function of these m–sequences is constant everywhere as the following Theorem indicates.

Theorem 6. *The periodic crosscorrelation function $R_{x,y}(\tau)$ of the m–sequences x and y, with relatively prime least periods N_1 and N_2, and with minimal polynomials $g(z)$ and $f(z)$ respectively, equals $1/N$ for all $\tau \in \mathbb{Z}_N$, where $N = N_1 N_2$.*

As a result, the autocorrelation function and the higher order moments of sequence $w = x \oplus y$ of least period N, where x and y satisfy the assumptions of Theorem 6, are given by Theorems 2 and 3 by substituting $R_{x,y}$ with $1/N$. Therefore, R_w takes four values which appear a specific number of times in one period and also at specific positions determined by the least periods of x and y. Similar remarks hold for the higher order moments of w. In addition, sequence w is always balanced since $\sum_{t=0}^{N-1}(-1)^{w_t} = NR_{x,y}(0) = 1$. The set of all trinomial pairs T_{gf} of sequence w can be explicitly expressed as follows ([5]):

$$T_{gf} = \{(\tau_1^w, \tau_2^w) \in \mathbb{Z}_N^2 : \ \tau_i^w = \tau_i^x N_2^{\varphi(N_1)} + \tau_i^y N_1^{\varphi(N_2)} \bmod N \\ \text{for } i = 1, 2, \text{ where } (\tau_1^x, \tau_2^x) \in T_g \text{ and } (\tau_1^y, \tau_2^y) \in T_f\}$$

where $\varphi(\cdot)$ is the Euler's totient function ([4]). Moreover, it holds that $|T_{gf}| = |T_g||T_f|$. A direct implication of the above expression is that τ_i^w runs through all elements of $\mathbb{Z}_N$ which are not zero or a multiple of N_i, exactly once. It can also be shown that sequence w has no trinomial pairs in $\mathbb{Z}_{N_1}^2$ or $\mathbb{Z}_{N_2}^2$, if and only if x and y have no common trinomial pairs in $\mathbb{Z}_{N_1}^2$ and $\mathbb{Z}_{N_2}^2$ respectively. This usually holds in practice, and hence the trinomial pairs of w are in most cases positioned quite far from the origin. Since this property can easily be generalized to higher order moments of w, we conclude that this class of binary sequences is almost ideal for simulating higher order white noise signals in identification problems where peaks in higher order moments of the excitation signal,

		m–sequences		Gold sequences	
	True Value	Mean	Variance	Mean	Variance
a1	1.4	1.7551	5.3229	1.3965	6.8×10^{-1}
a2	-0.48	-3.7441	343.4219	-0.5984	7.9262
b0	1	0.9785	1.0×10^{-4}	1.0104	2.0×10^{-4}
b1	0.5	0.1752	5.1383	0.4450	7.2×10^{-1}
c11	0.05	0.0288	2.0×10^{-4}	0.1128	2.0×10^{-4}
c12	0.1	0.1026	2.4×10^{-3}	0.1147	1.7×10^{-3}
c22	0.2	0.1903	1.2×10^{-2}	0.1527	6.4×10^{-3}

Table 1: $N = 4095$, 40 runs, SNR=20dB, maximal versus Gold sequences

which are far from the origin, do not affect the identification procedure.

5 SIMULATIONS

In this section we demonstrate the quality and efficiency of the proposed binary sequences in simulating higher order white noise signals, by providing simulation results in comparison with m–sequences, for the identification of two bilinear input–output models of the form:

$$z(n) = \sum_{i=1}^{k_1} a_i z(n-i) + \sum_{i=1}^{k_2} \sum_{j=i}^{k_3} c_{ij} z(n-i) u(n-j) + \\ + \sum_{i=0}^{k_4} b_i u(n-i), \qquad y(n) = z(n) + \eta(n)$$

using the cumulant based algorithm of [11]. $y(n)$ is the measured output of the model and $u(n)$ is the higher order white noise input signal. The measurement noise $\eta(n)$ is assumed to be a zero mean random process independent from $u(n)$, and in all simulations was a Gaussian IID random process. If $p_1, p_2, \ldots, p_{k_1}$ denote the roots of the polynomial $a(z) = z^{k_1}(1 - \sum_{i=1}^{k_1} a_i z^{-i})$, then the closer the p_i's are to the unit circle the larger is the error induced in the identification algorithm of [11] by higher order moment peaks of $u(n)$ even if they are positioned far from the origin ([5]). In the simulations, we used those m–sequences of a given data length for which their trinomial pairs are positioned as far from the origin as possible.

We consider two bilinear models, one with $k_1 = 2$, $k_2 = k_3 = 2$, $k_4 = 1$ and with the roots of $a(z)$ being equal to 0.6 and 0.8, and one with $k_1 = 2$, $k_2 = k_3 = 3$, $k_4 = 1$ and with the roots of $a(z)$ being equal to 0.4 and 0.9. For the first model simulation results are provided in Table 1 at an SNR ratio of 20db comparing the best m–sequences of least period 4095 in the sense described above, with Gold sequences obtained from Theorem 4 with $N = 4095$ ($n = 12$), $e = 4$, $k = 8$ and $d = 2^{2k} - 2^k + 1$. For the second model simulation results are given in Table 2 at an SNR ratio of 20db comparing the best m–sequences of least period 16383 with sequences obtained from two m–sequences with relatively prime least periods 31 and 511. We observe that most of the parameter estimates obtained with m–sequences are biased (especially the estimate of a_2) and with much larger variances compared with the variances obtained with Gold sequences and sequences resulting from two m–sequences, which yield almost unbiased estimates of all model parameters. The above results clearly indicate that the proposed binary sequences are almost ideal for simulating higher order white noise signals.

6 CONCLUSIONS

In this paper we studied the generation of binary sequences that exhibit the characteristics of higher order white noise signals and are obtained from appropriately selected pairs of m–sequences of either the same or relatively prime least periods. The statistics of these sequences depend on the crosscorrelation function of the two component m–sequences which is either three–valued or constant everywhere. The number of their higher order moment peaks is significantly reduced or vanished, as it is also the case of dual–BCH sequences. The quality of these sequences in simulating higher order white noise signals was illustrated by simulations in the identification of bilinear input–output models.

		m–sequences		sequences obtained from 2 m-sequences	
	True Value	Mean	Variance	Mean	Variance
a1	1.3	1.3953	5.1×10^{-1}	1.2969	1.4×10^{-1}
a2	-0.36	-1.0572	6.8023	-0.4261	8.1×10^{-1}
b0	1	0.9975	1.9×10^{-5}	0.9983	2.3×10^{-5}
b1	0.5	0.4091	5.2×10^{-1}	0.5050	1.4×10^{-1}
c11	0.05	0.0425	3.0×10^{-4}	0.0467	2.0×10^{-4}
c12	0.1	0.1510	2.1×10^{-3}	0.0915	1.4×10^{-3}
c13	0.2	0.1809	2.8×10^{-3}	0.1881	1.4×10^{-3}
c22	-0.15	-0.2005	2.5×10^{-3}	-0.1361	1.9×10^{-3}
c23	0.07	0.0472	9.1×10^{-3}	0.0760	9.7×10^{-3}
c33	0.3	0.3867	3.0×10^{-2}	0.3176	9.4×10^{-3}

Table 2: 40 runs, SNR=20dB, maximal of least period 16383 versus sequences obtained from two maximal and having least period 15841

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