Channel Precoder with Nonuniform Sampling Equalizer

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ABSTRACT

A class of channel precoder constructed with equalizer with minimal implementation complexity, and no channel noise amplification effect is proposed in this paper. The derivation on the formulation of the precoder and equalizer pairs are presented. Where the applied equalizer is a simple nonuniform sampling system. Because of the allpass nature of the equalizer, the performance of this class of channel precoder will not be suffered from channel noise amplification effect. The existence of such precoder, equalizer pairs are derived, which is shown to be compatible with that of ordinary linear precoder.

1 Introduction

While the data rate of the telecommunication system expands rapidly, the effect of multipath fading becomes more and more serious. Therefore, eliminating the frequency distortion caused by the intersymbol interference (ISI) problem in multipath fading turns out to be an essential process for most of the systems. Channel equalization is one of the technique used for compensating the frequency distortion induced by ISI. The channel equalization can be performed in the transmitter (precoding) or the receiver (post-equalization). There are many research results for the post-equalization technique, but most of these techniques have a common problem. When the post-equalizer trying to equalize a signal that falls in deep fading by increasing the amplification in the fading frequency. As a result, the noise in the corresponding frequency spectrum will also be amplified. This will reduce the signal-to-noise ratio of the equalized signal, and hence increase the error rate of the received information. However, if the transmitter can have the knowledge of the channel transfer function, precoding technique can be applied to effectively reduce this noise enhancement effect.

The precoding techniques can also be used to simplify the equalization process in the receiver. The post-equalization technique usually requires a lot of processing power in the receiver, so if post-equalization techniques is used, the complexity and power consumption in the receiver will usually be high. This will directly affect the cost, size and bettary life of the mobile receiver significantly. By reshaping the transmitted signal with some special precoding techniques, the computation required in the equalizer can be reduced substantially.

In this paper, a class of precoding technique is proposed. The equalizer of this class of precoder is constructed by samplers only. As a result, the equalizer of this class of precoder achieve minimal implementation complexity. Moreover, the noise enhancement effect associated with channel equalization will be eliminated. This is because the equalizers are allpass functions, and has unit gain. This paper begins by introducing perfect reconstruction transmultiplexor based on nonuniform sampling. The proposed precoder will be derived in Section 3. Although the equalizer of this class of precoder is constrained to be simple nonuniform sampling system, the existence criteria of such precoder, equalizer pairs are shown to be compatible with that of ordinary linear precoder as shown in Section 4. The paper concluded with the formulation on the precoder filters with a given channel response in Section 5.

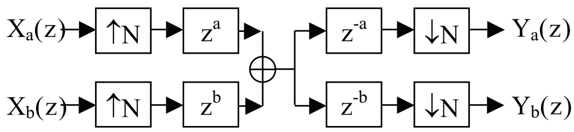

Figure 1: Two channels perfect reconstruction tran-multiplexor.

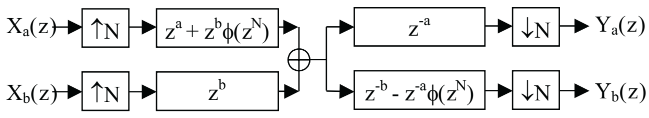

Figure 2: Modified two channels perfect reconstruction transmultiplexor.

2 Transmultiplexor Structure

Fig.1 shows a simple transmultiplexor structure with two channels. The transmultiplexor is known as perfect reconstruction when $Y_a(z) = X_a(z)$ and $Y_b(z) = X_b(z)$. Observed from Fig.1, perfect reconstruction is possible if and only if,

$$(a-b) \bmod N \neq 0 \tag{1}$$

If we modify one of the synthesis filter from z^a to $z^a + z^b\phi(z^N)$, and the analysis filter from z^{-b} to $z^{-b} - z^{-a}\phi(z^N)$, we obtain a new transmultiplexor, where the intermediate signal before downsampling is given by

$$U_a(z) = X_a(z^N)[1 + z^{b-a}\phi(z^N)] + X_b(z^N)z^{b-a} \tag{2}$$

$$U_b(z) = X_b(z^N)[1 - z^{b-a}\phi(z^N)] + X_a(z^N)z^{a-b}[1 - z^{-1}\phi^2(z^N)] \tag{3}$$

It can be easily observed that downsampling $U_a(z)$ by N will recover $X_a(z)$ if and only if eq.(1) is satisfied. Similarly, $X_b(z)$ can be recovered from $U_b(z)$ with the same

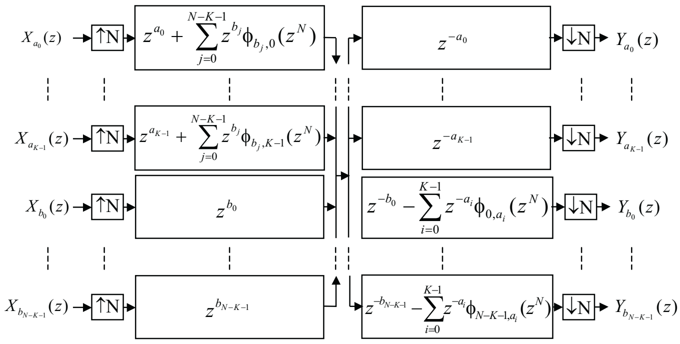

Figure 3: Extended N channel perfect reconstruction transmultiplexor.

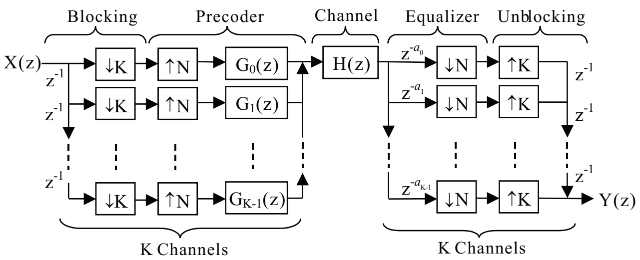

Figure 4: Structure of Proposed Precoder-Equalizer.

condition. Further noticed that the above perfect reconstruction transmultiplexor is structurally guaranteed and independent with the choice of $\phi(z)$ [8]. This simple two channels perfect reconstruction transmultiplexor can be extended to an N channel system. Let A be a subset of integers chosen from $R : \{0, 1, 2, \cdots, N-1\}$ with K elements. The extended perfect reconstruction N channel transmultiplexor is shown in Fig.3, where $a_i \in A$, $0 \le i \le K-1$, and $b_i \in R \setminus A$, $0 \le j \le N-K-1$. It can be verified that the transmultiplexor in Fig.3 is structurally guaranteed to have perfect reconstruction [8]. This structure forms the base of our discussion.

3 Proposed System

Fig.4 shows the proposed system which consists of the blocking and unblocking structures at the two ends of the system, and a precoder-equalizer core. Noticed that the equalizer is constructed by sampling the delayed channel input signal at time a_k, where $0 \le a_k \le N-1$ and $0 \le k \le K-1$. Without loss of generality, we assumed that $a_0 < a_1 < \ldots < a_{K-1}$.

Since the blocking and unblocking structures in the proposed system is known to be perfect reconstruction [2]. Therefore, the proposed system is perfect reconstruction if and only if the precoder-equalizer pair is perfect reconstruction. Fig.5 redraw the extracted precoder-equalizer pair from Fig.4. Comparing Fig.5 with Fig.3, it can be observed that the proposed precoder-equalizer structure is compatible with the N channel perfect reconstruction transmultiplexor discussed in Section 2 with the following modifications

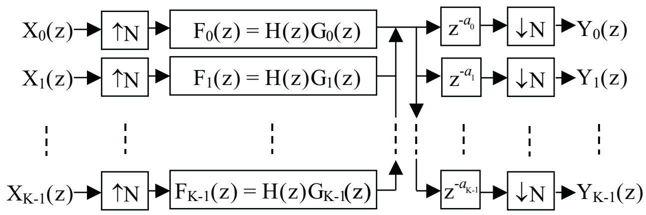

Figure 5: Transmultiplexor core of the proposed system.

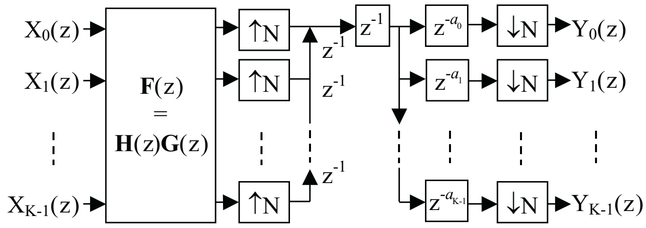

Figure 6: Polyphase representation of the transmultiplexor core of the proposed system.

1) Ignore all channels for X_{b_j} for $0 \le j \le N-K-1$. One way to achieve this is to assume $X_{b_j} = 0$.
2) The product of the procoding filter and channel response satisfies

$$H(z)G_i(z) = z^{a_i} + \sum_{j=0}^{N-K-1} z^{b_j} \phi_{b_j,i}(z^N), \quad \text{for } 0 \le j \le N-K-1 \tag{4}$$

When all of the above conditions are satisfied, the transmultiplexor core in Fig.5 is perfect reconstruction, and hence the proposed system in Fig.4 will be perfect reconstruction. As a result, the analysis of the proposed system can be simplified to the analysis of the transmultiplexor core in Fig.5. Noted that although the considered subband filter $G_i(z)$, $0 \le i \le K-1$ will be noncausal, the FIR nature of $G_i(z)$ allows us to delay the filter with finite number of delays to make it causal. As a result, without loss of generality, we will consider the design and analysis of a noncausal $G_i(z)$ for simplicity.

Consider

$$F_i(z) \triangleq H(z)G_i(z) \tag{5}$$

The proposed transmultiplexor core in Fig.5 can be represented by the polyphase matrix $\mathbf{F}(z)$ as shown in Fig.6, where $\mathbf{F}(z)$ is defined as

$$\mathbf{F}(z) = \mathbf{H}(z)\mathbf{G}(z), \tag{6}$$

and $\mathbf{H}(z)$ and $\mathbf{G}(z)$ are the polyphase representation of the channel filter $H(z)$ and subband filters $G_i(z)$, $0 \le i \le K-1$, and are given by

$$\mathbf{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{N-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \cdots & \vdots \\ H_{N-2}(z) & H_{N-3}(z) & \ddots & z^{-1}H_{N-1}(z) \\ H_{N-1}(z) & H_{N-2}(z) & \cdots & H_0(z) \end{bmatrix}, \tag{7}$$

$$\mathbf{G}(z) \triangleq \begin{bmatrix} G_{0,0}(z) & \cdots & G_{0,K-1}(z) \\ \vdots & \ddots & \vdots \\ G_{N-1,0}(z) & \cdots & G_{N-1,K-1}(z) \end{bmatrix}, \quad (8)$$

where $H_i(z)$, $0 \le i \le N-1$ and $G_{i,k}(z), 0 \le k \le N-1$, $0 \le i \le K-1$ are the Type-2 N polyphase components of $H(z)$ and $G_i(z)$ respectively. Noticed that with

$$\mathbf{F}(z) = \begin{bmatrix} F_{0,0}(z) & \cdots & F_{0,K-1}(z) \\ \vdots & \ddots & \vdots \\ F_{N-1,0}(z) & \cdots & F_{N-1,K-1}(z) \end{bmatrix}, \quad (9)$$

the proposed transmultiplexor core is perfect reconstruction if and only if eq.(4) is satisfied, which is equivalent to have

$$F_{j,i}(z) = \begin{cases} 1 & , j = a_i \\ \phi_{b_p,i}(z) & , j = b_p, \text{ for } 0 \le p \le N-K-1 \\ 0 & , otherwise \end{cases} \quad (10)$$

Since $\phi_{b_p,i}(z)$ can be arbitrary chosen, $F_{j,i}(z)$ can have any form when $j = b_p$. Let,

$$\mathbf{\Gamma} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \quad (11)$$

where '1' in $\mathbf{\Gamma}(z)$ are located at the a_i-th columns for $0 \le i \le K-1$. Hence,

$$\mathbf{\Gamma}\mathbf{F}(z) = \mathbf{I}_K \quad (12)$$

where $\mathbf{I}_K$ is a $K \times K$ identity matrix. By (12),

$$\mathbf{\Gamma}\mathbf{H}(z)\mathbf{G}(z) = \mathbf{I}_K \quad (13)$$

As a result, when $\mathbf{\Gamma}(z)\mathbf{H}(z)$ has FIR inverse, the proposed precoder exists, and equals to the FIR inverse of $\mathbf{\Gamma}(z)\mathbf{H}(z)$. Noncausal filters $G_i(z)$, $0 \le i \le K-1$, can be obtained. The causal FIR precoder can be found easily by adding an appropriate constant delay to each precoding filter, $G_i(z)$.

4 Existence of Precoder-Equalizer Pair

The proposed precoder-equalizer pair exist if and only if $\mathbf{\Gamma}\mathbf{H}(z)$ has FIR inverse. The necessary and sufficient condition on $H(z)$ to has a general precoder-equalizer pair has been derived in [3]. Since our proposed system has a constrained structure for the equalizer, there will be extra constrains on the sufficient condition for the existence of FIR inverse of the proposed system.

Let $\mathbf{W}_N$ be a $N \times N$ DFT matrix, i.e., $\mathbf{W}_n \triangleq (W_N^{nk})_{0 \le n,k \le N-1}$, where $W_N = e^{-j2\pi/N}$. Let $\Lambda(z)$ be a diagonal matrix

$$\Lambda(z) \triangleq \text{diag}(1, z^{-1}, \cdots, z^{-(N-1)})$$

Derived from eq.(7), we can see that

$$[H(z), z^{-1}H(z), \ldots, z^{-(N-1)}H(z)] = [1, z^{-1}, \ldots, z^{-(N-1)}]\mathbf{H}(z^N) \quad (14)$$

By replacing z by zW_N^n for $n = 0, 1, \cdots, N-1$ in eq.(14),

$$\hat{\mathbf{H}}(z) = \mathbf{W}_N^* \Lambda(z) \mathbf{H}(z^N) \quad (15)$$

where

$$\hat{\mathbf{H}}(z) = \begin{bmatrix} H(z) & z^{-1}H(z) & \cdots & \cdots & z^{-(N-1)}H(z) \\ H(zW_N) & z^{-1}W_N^{-1}H(zW_N) & \cdots & \cdots & z^{-(N-1)}W_N^{-(N-1)}H(zW_N) \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ H(zW_N^{N-1}) & z^{-1}W_N^{-(N-1)}H(zW_N^{N-1}) & \cdots & \cdots & z^{-(N-1)}W_N^{-(N-1)(N-1)}H(zW_N^{N-1}) \end{bmatrix} \quad (16)$$

and $(\cdot)^*$ denotes conjugate. Let $\mathbf{V}(z)$ be the following diagonal matrix

$$\mathbf{V}(z) \triangleq \text{diag}(H(z), H(zW_N), \cdots, H(zW_N^{N-1})). \quad (17)$$

The matrix $\hat{\mathbf{H}}(z)$ in eq.(16) can be written as

$$\hat{\mathbf{H}}(z) = \mathbf{V}(z)\mathbf{W}_N^* \Lambda(z). \quad (18)$$

By combining eq.(15) and (18), $\mathbf{H}(z^N)$ can be diagonalized.

$$\mathbf{H}(z^N) = [W_N^* \Lambda(z)]^{-1} \mathbf{V}(z) W_N^* \Lambda(z) \quad (19)$$

The condition for the matrix $\mathbf{\Gamma}\mathbf{H}(z)$ has FIR inverse is equivalent to $\mathbf{\Gamma}\mathbf{H}(z^N)$ having FIR inverse. The existence problem can be reformulated as $\mathbf{\Gamma}\mathbf{H}(z^N)$ FIR inverse existence problem. Multiply $\mathbf{\Gamma}$ to eq.(19),

$$\begin{aligned} \mathbf{\Gamma}\mathbf{H}(z) &= \mathbf{\Gamma}[\mathbf{W}_N^* \Lambda(z)]^{-1} \mathbf{V}(z) \mathbf{W}_N^* \Lambda(z) \\ &= \text{diag}[z^{a_0}, z^{a_1}, \ldots z^{a_{K-1}}] \\ &\quad \times \mathbf{A}(z) \mathbf{V}(z) \mathbf{W}_N^* \Lambda(z) \end{aligned}$$

where

$$\mathbf{A}(z) = \begin{bmatrix} 1 & W_N^{a_0} & \cdots & W_N^{(N-1)a_0} \\ 1 & W_N^{a_1} & \cdots & W_N^{(N-1)a_1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{a_{K-1}} & \cdots & W_N^{(N-1)a_{K-1}} \end{bmatrix} \quad (20)$$

Since $\text{diag}[z^{a_0}, z^{a_1}, \ldots z^{a_{K-1}}]$, W_N^* and $\Lambda(z)$ have FIR inverse, $\mathbf{\Gamma}\mathbf{H}(z)$ has FIR inverse equivalent to $\mathbf{A}(z)\mathbf{V}(z)$ has FIR inverse.

Let S be the set of all zeros of $H(z)$: $S \triangleq \{z_1, z_2, \ldots, z_P\}$ with $H(z_i) = 0$, $1 \le i \le P$. [3] has proved that a general precoder-equalizer pair with $K = N-1$ exist if and only if

$$S_p \cap S_q = \phi \quad (21)$$

for $0 \le p \ne q \le N-1$, where $S_i \triangleq W_N^i S$ for any integer i. Therefore, we can always set $K = N-1$ by adjusting the value N such that $S_p \cap S_q = \phi$ for $0 \le p \ne q \le N-1$.

Assume the eq.(21) is satisfied, so $N = K - 1$. Because $\mathbf{A}(z)\mathbf{V}(z)$ is a $K \times N$ matrix, $\mathbf{A}(z)\mathbf{V}(z)$ has FIR inverse if and only if the greatest common divisor (gcd) of determinants of all $K \times K$ submatrices of the $\mathbf{A}(z)\mathbf{V}(z)$ is cz^{-d} for a nonzero constant c and an integer d [2]. Consider a $K \times K$ submatrix of $\mathbf{A}(z)\mathbf{V}(z)$. The determinant of such a matrix is given by

$$\begin{aligned}
& \det\{[(\mathbf{W}_N)_{\{a_0,\ldots,a_{K-1}\},\{1,\ldots,p-1,p+1,\ldots,N\}}] \\
& \quad \times \operatorname{diag}(H(z),\ldots,H(zW_N^{p-1}),H(zW_N^{p+1}),\ldots \\
& \quad\quad \ldots,H(zW_N^{N-1}))\} \\
= \; & W_N^{N-p-1}\det\{[(\mathbf{W}_N)_{\{a_0,\ldots,a_{K-1}\},\{1,\ldots,N-1\}}]\} \\
& \quad \times \prod_{i=0,i\neq p}^{N-1} H(zW_N^i) \\
= \; & C_p \prod_{i=0,i\neq p}^{N-1} H(zW_N^i)
\end{aligned}$$

Since $\det\{[(\mathbf{W}_N)_{\{a_0,\ldots,a_{N-2}\},\{1,\ldots,N-1\}}]\}$ is a Vandermonde's determinant for the sampling sequence $\{a_0, a_1, \ldots, a_{K-1}\}$, C_p is nonzero. As a result, if $H(z) \neq 0$, $\forall z > 0$, the $K \times K$ submatrices of $\mathbf{A}(z)\mathbf{V}(z)$ can be made equals to cz^{-d} by adjusting N. Therefore, *the precoder for our proposed equalizer structure can always be found for any sampling sequence in the equalizer.* Noticed that similar condition exist in the general linear precoder-equalizer pair as derived in [3]. Further noticed that the minimum N that can be used to construct FIR precoder with our proposed equalizer structure is the same as that of the general linear precoder-equalizer pair as discussed in [3]. Therefore, we considered the FIR equalizable condition our proposed precoder structure to be compatible with that of the general linear precoder considered in [3] various literature.

5 Finding Precoder $\mathbf{G}(z)$

If $\Gamma\mathbf{H}(z)$ has FIR inverse, it can be easily see that $\mathbf{G}(z)$ can be found by,

$$\mathbf{G}(z) = [\Gamma\mathbf{H}(z)]^{\dagger} \tag{22}$$

where $(\cdot)^{\dagger}$ denotes the pseudoinverse. The pseudoinverse can be found by decomposing the $K \times N$ polynomial matrix $\Gamma\mathbf{H}(z)$ into a product of three polynomial matrices $\mathbf{P}(z)$, $\mathbf{D}(z)$ and $\mathbf{Q}(z)$.

$$\Gamma\mathbf{H}(z) = \mathbf{P}(z)\mathbf{D}(z)\mathbf{Q}(z), \tag{23}$$

where $\mathbf{P}(z)$ and $\mathbf{Q}(z)$ are $K \times K$ and $N \times N$ unimodular matrices, respectively, and

$$\mathbf{D}(z) = \left[\operatorname{diag}[z^{-d_0}, z^{-d_1}, \ldots, z^{-d_{K-1}}] \;\; 0_{K\times(N-K)}\right]$$

for K integers $d_0, d_1, \ldots, d_{K-1}$. Since $\mathbf{P}(z)$, $\mathbf{D}(z)$ and $\mathbf{Q}(z)$ has FIR inverse, the polynomial matrix of the precoder $\mathbf{G}(z)$ is given by

$$\mathbf{G}(z) = \mathbf{Q}^{-1}(z)\begin{bmatrix} \operatorname{diag}[z^{d_0}, z^{d_1}, \ldots, z^{d_{K-1}}] \\ 0_{(N-K)\times K} \end{bmatrix}\mathbf{P}^{-1}(z) \tag{24}$$

6 Conclusions

A class of precoder-equalizer pair is proposed where the equalizer is a simple delayed sampling system. Since the sampler is an allpass system and has unit gain, the channel noise amplification problem associated with channel equalizer is eliminated in the proposed system. Moreover the equalizer has minimal implementation complexity, and thus is very suitable for mobile applications. The formulation and structure of the proposed system is derived. It is proved that the proposed precoder-equalizer pair is applicable in all FIR equalizable channels for ISI free communications. Other advantages of the proposed system, such as transmitter power control, signal spectrum utilization efficiency etc. will be our future work.

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