A MINIMAX-CONSTRAINED SUPERRESOLUTION ALGORITHM FOR REMOTE SENSING IMAGERY

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ABSTRACT

Superresolution algorithms use several blurred, undersampled and noisy images of a scene to reconstruct a higher resolution version. In this paper we apply the superresolution concept to the remote sensing scenario, and develop a novel superresolution algorithm based on quadratic programming, and compare it with existing methods. The proposed algorithm achieves PSNR performance similar to state-of-the-art techniques, providing additional capabilities in terms of uniqueness of the solution and user-defined bounds for the superresolution problem.

1 INTRODUCTION

Superresolution (SR) imaging consists in exploiting multiple blurred, displaced, decimated and noisy low-resolution (LR) pictures of the same scene, in order to build a high resolution (HR) image or image sequence. This research field has recently been very active, and a number of SR applications have been proposed mainly in the multimedia field (see e.g. [1]).

A SR system consists of three main tasks, namely estimating the blur, estimating the motion of the available pictures with respect to a reference one, and combining all pictures to obtain a deblurred HR image. The blur point spread function is usually assumed known; otherwise it can be measured or estimated [2]. The motion characteristics between adjacent frames heavily depend on the application. In the case of multimedia video sequences it is necessary to assume independent object motion within the scene [1]. In some applications, such as videosurveillance, simpler motion models are also suitable, such as global translational motion due to camera panning at constant velocity. In this case, the motion estimation problem is largely alleviated, and faster algorithms can be devised due to the 2-D shift-invariant image acquisition model [3, 5].

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As for picture combination, several techniques have been proposed, based on different prior image models, such as the maximum likelihood (ML) and maximum a posteriori (MAP) estimators [6]. These techniques attempt to minimize an average quadratic error measure on the LR data, such as mean squared error (MSE), and have been shown to provide visually pleasant HR reconstructions results in multimedia applications.

Although SR has been mainly developed for multimedia, other applications exist where the image acquisition process is characterized by a certain degree of redundancy. As an example, SR has been applied in [3] to forward-looking infrared data for aerial videosurveillance, where adjacent frames may exhibit overlap regions which can be used to improve the native sensor resolution. Another possible application is in satellite remote sensing; in fact, orbiting satellites image the same region at regular intervals, in such a way that several scenes of the same region, taken at different times, are available at the ground station, and can possibly be used for SR. However, all the techniques presented so far in a remote sensing context, e.g. [3, 4], deal with visual inspection of remote sensing images. On the other hand, especially in case of automatic feature extraction from remote sensing images, there is a strong need of image quality assessment; therefore, in case SR is carried out, the quality of the HR interpolated images must be carefully evaluated. In general, the image quality issue is a very well-known one in the remote sensing community, which has led to the consideration that, since every pixel of an optical remote sensing image is a radiance *measurement*, quality should be guaranteed at the single pixel level rather than globally through an "average" quality metric such as peak signal-to-noise ratio (PSNR). Therefore, if processing is carried out on the data so as to modify pixel values, it is highly desirable that the maximum deviation from the original data can be bounded by a user-specified value. This bounded error concept can be formalized as a minimax estimation problem, or equivalently as an optimization problem in $\mathcal{L}_\infty$ norm, and has already been applied e.g. to the compression problem [7].

In this paper we propose a SR method, suitable for remote sensing imagery, which applies the bounded error concept to the formation of a HR image from multiple LR ones. A main difference with respect to the compression problem lies in the fact that in the SR application the original HR image is not available for comparison; therefore it is not possible to tightly enforce a user-defined bound on the absolute difference between the reconstructed HR image and the ground truth. The proposed algorithm allows the user to set two different bounds on the HR image. The first one measures the compliance of the HR data with the observed LR images through the image formation model; the second one addresses the arbitrary $\mathcal{L}_\infty$ closeness of the HR image to a given smooth HR interpolation (e.g. bilinear or spline).

2 PROPOSED ALGORITHM

In the following we outline the SR algorithm proposed in this paper. In particular, in Sect. 2.1 we define the image formation model used in the following; in Sect. 2.2 we detail the proposed technique named quadratic programming superresolution (QPSR), while in Sect. 2.3 we emphasize its relation with other SR algorithms.

2.1 Image formation model

We consider a fairly general and widely used image formation model. It is assumed that N LR images $\underline{Y}_k$, $k = 1, \ldots, N$, are observed, each column vector $\underline{Y}_k$ being the raster-scan reading of an $M \times M$ image. Each of these images is obtained by arbitrary warping, linear space-variant blurring and decimation of an original HR image $\underline{X}$ of size $L \times L$. A very general relation between each $\underline{Y}_k$ image and $\underline{X}$ is:

$$\underline{Y}_k = \mathbf{D}_k \mathbf{C}_k \mathbf{F}_k \underline{X} + \underline{E}_k \quad \text{for} \quad 1 \leq k \leq N \tag{1}$$

$\mathbf{F}_k$ being an $L^2 \times L^2$ matrix describing the geometric relation (e.g. translation, rotation, ...) between $\underline{Y}_k$ and $\underline{X}$, $\mathbf{C}_k$ an $L^2 \times L^2$ blur matrix, not necessarily structured, and $\mathbf{D}_k$ an $M^2 \times L^2$ matrix defining the subsampling operator. $\underline{E}_k$ is a zero-mean Gaussian noise vector with autocorrelation matrix $\mathbf{W}_k$. In the following we will assume white noise stationary among all observations, i.e. $\mathbf{W}_k = \sigma^2 \mathbf{I}$ and σ^2 independent of k. Moreover, in this paper we consider the case of global translational motion and 2-D linear shift-invariant blur common to all measurements, so that $\mathbf{C}_k = \mathbf{C}\ \forall k$ is a block-Toeplitz matrix.

The model equations for each image can be grouped to obtain a complete observation model as follows:

$$\underline{Y} = \mathbf{H}\underline{X} + \underline{E} \tag{2}$$

being $\underline{Y} = [\underline{Y}_1^T, \ldots, \underline{Y}_N^T]^T$, $\underline{E} = [\underline{E}_1^T, \ldots, \underline{E}_N^T]^T$, and $\mathbf{H}$ obtained by columnwise stacking $\mathbf{D}_1\mathbf{C}_1\mathbf{F}_1$, $\mathbf{D}_2\mathbf{C}_2\mathbf{F}_2$, $\ldots$, $\mathbf{D}_N\mathbf{C}_N\mathbf{F}_N$.

It is easily recognized that the model in Eq. 2 is a classical image restoration problem, which can be solved by means of ML, MAP and projection onto convex sets (POCS) techniques [1] amongst others. All these techniques are based on the remark that inverting Eq. 2 is a highly ill-posed problem, which needs to be regularized to obtain meaningful solutions. In the most classical formulation the ML estimate is formulated as the solution of a constrained least squares minimization problem with cost function

$$\Phi(\hat{\underline{X}}) = ||\underline{Y} - \mathbf{H}\hat{\underline{X}}||_2^2 + \lambda ||\mathbf{S}\hat{\underline{X}}||_2^2 \tag{3}$$

where $\hat{\underline{X}}$ is the estimated HR image and $||\cdot||_2^2$ denotes the squared $\mathcal{L}_2$ norm. The cost function has two terms weighted by the Lagrange multiplier λ. The leftmost term enforces compatibility between $\hat{\underline{X}}$ and the image formation model in terms of MSE of the residual image; the rightmost one enforces global smoothness of the solution by penalizing high values of the functional $\mathbf{S}\hat{\underline{X}}$, with $\mathbf{S}$ a highpass operator, e.g. the 2-D Laplacian. The optimal value of λ can be found by bisection search imposing that $||\underline{Y} - \mathbf{H}\hat{\underline{X}}||_2^2 = ||\underline{E}||_2^2$ within a given tolerance.

2.2 Superresolution by quadratic programming

Although the ML algorithm provides visually good HR reconstructions, it solves the restoration problem by minimizing an average quadratic cost function. However, as stated, in remote sensing applications a minimax solution should be sought. However, unlike the image compression problem, in the case of SR the original image is *not* available; therefore our proposed approach is to impose a maximum distance between the reconstructed data and a reasonable approximation of the original HR data obtained by interpolating the observed LR data. The maximum distance should reflect the confidence that the user has in this approximation. A similar reasoning can be made for the compliance of the reconstructed image with the image formation model.

In particular, the SR algorithm proposed in this paper formulates the image SR problem in the following way. Suppose that $\underline{X_0}$ is a smooth HR approximation to $\underline{X}$, e.g. obtained by nearest neighbor, bilinear or spline interpolation; assume also that the user has a confidence in that interpolation, which can be quantified as a maximum desired lower or upper deviation ($\underline{U}_l^H$ and $\underline{U}_h^H$ respectively) of the solution $\hat{\underline{X}}$ from the image $\underline{X_0}$. Moreover, suppose that the user has a certain confidence in the accuracy of the image formation model of Eq. 2, which can be expressed in terms of maximum lower and upper bounds on the residual images ($\underline{U}_l^L$ and $\underline{U}_h^L$ respectively). In our approach we solve the following optimization problem:

$$\begin{array}{ll} \text{minimize} & ||\mathbf{S}\hat{\underline{X}}||_2^2 \\ \text{subject to} & \underline{U}_l^L < \underline{Y} - \mathbf{H}\hat{\underline{X}} < \underline{U}_u^L \\ & \underline{U}_l^H < \hat{\underline{X}} - \underline{X}_0 < \underline{U}_h^H \\ & 0 < \hat{\underline{X}} < 255 \end{array} \tag{4}$$

This is easily recognized as a large-scale quadratic programming problem with linear constraints and variable bounds, which can be solved by standard interior-point methods. In the case of a linear shift-invariant image formation model, the Hessian matrix $\mathbf{S}^T\mathbf{S}$ is positive definite; hence the problem is a convex one, and if a local minimizer exists, then it is the unique global minimizer for this problem. It is worth noticing that, while the $\underline{U}_l^H$, $\underline{U}_h^H$, $\underline{U}_l^L$, and $\underline{U}_h^L$ bounds are considered constant vectors in this work in order to obtain HR solutions with bounded deviations from a reference model, the proposed formulation can also encompass space-varying bounds and thus easily perform spatially adaptive regularization.

2.3 Relation to other methods

Several remarks can be made as to the relation between the proposed QPSR algorithm and other existing methods. The use of quadratic programming in the field of image restoration has already been proposed in [8], in an attempt to incorporate a positivity constraint in the solution. In this respect, our innovation is to impose variable bounds with respect to a trusted approximation to the solution, and to introduce a linear constraint related to the fidelity of the solution to the image formation model.

The main difference of QPSR with respect to the ML solution is its ability to incorporate non-quadratic constraints into the problem, thus allowing to set bounds on the solution at the HR and LR pixel level, including the positivity constraint; additionally, the computation of a regularization parameter is not required. The major advantages of QPSR with respect to POCS are the following. Firstly, while QPSR also applies an $\mathcal{L}_\infty$ norm constraint, in POCS the solution depends on the order the projection operators are applied; conversely, QPSR selects among all feasible solutions the one which minimizes the squared norm of a user-defined quadratic functional, e.g. maximizing the smoothness of the solution. Secondly, the POCS method as proposed in the literature does not enforce a constraint on the minimax closeness of the solution to a user-specified image.

3 EXPERIMENTAL RESULTS

In the following we report comparative results of several interpolation and SR algorithms on band 1 of a LANDSAT 7 ETM+ image. The results are obtained by simulating four 128×128 images with horizontal and vertical shifts of one HR pixel from an original 256×256 image portion. The noise standard deviation is $\sigma = 1$, and the blur kernel is 2×2 uniform with unit DC amplitude. The reconstruction aims at improving resolution of a factor 2 both horizontally and vertically. It must be remarked that in our simulations, even though the original image is not known by the SR algorithms, it is available for comparison, so that the comparative results reported in the following are between the SR and original image, not the reference $\underline{X}_0$. We consider nearest neighbor interpolation, bilinear interpolation, third order spline interpolation, the ML algorithm, the POCS algorithm and the proposed QPSR algorithm. In particular, we impose on the QPSR solution a maximum distance of ± 7 with respect to the third order spline interpolation, and a bound of ± 4 on the LR residual image. The POCS algorithm is run with a bound of ± 2 on the residual image, and is also initialized with a third order spline interpolation. A 2-D Laplacian has been used as smoothness constraint for the ML and QPSR methods.

Method	PSNR (dB)	Max. err.
NN. int.	43.1	11
Bilinear int.	43.2	13
Spline int.	42.98	11
ML	45.68	8
POCS	45.09	7
QPSR	45.65	7

Table 1: Comparison between different SR techniques: PSNR and maximum error between reconstruction and original

The obtained HR images have been compared with the original, noise-free image. The achieved results, in terms of PSNR and maximum absolute error with respect to the original are reported in Tab. 1. It is worth noticing that all SR algorithms achieve better results than single frame interpolators, as to both PSNR and maximum error. The best algorithm in terms of PSNR is the ML one; the QPSR method nearly achieves the same performance, while the POCS result is slightly worse. However, QPSR and POCS achieve a lower maximum error than ML.

Visual results are reported in Fig. 3 for the ML and QPSR algorithms. As can be seen, QPSR yields a sharper reconstruction than ML, and very similar to that of the POCS algorithm (which is not reported here). In summary, the best visual results are achieved by QPSR and POCS, with the important advantage of QPSR of providing a unique solution to the SR problem, along with a minimax constraint in terms of distance of the solution from a prototype one.

4 CONCLUSIONS

In this paper we have presented a novel image SR algorithm based on quadratic programming. The algorithm has complexity comparable to that of the ML and POCS techniques, and achieves similar performance in terms of PSNR and maximum absolute error of the reconstructed HR image with respect to the original. Additionally, it provides a unique solution to the SR problem, and allows to set user-defined bounds on the compatibility of the solution with the image formation model and with

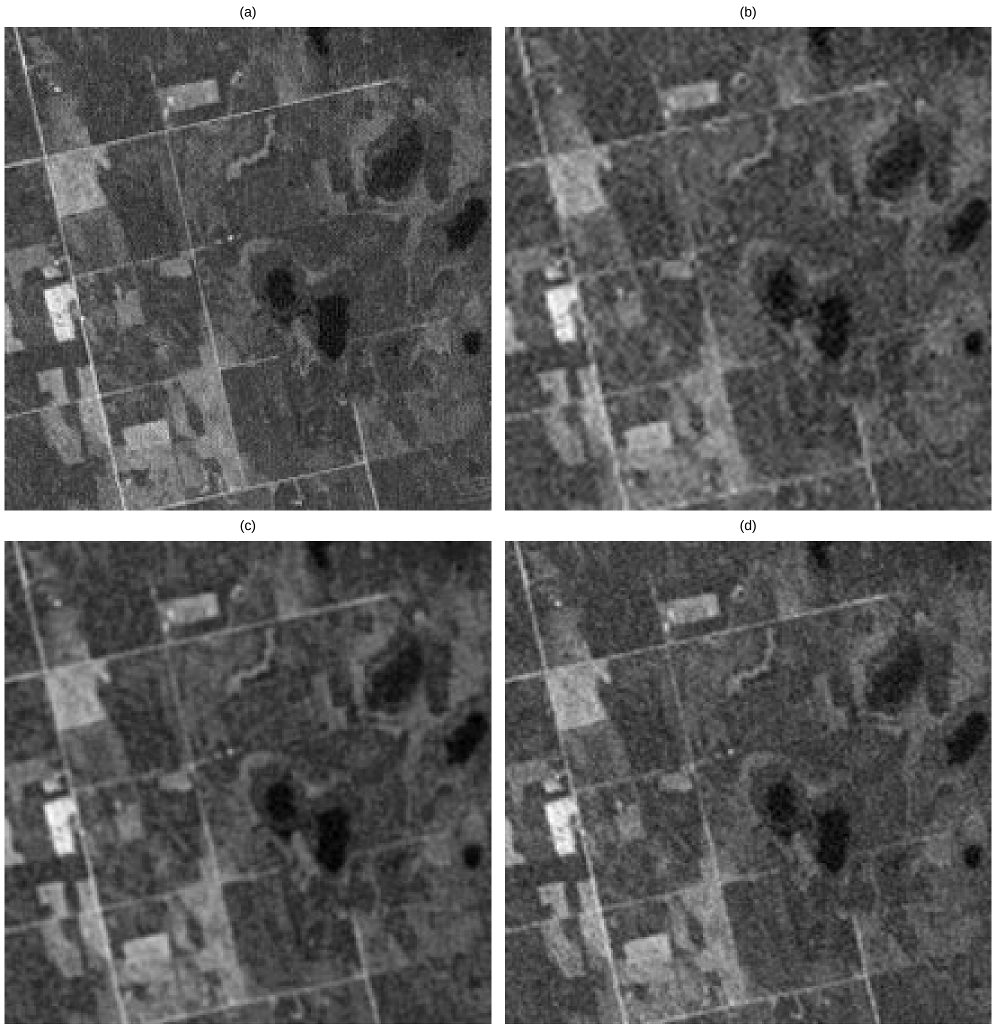

Figure 1: (a) Original HR image; (b) Reference interpolated image X_0; (c) ML reconstructed HR image; (d) QPSR reconstructed SR image

a user-provided reference solution, thus being a suitable choice for resolution enhancement of remote sensing images.

References

[1] A.M. Tekalp, *Digital video processing*, Prentice-Hall, 1995

[2] G. Pavlović, A.M. Tekalp, "Maximum likelihood parametric blur identification based on a continuous spatial domain model", *IEEE Trans. on Image Processing*, vol.1, n.4, Oct. 1992, pp. 496-504

[3] M.S. Alam, J.G. Bognar, R.C. Hardie, B.J. Yasuda, "Infrared image registration and high-resolution reconstruction using multiple translationally shifted aliased video frames", *IEEE Trans. on Instr. and Meas.*, vol.49, n.5, Oct. 2000, pp. 915-923

[4] M. Suess, M. Volker, J.J.W. Wilson, C.H. Buck, "Superresolution: range resolution improvement by coherent combination of repeat pass SAR images", *Proc. of EUSAR 98*, Berlin, Germany, 1998

[5] M. Elad, Y. Hel-Or, "A fast super-resolution reconstruction algorithm for pure translational motion and common space-invariant blur", *IEEE Trans. on Image Processing*, vol.10, n.8, Aug. 2001, pp. 1187-1193

[6] M. Elad, A. Feuer, "Restoration of a single superresolution image from several blurred, noisy and undersampled measured images", *IEEE Trans. on Image Processing*, vol.6, n.12, Dec. 1997, pp. 1646-1658

[7] B. Aiazzi, L. Alparone, S. Baronti, "Information preserving storage of remote sensing data: virtually lossless compression of optical and SAR images", *Proc. of IEEE IGARSS 2000*

[8] A.J. Levy, "A fast quadratic programming algorithm for positive signal restoration", *IEEE Trans. on Acoust., Speech and Signal Proc.*, vol.ASSP-31, n.6, Dec. 1983, pp.1337-1341