USING A NORMALISED LMF ALGORITHM FOR CHANNEL EQUALISATION WITH CO-CHANNEL INTERFERENCE

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ABSTRACT

A new normalised LMF (XE-NLMF) algorithm is proposed for channel equalisation to improve the convergence speed and bit error rate performance. The analysis of the LMF algorithm's initial convergence in the mean has facilitated the normalisation which is bounded by the error power and signal power. Hence, a combination of the signal power and error power, by means of mixing, is used to normalise the step size. The proposed normalisation improves the stability of the mean fourth cost function and achieves a faster convergence in both co-channel interference and white noise. The complexity of the normalised LMF algorithm is slightly higher than the NLMS algorithm but the convergence speed and bit error rate performance significantly outperform the NLMS algorithm.

1. Introduction

Adaptive channel equalisation is essential to a communication system to ensure the integrity of the received signal, which is corrupted by the intersymbol interference (ISI), co-channel interference (CCI) and additive white Gaussian noise (AWGN). Future digital mobile communications are likely to be a multi-user system shared over a common channel. As the number of users and data traffic increases, the CCI will increase and the NLMS based channel equalisation will be non-optimum [2].

The NLMS is the most popular algorithm, because of it simplicity and robustness in adaptive equalisation. Many offspring algorithms have evolved from the LMS algorithm, such as the Sign-LMS algorithm. However, the modifications to the correlation multipliers often degrade the performance [3]. Except the least mean fourth (LMF) algorithm, under non-Gaussian additive noise, where the LMF algorithm outperforms the LMS algorithm [1]. In this work, the CCI is regarded as sub-Gaussian noise.

However, the statistical instability of LMF algorithm requires a smaller step size to ensure stable convergence, which will decelerate the convergence speed [5][10]. Recently, by means of switching and mixing the norm of cost functions, the LMMN [4] and CFA algorithms [6] have been designed to improve the convergence speed and the adaptation accuracy.

In this work, a step size normalisation to the LMF algorithm is proposed to improve the stability and convergence speed. The proposed normalised LMF (XE-NLMF) algorithm is applied to the problem of channel equalisation with CCI and AWGN, as shown in Figure 1.

Section 2 discusses the mean and mean square weight error convergence of the LMF algorithm. Section 3 analyses the initial stability of LMF algorithm, which is dependent upon the initial condition of error power. Simulation results in Section 4 show that the XE-NLMF algorithm significantly outperforms the NLMS and LMF algorithms in the presence of AWGN and CCI. Lastly, Section 5 states the conclusions made in this paper.

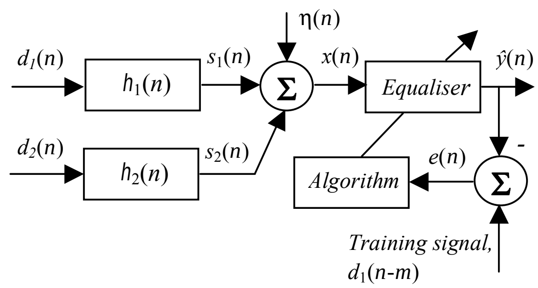

Figure 1: A communication system affected by a dispersive channel (ISI), CCI and AWGN.

2. Overview of LMF Algorithm

Consider a time-invariant adaptive system identification structure. The LMF algorithm is obtained by applying the partial differentiation to each filter weight [1] and the recursive equation for LMF algorithm is updated as follows [1]:

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \gamma e^3(n)\boldsymbol{X}(n), \qquad (1)$$
$$e(n) = d(n) - \boldsymbol{X}^T(n)\boldsymbol{W}(n).$$

Where, the desired signal is $d(n) = \boldsymbol{X}^T(n)\boldsymbol{W}^*(n) + \eta(n)$. $\boldsymbol{X}(n)$ is the input vector, $\boldsymbol{X}(n) = [x(n), x(n\text{-}1), \ldots, x(n\text{-}N\text{+}1)]^T$ and independent of the zero mean white noise, $\eta(n)$. $\boldsymbol{W}^*(n)$ is the unknown FIR filter weights and $\boldsymbol{W}(n)$ is the adaptive filter weight vectors, $\boldsymbol{W}(n) = [w(n), w(n\text{-}1), \ldots, w(n\text{-}N\text{+}1)]^T$, T represents the transpose of a vector and N is the filter order. The cubic error in (1) is a non-linear modification to the LMS algorithm. By defining the weight deviation error as $\boldsymbol{V}(n) = \boldsymbol{W}(n) - \boldsymbol{W}^*(n)$, the different equation in the mean convergence of weight error is derived as:

$$E\{\boldsymbol{V}(n+1)\} = [\mathbf{I} - 3\gamma E\{\eta^2(n)\}\boldsymbol{R}]E\{\boldsymbol{V}(n)\}, \tag{2}$$

where $E\{\cdot\}$is the statistical expectation, $\mathbf{I}$ is a unity identity matrix and $\mathbf{R} = E\{\boldsymbol{X}(n)\boldsymbol{X}^T(n)\}$. According to the analysis of the mean square weight error convergence in [1], a sufficient condition for convergence is satisfied when:

$$0 < \gamma < \frac{E\{\eta^2(n)\}}{10NE\{x^2(n)\}E\{\eta^4(n)\}}. \tag{3}$$

The condition in (3) is bounded by the covariance of $E\{x^2(n)\}$ and the 4th order moment (kurtosis) of the additive noise. Since the kurtosis of a sub-Gaussian signal is smaller than that of Gaussian signal, the LMF algorithm will therefore perform better in sub-Gaussian additive noise [1].

3. Normalised LMF Algorithm

To facilitate the normalisation, the initial instability of the LMF algorithm is reassessed. The instability of LMF algorithm is dependent on the initial convergence and has been studied in [8]. Using the Gaussian moment factoring theorem [7] and without expanding the cubic error term in (1), the weight error difference equation is simplified to (see *Appendix*):

$$E\{\boldsymbol{V}(n+1)\} = [\mathbf{I}-2\gamma E\{e^2(n)\}\boldsymbol{R}\,]\,E\{\boldsymbol{V}(n)\}\Big|_{\|\boldsymbol{W}(n)\|<<\|\boldsymbol{W}^*(n)\|}. \tag{4}$$

The difference equation in (4) is analogous to (2) and after convergence is reached the $E\{e^2(n)\}$ will be very closed to $E\{\eta^2(n)\}$[7]. A general condition for (4) to hold is

$$\gamma < \frac{1}{2\sigma_e^2\lambda_{max}}\Big|_{\|\boldsymbol{W}(n)\|<<\|\boldsymbol{W}^*(n)\|}. \tag{5}$$

Where, λ_{max} is the maximum eigenvalue of $\boldsymbol{R}$ and σ_e^2 is the covariance of $E\{e^2(n)\}$. $\|\cdot\|$ is the Euclidean norm and $\|\boldsymbol{W}(n)\| << \|\boldsymbol{W}^*(n)\|$ represents the condition of the initial distance between $\|\boldsymbol{W}(n)\|$ and the optimum $\|\boldsymbol{W}^*(n)\|$. Note that the step size in (5) is time-varying, where the error power, σ_e^2, is large during the initial adaptation and becomes minimum after convergence is reached. Hence, the condition in (5) shows that the stability of the LMF algorithm depends on the initial condition, which is the initial σ_e^2.

Since the maximum bounded step size in (4) is a time-varying function, normalising the step size will gain a faster convergence speed. To normalise the LMF algorithm, γ should approximately equate to the second term in (4), $E\{e^2(n)\}\boldsymbol{R}$. Let the $\boldsymbol{R} = \sigma_x^2(n)\mathbf{I}$, equation (4) can be rewritten as:

$$E\{\boldsymbol{V}(n+1)\} = [\mathbf{I} - 2\gamma\sigma_e^2\,\sigma_x^2\,\mathbf{I}]E\{\boldsymbol{V}(n)\}\Big|_{\|\boldsymbol{W}(n)\|<<\|\boldsymbol{W}^*(n)\|}. \tag{6}$$

Since the estimation of $\sigma_e^2\,\sigma_x^2\,\mathbf{I}$ is not practically available, the following approximation is made:

$$\sigma_e^2\,\sigma_x^2\,\mathbf{I} \le [\sigma_e^2 + \sigma_x^2]\mathbf{I}. \tag{7}$$

For inequality (7) to be valid, the conditions to be justified are $|\sigma_e| < 1$ and $|\sigma_x| < 1$, which is valid as the norm of error power is always less than unity. A tighter condition for stability becomes:

$$\gamma < \frac{1}{2(\sigma_x^2 + \sigma_e^2)}\Big|_{\|\boldsymbol{W}(n)\|<<\|\boldsymbol{W}^*(n)\|}. \tag{8}$$

Thus, the inequality in (7) facilitates the normalisation and we propose to normalise the step size as follows:

$$\hat{\gamma}(n) = \frac{\gamma_{xe}}{\delta + \lambda_1(\boldsymbol{X}^T(n)\boldsymbol{X}(n)) + \lambda_2(\boldsymbol{e}^T(n)\boldsymbol{e}(n))}. \tag{8}$$

Where $\sigma_e^2 = \boldsymbol{e}^T(n)\boldsymbol{e}(n)$ and $\sigma_x^2(n) = \boldsymbol{X}^T(n)\boldsymbol{X}(n)$, the vector of $\boldsymbol{e}(n) = [e(n), e(n-1), \ldots, e(n-N+1)]^T$. γ_{xe} is the new step size and δ is a small constant to avoid a numerical division problem. The parameters λ_1 and λ_2 are introduced as the mixed norm power to control the mixing between the signal power and error power, where $\lambda_1+\lambda_2 = 1$. A signal power normalisation, $\hat{\gamma}(n) = 1/(\boldsymbol{X}^T(n)\boldsymbol{X}(n))$, had been proposed in [9], but found to be non-optimum [5]. The recursive updating equation for the normalised LMF algorithm (XE-NLMF) is proposed as:

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \hat{\gamma}(n)e^3(n)\boldsymbol{X}(n). \tag{9}$$

Applying the weight error deviation, the difference equation for the weight error is defined as follows:

$$E\{\boldsymbol{V}(n+1) = [\mathbf{I} - 3E\{\hat{\gamma}(n)\}E\{\eta^2(n)\}\boldsymbol{R}\,]E\{\boldsymbol{V}(n)\}, \tag{10}$$

$$E\{\boldsymbol{V}(n+1)\} = [\mathbf{I} - \frac{3\gamma_{xe}E\{\eta^2(n)\}\boldsymbol{R}}{\lambda_1\sigma_x^2 + \lambda_2\sigma_e^2}]E\{\boldsymbol{V}(n)\}. \tag{11}$$

For $\mathrm{tr}[\boldsymbol{R}] = N\sigma_x^2$, then, a general condition for equation (11) to hold is

$$\gamma_{xe} < \frac{\lambda_1\sigma_x^2 + \lambda_2\sigma_e^2}{3N\sigma_\eta^2\sigma_x^2}. \tag{12}$$

The error is usually larger during the initial adaptation and gradually decreases toward a minimum. Therefore, the signal power, σ_x^2, will act as a threshold to avoid large step size when σ_e^2 converges to a minimum. The combination of $\lambda_1\sigma_x^2 + \lambda_2\sigma_e^2$ has the advantage of normalising the input signal power and an improved stability where the σ_e^2 will dampen down the outlier distribution of $e^3(n)$ in the recursive updating equation of XE-NLMF algorithm.

4. Simulation Results

To study the performance of the XE-NLMF algorithm, channel equalisation under CCI and AWGN is considered, as shown in Figure 1. The NLMS algorithm is set up to benchmark the performance of the XE-NLMF algorithm:

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \frac{\alpha\, e(n)\boldsymbol{X}(n)}{\delta + \boldsymbol{X}^T(n)\boldsymbol{X}(n)}. \qquad (13)$$

A non-minimum phase channel with impulse responses of $h_1(n) = 1{+}0.2(n{-}1)$ and a CCI channel with impulse responses of $h_2(n) = \beta(1{+}0.4(n{-}1))$ is considered for the transmitted binary sequences $\{+1,-1\}$ of $d_1(n)$ and $d_2(n)$ that are mutually independent. An equaliser of the order $N = 8$ is considered and with reference to the AWGN power, σ_η^2, the parameter of signal to noise ratio (SNR), signal to interference (SIR), can be calculated as, SNR = $\sigma_{s_1}^2/\sigma_\eta^2$, SIR = $\sigma_{s_1}^2/\sigma_{s_2}^2$ and SINR = $\sigma_{s_1}^2/(\sigma_\eta^2+\sigma_{s_2}^2)$. Here the $\sigma_{s_1}^2$ and $\sigma_{s_2}^2$ represent the observed signal power and the co-channel signal power respectively. By changing the factor β, the SIR can be varied accordingly. The convergence curves in MSE are averaged for 1000 independent simulation to ensure stability. The Mean Square Error (MSE) is calculated as MSE(dB) = $10\log_{10}(E\{e^2(n)\})$.

Experiment 1. Channel Equalisation with AWGN.

Consider SNR = 20dB and no co-channel interference, $\beta = 0$. The convergence curves in Figure 2 show that the XE-NLMF algorithm converges faster than both the NLMS algorithm and LMF algorithms. The step size for NLMS is $\alpha = 0.3$, which has a comparable steady state MSE to the XE-NLMF algorithm. To assure stability, the step size for LMF is $\gamma = 0.06$. Increasing the NLMS algorithm's step size can increase the convergence speed, but it will degrade the steady state MSE, hence the XE-NLMF algorithm outperforms the NLMS algorithm in AWGN. The significant improvement in convergence speed demonstrates that the normalisation to LMF algorithm has improved the stability. Figure 3 shows the bit error rate performance where the XE-NLMF is slightly better than the NLMS algorithm.

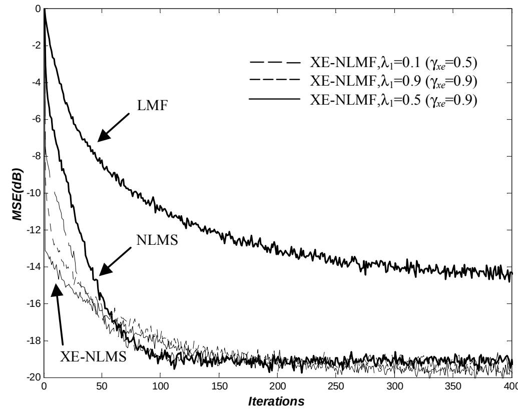

Figure 2: Convergence curves under AWGN.

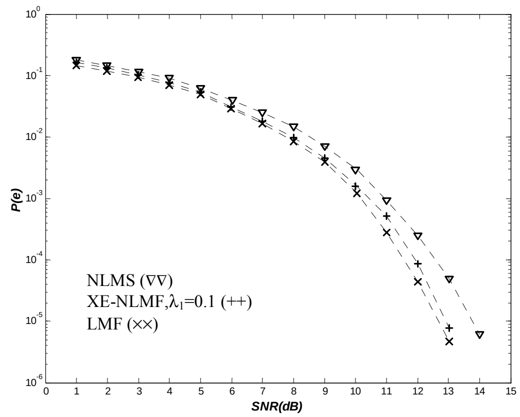

Figure 3: SIR=0dB. BER Performances in SNR.

Experiment 2. Channel Equalisation with CCI.

With the same set up as for Experiment 1, the algorithm are now tested with CCI of SIR = 15dB and AWGN of SNR = 20dB, SNIR = 14dB. The parameters used for XE-NLMF are $\gamma_{xe} = 0.9$, $\lambda_1 = 0.5$. For the fastest convergence, the step size for the NLMS and LMF are 0.3 and 0.06 respectively. The convergence curves are shown in Figure 4 that the XE-NLMF algorithm converges significantly faster than the NLMS and LMF algorithm.

For the bit error rate results in SIR, the step size for XE-NLMF, NLMS and LMF are chosen to produce the best bit error rate performance and the AWGN is set at SNR=20dB. The results in Figure 5 show that the XE-NLMF algorithm produces a better bit error rate performance than the NLMS and the LMF algorithms. In terms of CCI, the XE-NLMF achieves more improvement than in AWGN.

5. Conclusions

A step size normalisation to the LMF algorithm has been introduced, which uses the mixed norm of the signal power and error power to improve the stability and convergence speed. Meanwhile, the convergence in the mean weight error has been analysed for the LMF algorithm and the initial instability is found to be dependent on the initial error power.

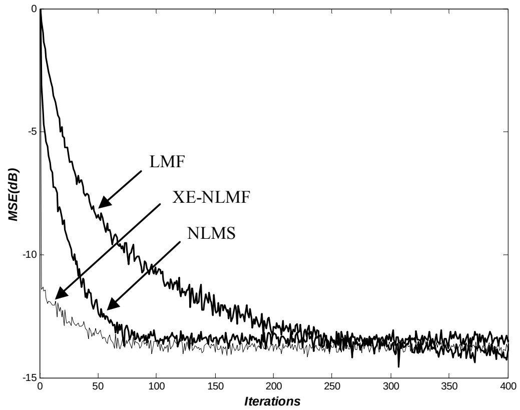

Figure 4: Convergence curves under CCI.

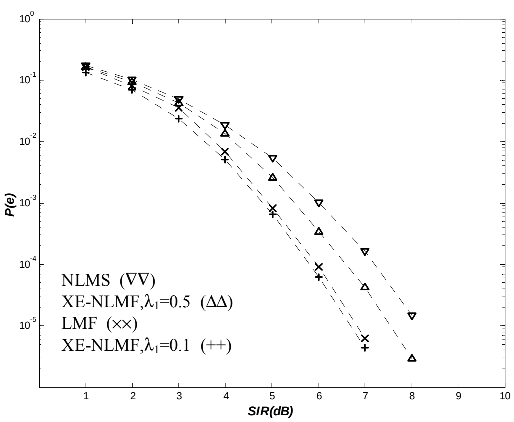

Figure 5: SNR = 20dB; BER Performance in SIR.

The simulation results show that the BER performance of the LMF is superior to the NLMS, however the convergence speed is slow. With the step size normalisation, the XE-NLMF algorithm achieves both fast convergence and better BER performance compared to the NLMS and LMF algorithms. With a small overhead complexity increase, the XE-NLMF algorithm can outperform the NLMS algorithm in both AWGN and CCI noise.

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Appendix.

It is assumed that the $\boldsymbol{X}(n)$ is correlated to $e(n)$ during *initial adaptation*, $E\{x(n)e(n)\} \neq 0$. The Gaussian moment factoring theorem [7][8] is used to simplify the higher order term that the $x(n)$ and $e(n)$ are Gaussian distributed,

$$E\{\boldsymbol{V}(n+1)\} = E\{\boldsymbol{V}(n)\} + \gamma E\{e^3(n)\boldsymbol{X}(n)\}. \quad (A.1)$$

Using the Gaussian moment factoring theorem,

$$E\{e^3(n)\boldsymbol{X}(n)\} = 2[E\{e^2(n)\}E\{e(n)\boldsymbol{X}(n)\}], \quad (A.2)$$

and $E\{\boldsymbol{V}(n+1)\} = E\{\boldsymbol{V}(n)\} + 2\gamma[E\{e^2(n)\}E\{\boldsymbol{X}(n)e(n)\}]$.

Since, $e(n) = \eta(n) - \boldsymbol{X}^T(n)\boldsymbol{V}(n)$,

$$E\{\boldsymbol{V}(n+1)\} = E\{\boldsymbol{V}(n)\} + 2\gamma[E\{e^2(n)\}E\{\boldsymbol{X}(n)[\eta(n) - \boldsymbol{X}^T(n)\boldsymbol{V}(n)]\}]. \quad (A.3)$$

Follow the assumption that $x(n)$ and $\eta(n)$ are independent, then, ($A.3$) is simplifies to

$$E\{\boldsymbol{V}(n+1)\} = E\{\boldsymbol{V}(n)\} - 2\gamma E\{e^2(n)\}E\{\boldsymbol{X}(n)\boldsymbol{X}^T(n)\}E\{\boldsymbol{V}(n)\}, \quad (A.4)$$

$$E\{\boldsymbol{V}(n+1)\} = [\,\mathbf{I} - 2\gamma E\{e^2(n)\}\boldsymbol{R}\,]E\{\boldsymbol{V}(n)\}\Big|_{\|\boldsymbol{W}(n)\| \ll \|\boldsymbol{W}^*(n)\|}. \quad (A.5)$$

($A.5$) represents the mean weight error difference equation during the initial adaptation.