Non–Minimum Phase AR Identification using Blind Deconvolution Methods

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ABSTRACT

Identification of the parameters of non–minimum phase AR processes is formulated in the framework of the Super Exponential blind deconvolution scheme described in [4]. We show that the vector of the AR parameters lies in the range of a suitable (rank one) matrix formed using second and higher order statistics measured from the received data. The resulting algorithm is similar to the blind deconvolution scheme presented in [5], which here is shown to be directly obtained from the optimality criterion underlying the Super Exponential blind deconvolution algorithm.

1 Introduction

In this contribution, we address the problem of the estimation of the parameters of autoregressive (AR) discrete–time processes (series), *i.e.* processes observed at the output of an “all-pole” linear and shift–invariant (LSI) transformation driven by i.i.d. series.

When the AR modeling is for purposes of power spectral density (PSD) estimation, the phase of the underlying LSI transformation is not of interest and the celebrated Yule–Walker equations solve the minimum (and the maximum) phase case(s).

On the other hand, estimation of the phase of such IIR transformations has stimulated the interest of resarchers since [2], and it still receiving great attention nowadays (see, for instance, the contributions to the last IEEE Workshop on Higher Order Statistics [1]).

Herein, an estimation procedures of parameters of (possibly non–causal) non–Gaussian AR series is described which explicitely makes use of higher order statistics.

In particular, the structure of a rank–one matrix of higher order cross–cumulants between the AR series and the driving i.i.d. noise is exploited in an iterative blind deconvolution scheme, since all-pole stable IIR transformation are inverted (deconvolved) by FIR filters of suitable orders whose output is nothing else that the driving i.i.d. non–Gaussian noise.

The resulting algorithm is derived using the paradigm illustrated in [4] to obtain the so–called Super Exponential blind deconvolution algorithm and it results to be expressed as an iterative search of the unique eigenvector of a suitable matrix involving both second–order and higher order statistics of the received data.

Interestingly enough, it assumes the same form of the so-called EigenVector Algorithm described in [5], which here is revisited and rederived in the framework of the SE blind deconvolution.

2 Non–Minimum Phase AR Identification

Let us consider the following AR model

$$x[n] = \{H(q)\} \cdot w[n] \qquad (1)$$

where $w[n]$ is a zero–mean, non–Gaussian i.i.d. series with variance σ_w^2, $\{H(q)\}$ is the linear and shift–invariant operator[1] associated to the stable and (possibly) non–causal, Linear and Shift Invariant (LSI) transformation described by the following all-pole transfer function of order N

$$H(z) \stackrel{\text{def}}{=} \sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n} = \frac{1}{\displaystyle\sum_{k=0}^{N} a_k \cdot z^{-k}}$$

normalized to have a unit norm denominator polynomial, *i.e.* $\sum_{k=0}^{N} a_k^2 = 1$.

The inverse system exists if poles are not allowed onto the unit circle; it is given by a FIR equalizer of order N

$$F(z) \stackrel{\text{def}}{=} \sum_{n=0}^{N} f[n] \cdot z^{-n} = \sum_{n=0}^{N} a_n \cdot z^{-n}$$

Within a (complex) scalar factor, it is the (unique) stable point of the so-called Super Exponential (SE) blind deconvolution algorithm described in [4]. With reference to fig.1, the SE blind deconvolution algorithm iteratively obtains an estimate $\mathbf{f}_i$ (at iteration i) of the equalizer coefficients such that (the magnitude of) a higher order cumulant measured at the output of the equalizer $s[n]$ is maximized, under a quadratic constraint on $f_i[n]$. Convergence of the SE algorithm is understood from the fact that the overall channel+equalizer

[1] For the sake of readibility, we have indicated by q the “unit delay” operator, *i.e.* $\{q^k\} \cdot x[n] \stackrel{\text{def}}{=} x[n-k]$

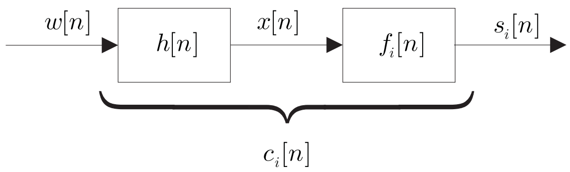

Figure 1: *Super Exponential blind deconvolution scheme.*

impulse response $c_i[n] = h[n] * f_i[n]$ is "spiked and spiked" as the iterations proceed.

For the sake of comprehension of the following developments, the SE algorithm relative to the maximization of the fourth order cumulant is briefly summarized in the case of real channel and signals. The input series $w[n]$ is supposed to have non–zero fourth–order cumulant κ_w^4.

The equalizer coefficients are obtained iteratively solving the following system of equations:

$$\begin{aligned} \mathbf{R}_{\mathrm{x}} \cdot \mathbf{f}_i' &= \boldsymbol{\kappa}^{(3,1)}_{\mathrm{s}_{i-1}\mathrm{x}} \\ \mathbf{f}_i &= \mathbf{f}_i' \cdot \left(\mathbf{f}_i'^{\mathrm{T}} \cdot \mathbf{f}_i' \right)^{-\frac{1}{2}} \end{aligned} \tag{2}$$

where $||\mathbf{f}_i||_l = f_i[l]$ is a unit norm vector which collects the equalizer coefficients at iteration i, $||\mathbf{R}_{\mathrm{x}}||_{km} = R_x[k-m] = \mathrm{E}\left\{x[k] \cdot x[m]\right\}$ is the covariance matrix of the observed AR series $x[n]$, the deconvolved series has been denoted by

$$s_i[n] = \sum_{l=0}^{N} f[l] \cdot x[n-l]$$

and the vector $||\boldsymbol{\kappa}^{(3,1)}_{\mathrm{s}_{i-1}\mathrm{x}}||_m = \kappa^{(3,1)}_{s_{i-1}x}[m]$ collects the diagonal slice of cross-cumulants of order $(3,1)$

$$\kappa^{(3,1)}_{s_{i-1}x}[m] \stackrel{\mathrm{def}}{=} \mathrm{cum}\Big(s_{i-1}[n], s_{i-1}[n], s_{i-1}[n], x[n-m] \Big)$$

To exploit the structure of the solution $\mathbf{f}_i$ at convergence, let us express the generic term of the r.h.s. of (2) as follows:

$$\begin{aligned} \kappa^{(3,1)}_{s_{i-1}x}[m] &= \mathrm{cum}\Big(s_{i-1}[n], s_{i-1}[n], s_{i-1}[n], x[n-m] \Big) \\ &= \sum_{l=0}^{N} f[l] \cdot \mathrm{cum}\Big(s_{i-1}[n], s_{i-1}[n], x[n-l], x[n-m] \Big) \end{aligned}$$

This expression allows for rewriting (2) in a form which we will exploit in the sequel. Introducing the matrix$\mathbf{K}^{(2,2)}_{\mathrm{s}_{i-1}\mathrm{x}}$ of cross–cumulants of order $(2,2)$

$$||\mathbf{K}^{(2,2)}_{\mathrm{s}_{i-1}\mathrm{x}}||_{l,m} = \mathrm{cum}\Big(s_{i-1}[n], s_{i-1}[n], x[n-l], x[n-m] \Big)$$

we can rewrite (2) as follows:

$$\begin{aligned} \mathbf{f}_i' &= \mathbf{R}_{\mathrm{x}}^{-1} \cdot \mathbf{K}^{(2,2)}_{\mathrm{s}_{i-1}\mathrm{x}} \cdot \mathbf{f}_{i-1} \\ \mathbf{f}_i &= \mathbf{f}_i' \cdot \left(\mathbf{f}_i'^{\mathrm{T}} \cdot \mathbf{f}_i' \right)^{-\frac{1}{2}} \end{aligned} \tag{3}$$

Due to the normalization, (3) constitutes the generic iteration of the power method [3] applied to the eigenequations

$$\lambda \cdot \mathbf{f} = \mathbf{R}_{\mathrm{x}}^{-1} \cdot \mathbf{K}^{(2,2)}_{\mathrm{wx}} \cdot \mathbf{f}' \tag{4}$$

where the matrix $\mathbf{K}^{(2,2)}_{\mathrm{wx}}$ is updated at each iteration, using the deconvolved series $s_i[n]$ as an estimate of $w[n]$.

Let us now prove the following:

Lemma I

The matrix of cross-cumulants $\mathbf{K}^{(2,2)}_{\mathrm{wx}}$ has rank one for AR series $x[n]$ and its generic element is given by

$$\kappa^{(2,2)}_{wx}[m,l] = \kappa_w^4 \cdot h[-l] \cdot h[-m].$$

Proof: Recalling the i.i.d. nature of the input series $w[n]$, the generic element of the matrix $\mathbf{K}^{(2,2)}_{\mathrm{wx}}$ takes the following form:

$$\begin{aligned} &\kappa^{(2,2)}_{wx}[m,l] \stackrel{\mathrm{def}}{=} \mathrm{cum}\Big(w[n], w[n], x[n-l], x[n-m] \Big) \\ &= \sum_{k,i=-\infty}^{\infty} h[i] \cdot h[k] \cdot \mathrm{cum}\Big(w[n], w[n], w[n-l-i], w[n-m-k] \Big) \\ &= \sum_{k,i=-\infty}^{\infty} h[i] \cdot h[k] \cdot \kappa_w^4 \cdot \delta[l+i] \cdot \delta[m+k] \\ &= \kappa_w^4 \cdot h[-l] \cdot h[-m] \square \end{aligned}$$

Interestingly, this implies that the useful solution of (4) is the eigenvector corresponding to the unique non–zero eigenvalue of the matrix $\mathbf{R}_{\mathrm{x}}^{-1} \cdot \mathbf{K}^{(2,2)}_{\mathrm{wx}}$.

The previous Lemma allows for proving the following:

Proposition I

The (unique) non–zero eigenvalue λ of the matrix

$$\mathbf{R}_{\mathrm{x}}^{-1} \cdot \mathbf{K}^{(2,2)}_{\mathrm{wx}}$$

is equal to the ratio κ_w^4 / σ_w^2.

Proof: Let us first write the eigen–equations (4) as follows

$$\mathbf{R}_{\mathrm{x}} \cdot (\lambda \cdot \mathbf{f}) = \mathbf{K}^{(2,2)}_{\mathrm{wx}} \cdot \mathbf{f} \tag{5}$$

where, due to Lemma I, λ assumes the role of the unique eigenvalue of the matrix $\mathbf{R}_{\mathrm{x}}^{-1} \cdot \mathbf{K}^{(2,2)}_{\mathrm{wx}}$, since the matrix $\mathbf{R}_{\mathrm{x}}$ is full rank.

Now, from Lemma I, the generic element of the r.h.s. of (5) can be expressed as follows:

$$\begin{aligned} \sum_{l=0}^{N} f[l] \left(\kappa_w^4 \cdot h[-m] \cdot h[-l] \right) &= \kappa_w^4 \cdot h[-m] \sum_{l=0}^{N} f[l] \cdot h[-l] \\ &= \kappa_w^4 \cdot h[-m] \end{aligned}$$

where the condition $h[n] * f[n] = \delta[n]$ of perfect equalization (inversion) has been invoked in the last equality.

Introducing the vector of cross-correlations

$$||\mathbf{r}_{\mathrm{wx}}||_m = \mathrm{E}\{w[n] \cdot x[n-m]\} = \sigma_w^2 \cdot h[-m]$$

we can rewrite (5) as follows:

$$\mathbf{R}_{\mathrm{x}} \cdot (\lambda \cdot \mathbf{f}) = \frac{\kappa_w^4}{\sigma_w^2} \cdot \mathbf{r}_{\mathrm{wx}}$$

Within a scalar constant, these are the Wiener equations which obtains the inverse filter $\mathbf{f}$; due to the unit norm assumption of the AR model coefficients, *i.e.* $\sum_{l=0}^{N} f[l] = 1$, it readily follows $\lambda = \kappa_w^4/\sigma_w^2$, since $\mathbf{R}_{\mathrm{x}} \cdot \mathbf{f} = \cdot \mathbf{r}_{\mathrm{wx}}$.□

Stemming out on this result, we propose a different and new algorithm which is based on solving the eigenequations

$$\lambda \cdot \mathbf{f}_i = \mathbf{R}_{\mathrm{x}}^{-1} \cdot \mathbf{K}_{\mathrm{s}_{i-1}\mathrm{x}}^{(2,2)} \cdot \mathbf{f}_i \tag{6}$$

using the power method without updating the matrix of cross-cumulants at each iteration! Simulations show that faster convergence is gained w.r.t. the SE algorithm, even counting the computations needed to obtain the eigenvector in (6).

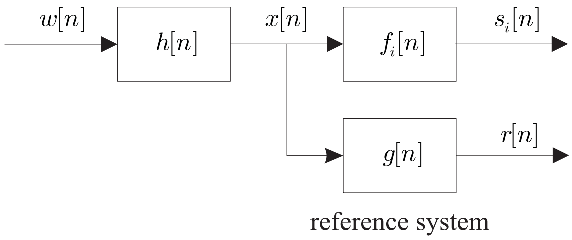

Figure 2: *EVA blind deconvolution scheme.*

It is worth mentioning that a so–called Eigenvector Algorithm for Blind Equalization (EVA), described in [5], results in the same equations (6) for the unknown equalizer coeeficients. It is based on a mathematical formulation, appeared in [6], where, with reference to fig.2, the output $r[n]$ of a *a priori* fixed "reference system" $g[n]$ is used to adjust the equalizer according to an "equalization quality" measured through the (magnitude of the) cross-cumulant $|\mathrm{cum}(s_i[n], s_i[n], r[n], r[n])|$, under a quadratic constraint on $f_i[n]$. Using the equalizer estimate at iteration i as a reference, *i.e.* $g[n] = f_i[n]$, the eigenequations (6) are obtained.

Quoting from [5], pg.14, *"(the algorithm of Shalvi and Weinstein [4]) turns out to be a special case of EVA ... note that [4] can be derived from EVA but not viceversa."*

Here, we have shown that, within the framework of all-pole IIR system identification, EVA is rather obtained form the paradigm described in [4], *i.e.* the maximization of the (absolute value) of an higher (than two) order cumulant measured at the output of the equalizer.

Inter alias, every extension or generalization of the Super Exponential algorithm is directly transposed to in a form similar to (6), where the matrix of the higher order cross-cumulants assumes a suitable form. For instance, using the framework of Bussgang deconvolution [7], a generalization of the Super Exponential algorithm is described in [8, 9], where optimal weighed linear combinations of higher order cumulants, tuned to the transmitted non-constant modulus constellation, are employed to minimize the final mean ISI.

3 Simulation Result and Conclusion

As an example, we consider the estimation of the parameters of a fourth order all–pole IIR filter having specular poles $re^{\pm j\theta}$ and $r^{-1}e^{\pm j\theta}$, for $r = 0.75$ and $\theta = 80$deg; the input series is drawn from a binary ± 1, zero–mean, i.i.d. series. All the statistics are measured from sampling at symbol rate, *i.e.* no redundancy due to fractional sampling is exploited. Figs. 3 and 4 report the mean Inter Symbol Interference (ISI) [2] obtained considering the inverse filter (equalizer) estimated from both the Super Exponential algorithm (labelled SW) and the described "Power Method" (labelled PM) for NSR values of 0 and -30dB, respectively. Each value of mean ISI has been obtained averaging over 25 Monte Carlo runs and using sample statistics drawn from 200 observations. We see that

[2] ISI is defined as $\mathrm{ISI} = \sum_n c^2[n] - 1$, being $c[n] \stackrel{\mathrm{def}}{=} h[n] * f[n]$ the overall (equalized) LSI transformation, normalized to have $c[0] = 1$.

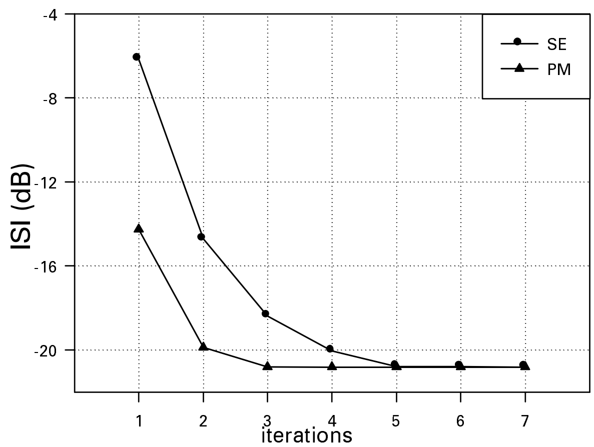

Figure 3: *Mean ISI vs. iterations (NSR=0). Real channel and binary symbols.*

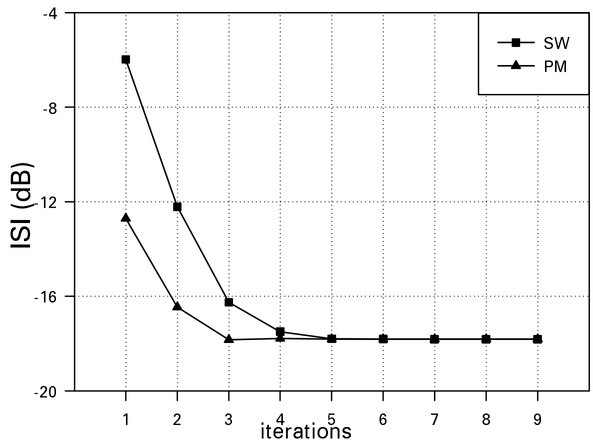

Figure 4: *Mean ISI vs. iterations (NSR=-30dB). Real channel and binary symbols.*

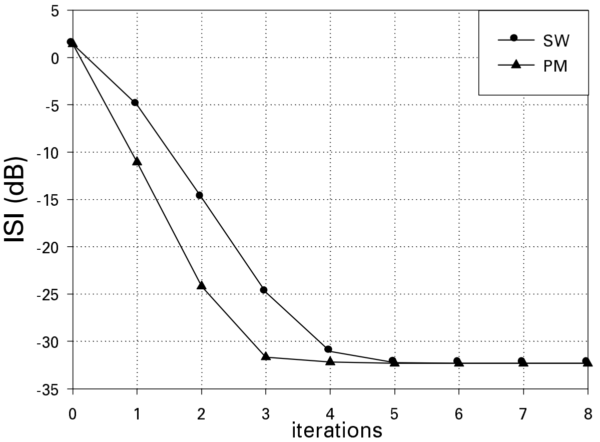

Figure 5: *Mean ISI vs. iterations (NSR=0). Complex channel and V27 symbols.*

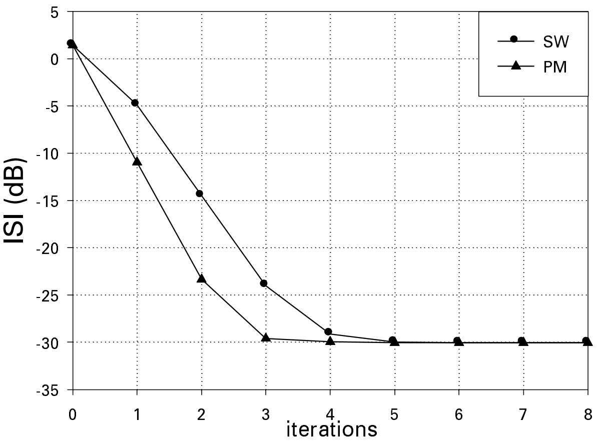

Figure 6: *Mean ISI vs. iterations (NSR=-30dB). Complex channel and V27 symbols.*

the estimation accuracy is roughly the same but the method herein described converges faster.

Another example concerning with a excitation sequence drawn from a complex valued constellation is reported in figs.5,6,7 and 8, where a fourth–order, complex all–pole IIR channel is considered; the poles are $\pm re^{j\phi_1}$ and $\pm r^{-1}e^{j\phi_2}$ for $r = 0.8$, $\phi_1 = \pi/6$ and $\phi_2 = \pi/3$. In figs.5 and 6 a V27 (Constant Modulus) constallation is considered, whereas in figs.7 and 8 the input series is drawn from a V29 (non Constant Modulus) constellation; sample statistics are drawn from 500 observations. These latter figures show the accuracy obtained using the power method with fourth–order cumulants (labelled PM4) in comparison with the power method with a linear combination of fourth–, sixth– and eighth–order cumulants (labelled PM468), whose weights are optimized to minimize the ISI [8, 9]. It is worth noting that sensible reduction of ISI can be gained using higher order cumulants when non Constant Modulus constellation are considered, as indicated in the statistical analysis performed in [4, 8, 9].

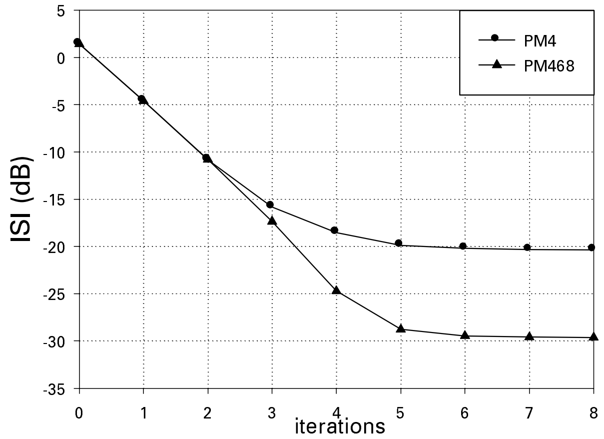

Figure 7: *Mean ISI vs. iterations (NSR=0). Complex channel and V29 symbols.*

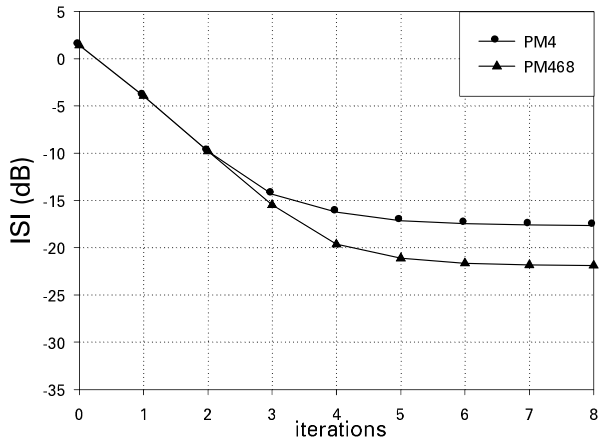

Figure 8: *Mean ISI vs. iterations (NSR=-30dB). Complex channel and V29 symbols.*

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