A New Operator for Image Processing Based on Lp-Filter Approximation

M. Tabiza, Ph. Bolon
LAMII/CESALP, Universit de Savoie,
B.P. 806 - F.74016 Annecy Cedex, France
(CNRS GDR ISIS-G720)
e-mail:{bolon,tabiza}@esia.univ-savoie.fr

ABSTRACT

In this paper, we define and analyse some properties of a class of order filters. These filters can be regarded as adaptive L-Filters which can be tuned by setting only one parameter instead of N, where N is the filter size. Deterministic and statistical properties are discussed. Experimental results obtained on both synthetic and real images show that the noise reduction effect of the new filter is similar to that of optimal L-Filters and optimal Lp-Filters whereas the edge preservation is improved.

1 INTRODUCTION

The Lp norm approach to image filtering provides alternatives to the least square (L2) technique. Furthermore, the Lp norm method ($p \neq 2$) performs better than the least square method for all non-Gaussian noise distributions [1, 2, 4]. However, some difficulties arise in the computation when parameter p is not an integer value. We have shown [5] that an appropriate linearization with respect to the order statistics means provides an easy way to evaluate the Lp-filter performances. Close to the optimal p, the Lp-filter and its L-Filter approximation give similar output and have similar output variances. The corresponding L-Filter does not only depend on p but also on the order statistics expected values. In this paper, we use the same structure and apply it to design a new operator for signal or image processing. Unlike the corresponding L-Filter, the proposed filter coefficients depend only on the parameter p and do not require the computation of the order statistics expected values. It is well-known that the parameter p of Lp-filters is basically related to the noise distribution. The new operator preserves this property. The new operator can then be adapted to various noise distributions by tuning only one parameter (namely p). Deterministic and statistical properties of the new filter are analyzed and simulation results are given. Experimental results obtained on both synthetic and real images are shown.

2 THE NEW FILTER

2.1 Definition

Let x_k (resp. y_k) be the input signal sample (resp. output) at location k. We consider a window having size N=2n+1 centered at location k. The filter output y_k is defined by:

$$y_k = \sum_{i=1}^{N} c_i x_{(i)} \tag{1}$$

where $x_{(i)}$ is the i^{th} order statistic
and

$$c_j = \frac{|\delta(p) - \frac{|w_{(j)}|}{|w_{(1)}|}|^{p-2}}{\sum_{i=1}^{N} |\delta(p) - \frac{|w_{(i)}|}{|w_{(1)}|}|^{p-2}} \tag{2}$$

$w_{(i)} = x_{(N-i+1)} - x_{(i)}$ is the i^{th} quasi-range, $w_{(1)}$ is the range, $\delta(p) = 0.01\ tanh(2(p-1))$.

The proposed filter depends on a single parameter. It should be noticed that for p = 2, the filter is linear (Mean Filter).

2.2 Filter Analysis

The new filter can be regarded as a L-Filter approximation of the Lp-Filter [5].
In fact, the Lp-Filter output y_k is:

$$y_k = argmin\ \epsilon(y) \quad with \quad \epsilon(y) = \sum_{i=1}^{N} |y - x_{(i)}|^p \qquad p \geq 1 \tag{3}$$

By linearizing with respect to the order statistics expected values, we have: $y_k \approx \sum_{i=1}^{N} a_i x_{(i)}$
and

$$a_j = \frac{|\bar{x}_{(j)}|^{p-2}}{\sum_{i=1}^{N} |\bar{x}_{(i)}|^{p-2}} \tag{4}$$

where $\bar{x}_j$ is the i^{th} order statistic expected value. For $(1 < p < 2)$, this equation presents discontinuities in the distribution of coefficients a_j. To cope with this difficulty, an offset $\delta(p)$ is introduced (see equation (2), more details are given in [5]). The computation of (4) or (5) requires the probability density function of the i^{th} order statistic. The noise probability density function must be known or estimated.
Moreover, we have shown that the performances of this L-Filter are close to the optimal Lp-Filter and the optimal L-Filter [3].
Using the new filter is equivalent to estimate the noise order statistics expected values by the half of the quasi-range.

3 DETERMINISTIC PROPERTIES

In this section, we examine the effects of the new filter along a step edge and a ramp edge. The proposed filter is :

- Shift-invariant
- Scale-invariant

  These properties are a direct consequence of equation (2).

- Step edge response

Figure 1 shows a 1D step edge. Let q be the number of pixels of intensity h in the filter window.

$$\begin{aligned} q &= 0 && for && k < -(n+1) \\ q &= N && for && k \geq n \\ q &= n+k+1 && for && k \in [-n, +n] \end{aligned}$$

The filter output, at location k, is given by:

$$y = \begin{cases} \dfrac{u\,(\delta(p)+h)^{p-2}}{2\,u\,(\delta(p)+h)^{p-2} + (1-2\,u)^{p-2}} & for\ q < \frac{N+1}{2} \\ \dfrac{(2\,u-1)\,\delta(p)^{p-2} + (1-u)\,(\delta(p)+h)^{p-2}}{2\,(1-u)\,(\delta(p)+h)^{p-2} + (2\,u-1)\,\delta(p)^{p-2}} & for\ q \geq \frac{N+1}{2} \end{cases} \tag{5}$$

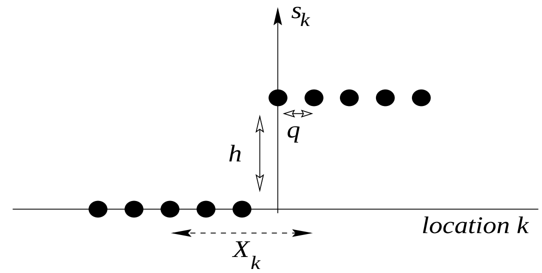

Figure 1: Step Edge

with $u = \frac{q}{N}$.

Figure 2 shows the response of the new filter and of the Lp-Filter for two values of parameter p. We

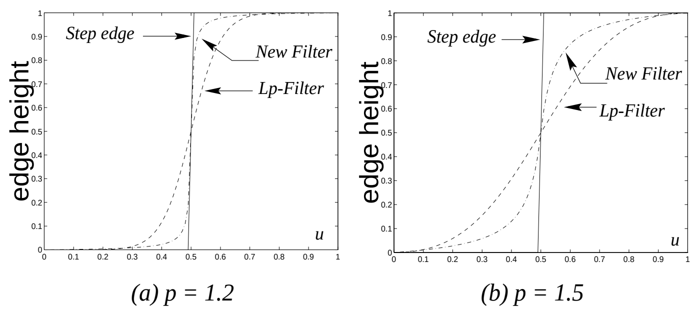

Figure 2: Filter response to a step edge.

can notice that for p ranging for 1.0 to 1.5 there is a good edge preservation. In these conditions, the new filter is better than the classical Lp-Filter.

- Ramp edge response

Figure 3 shows the response of the new filter and of the Lp-Filter for two values of parameter p. The

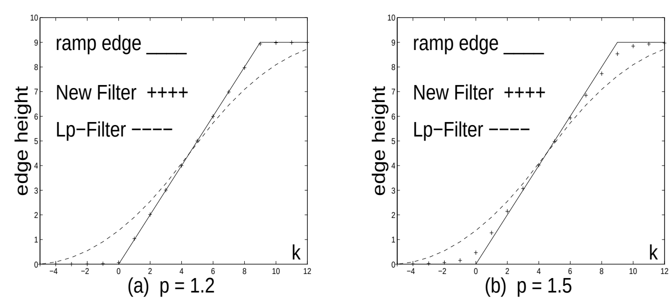

Figure 3: Filter response to a ramp edge.

new filter behavior in the presence of ramp edge is better than classical Lp-Filter.

4 STATISTICAL PROPERTIES

In this section, we consider the noise reduction effect which can be evaluated by means of the filter output variance, when the input is a noisy signal. We consider a constant signal s corrupted by a zero-mean white noise. Performances of the new filter are compared with those of the optimal Lp-Filter and of the optimal L-Filter (See Figure 4).

We remark that the proposed operator is close to

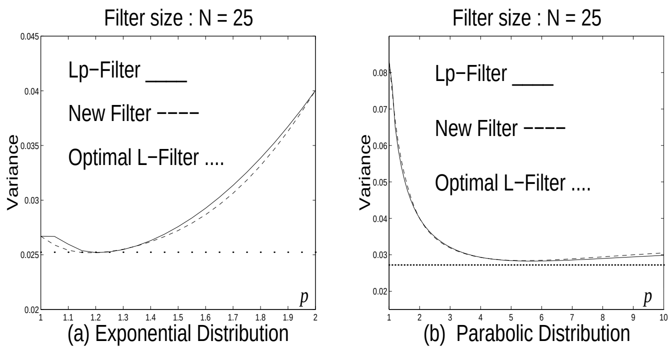

Figure 4: Output variance as a function of parameter p, Lp-Filter, Optimal L-Filter and the new filter.

the Lp-Filter. We can also notice that the output variance of the optimal L-filter and of the proposed filter are similar for each noise distribution. This study points out the fact that, for heavy tailed noise distributions, the noise reduction effect of the new filter is the same as that of the optimal L-Filter and of the optimal Lp-Filter whilst its behaviour near the edges is better. This property is no longer true for shallow-tailed distribution for which the parameter p is set to a value greater than 2.

5 EXPERIMENTAL RESULTS

In this section we present results obtained with synthetic and real images. They are compared with those obtained with the standard Lp-filter. Original, filtered and coutour images are displayed.

- Synthetic images

Figure 5 shows the results obtained with a synthetic image from the CNRS database (SAVOISE.gdr). This image is corrupted by an exponential noise. In this case, the optimal p value is 1.2 for a 5x5 filter size.

The new filter and the Lp-Filter have similar performances in terms of noise reduction. This result confirms the simulation study presented in the previous section.

- Real images

Figure 6 shows the results obtained with a real image from the CNRS database (MRI.gdr).

The effect of the new filter can be analysed in term of noise reduction and edge preservation. Image details are preserved and the noise is smoothed. Because of its better edge preservation property, the new filter yields a better edge map than the Lp-Filter. (see Figure 6d-6f).

6 CONCLUSION

A new operator for image pre-processing is introduced. The main interest of this filter is that it can be adapted to various noise distributions by tuning only one parameter (namely p) instead of N for a L-filter [3]. This filter preserves edges rather well for a large range of values of p ($1 \leq p < 1.5$). This allows to design filters adapted to various noise statistics. Moreover, the filter requires few operations, making it attractive for applications in image processing.

References

[1] J.Astola, Y. Neuvo, *"Optimal type median filters for exponential noise distribution"*, Signal processing, Vol.17, pp.95-104, oct. 1992.

[2] Ph. Bolon, *"Order filters, likelihood and optimality in image processing"*, Traitement du Signal, vol.9 no.3, oct. 1992, pp.225-250.

[3] A.C.Bovik, T.S.Huang, D.C.Munson, *"A generalization of median filtering using linear combinations of order statistics"*, IEEE Trans. Assp vol31-6, Dec.1983, pp.1342-1350.

[4] T.T.Pham and R.J.P. DeFigueiredo , *"Maximum likelihood estimation of a class of non-Gaussian densities with application to lp deconvolution"*, IEEE Trans. Acoust. Speech Signal Process. vol. 37, no. 1, 1989, pp. 73-82.

[5] M.Tabiza, Ph.Bolon , *"Performance Evaluation of Dα-Filters"*, Proceedings of EUSIPCO-96, Vol. 1, pp 49-52, Trieste.

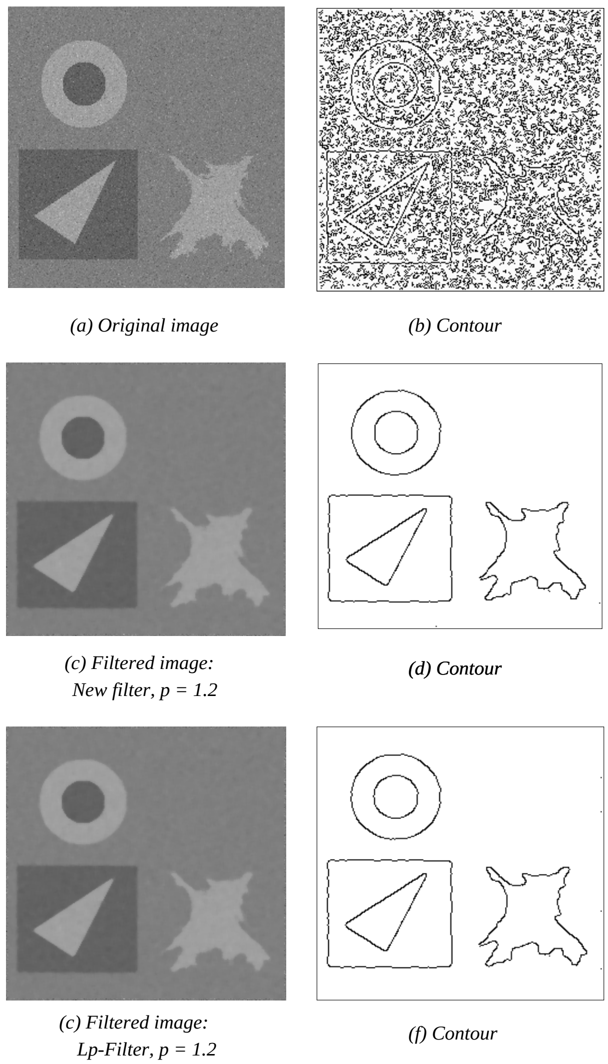

Figure 5:

(a) Original image SAVOISE.gdr corrupted by additive white exponential noise, σ^2=100;

(b) Deriche edge detector output of (a), $\lambda = 1.9$;

(c) Output of the new filter, size=5x5, $p = 1.2$;

(d) Deriche edge detector output of (c), $\lambda = 1.9$;

(e) Output of the Lp-filter,size=5x5, $p = 1.2$;

(f) Deriche edge detector output of (e), $\lambda = 1.9$.

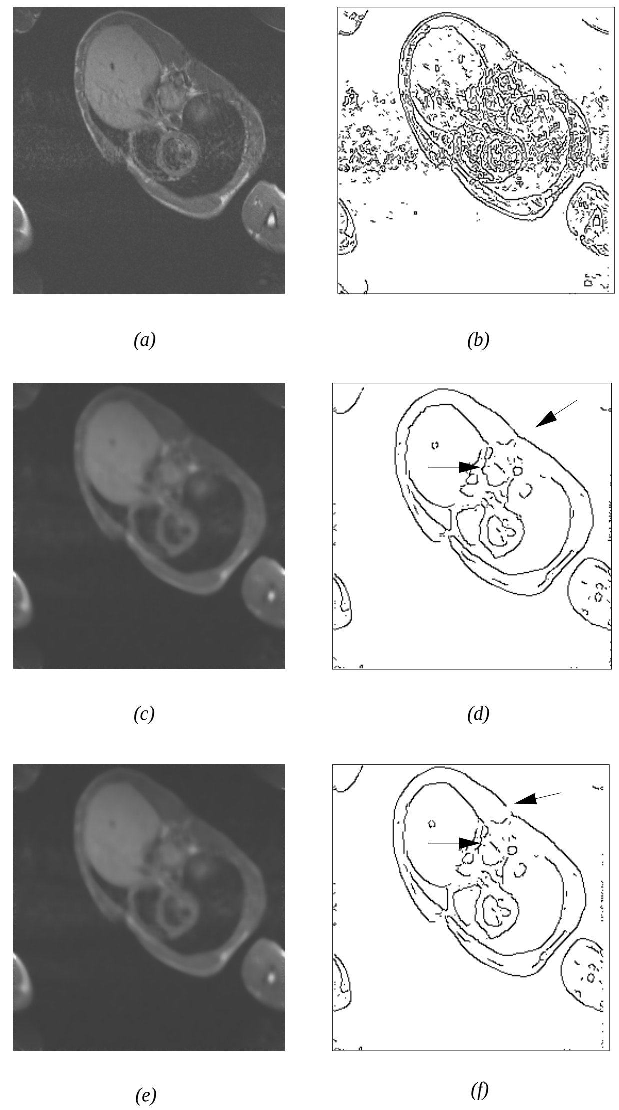

Figure 6:

(a) Original image MRI.gdr;

(b) Deriche edge detector output of (a), $\lambda = 2.1$;

(c) Output of the new filter, size=7x7, $p = 1.5$;

(d) Deriche edge detector output of (c), $\lambda = 2.1$;

(e) Output of the Lp-filter,size=7x7, $p = 1.5$;

(f) Deriche edge detector output of (e), $\lambda = 2.1$.