

# ABRLS ALGORITHM FOR TIME VARIANT CHANNEL EQUALISATION

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## ABSTRACT

This paper proposes a non-linear adaptive algorithm, the ABRLS algorithm, as an adaptation procedure for time variant channel equalisers. In the ABRLS algorithm, a coefficient matrix is updated based on the amplitude level of the received sequence. To enhance the tracking capability of the ABRLS algorithm, a parallel adaptation scheme is deployed which involves the structures of decision feedback equaliser (DFE). Computer simulations demonstrate that the novel ABRLS DFE provides a significant improvement related to the conventional RLS DFE on rapidly time variant communication channels.

## 1 INTRODUCTION

Two commonly used filter structures for communications channel equalisation are a linear transversal equaliser (LTE)[1] and a decision feedback equaliser (DFE)[2]. The computational complexity of the DFE is similar with that of the LTE. The DFE, however, can alleviate the noise enhancement involved in the LTE, and often provides better performance. This is due to the decision circuit in the structure of DFE, which provides a noise-free output if the decision is correct.

In time variant environments, rapid tracking is required for the equalisers. Although the least mean square (LMS) algorithm[5] is a simple and widely used adaptation procedure for the equaliser coefficients, its adaptation speed is not satisfied on a rapidly time variant channel. The alternative is the recursive least squares (RLS) algorithm[6]. The RLS algorithm has the potential to achieve more rapid adaptation than the LMS algorithm. In [3][4], the use of the DFE involving the coefficient adaptation with the RLS algorithm has been proposed on HF channels containing multipath characteristics with rapid time variation.

This paper proposes a novel technique for the adaptation of equaliser coefficients, the amplitude banded technique, to cope with time variant multipath channels. In the technique, amplitude information of the received sequence, which is directly associated with the channel impulse response, is used for the purpose of switching

the coefficients to be updated. Based on the principle of the technique, a novel adaptation algorithm, the amplitude banded recursive least squares (ABRLS) algorithm, is derived in which a non-linear adaptation process on a coefficient matrix is implemented.

This paper also presents a filter structure to make the ABRLS algorithm work effectively, which has a parallel adaptation scheme. By computer simulations, it is shown that the ABRLS algorithm provides superior performance in the structure of DFE by being aided by the standard RLS algorithm.

## 2 ABRLS ALGORITHM AND PARALLEL ADAPTATION

It is assumed that the channel model is given by

$$x_k = \sum_{i=0}^L h_i(k) u_{k-i} + n_k \quad (1)$$

where  $h_0(k), h_1(k), \dots, h_L(k)$  is the channel impulse response,  $u_k$  is the transmitted sequence, and  $n_k$  is a Gaussian white noise uncorrelated with  $u_k$ . The channel output  $x_k$  becomes the input for an equaliser.

Now let us consider that a tap coefficient vector  $\mathbf{c}(k)$  is updated by means of the RLS algorithm[6]. The input vector is  $\mathbf{x}(k)$ . When  $\mathbf{c}(k)$  and  $\mathbf{x}(k)$  are given by  $\mathbf{c}(k) = (c_0(k), c_1(k), \dots, c_{M_f+M_b}(k))^T$  and  $\mathbf{x}(k) = (x_k, x_{k-1}, \dots, x_{k-M_f}, \hat{u}_{k-d-1}, \hat{u}_{k-d-2}, \dots, \hat{u}_{k-d-M_b})^T$ , respectively, the RLS algorithm becomes the adaptation procedure for an  $M_f + M_b + 1$  length DFE, where  $\hat{u}_{k-d}$  is an estimate of the transmitted sequence delayed by  $d$  and  $M_f$  and  $M_b$  correspond to the order of the feedforward and feedback filters, respectively.

For the proposed algorithm, a  $Q$  by  $M_f + M_b + 1$  coefficient matrix  $\mathbf{C}_a(k)$  is prepared, elements of which are given by  $c_{ij}(k)$ ,  $i = 1, 2, \dots, Q$ ,  $j = 1, 2, \dots, M_f + M_b + 1$ . The  $\mathbf{C}_a(k)$  is initialised at  $k = 0$  where all the elements are set to zeros. For the adaptation, the elements of  $\mathbf{C}_a(k)$  are updated based on the operation of switching the elements to be updated. Among the  $Q$  by  $M_f + M_b + 1$  elements of  $\mathbf{C}_a(k)$ , only  $M_f + M_b + 1$  elements,  $c_{q(j)j}(k)$ ,  $j = 1, 2, \dots, M_f + M_b + 1$ , are selected for

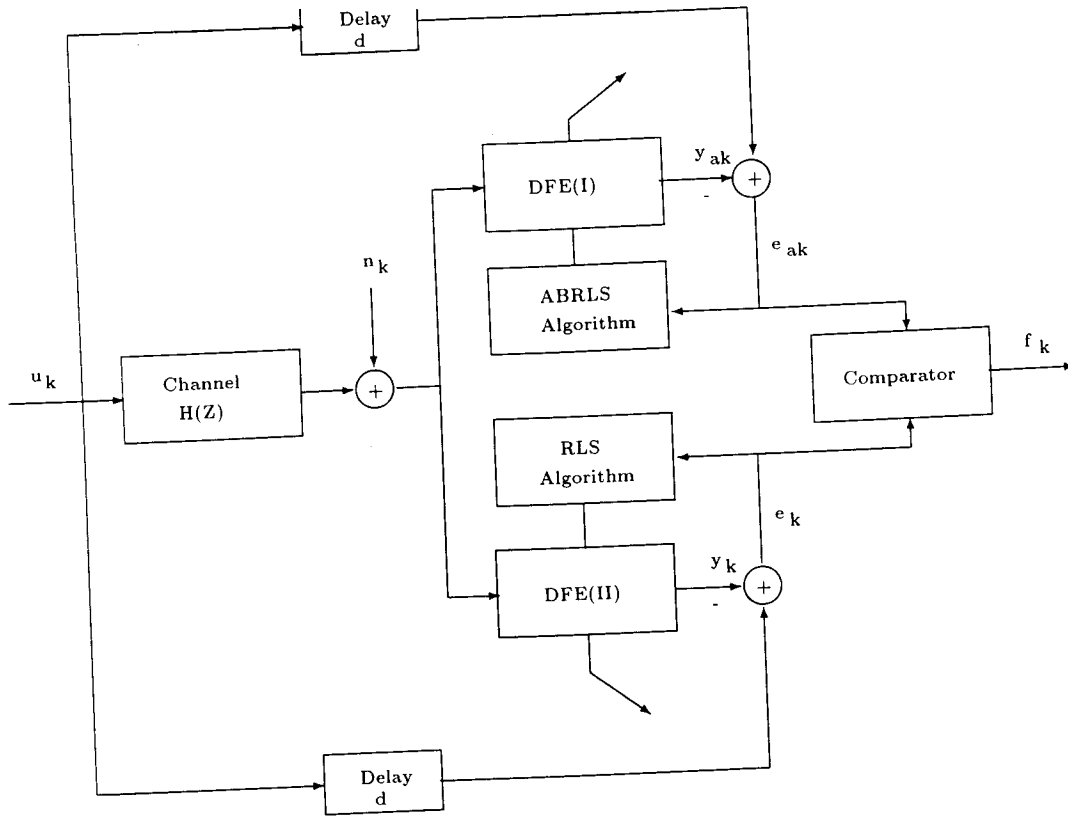


Figure 1: Configuration of the ABRLS DFE.

each iteration and a coefficient vector is set as  $\mathbf{c}_a(k) = (c_{q(1)1}(k), c_{q(2)2}(k), \dots, c_{q(M_f+M_b+1)M_f+M_b+1}(k))^T$  where  $q(j)$  is an integer. The  $q(j)$  is determined based on the amplitude level of each element  $x_j(k)$  of the input vector  $\mathbf{x}(k)$  for  $j = 1, 2, \dots, M_f + M_b + 1$  as follows:

- if  $|x_j(k)| \geq A_{max}(1 - 1/Q)$ , then  $q(j) = 1$
- if  $A_{max}(1 - 1/Q) > |x_j(k)| \geq A_{max}(1 - 2/Q)$ , then  $q(j) = 2$
- if  $A_{max}(1 - 2/Q) > |x_j(k)| \geq A_{max}(1 - 3/Q)$ , then  $q(j) = 3$
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- if  $A_{max}/Q > |x_j(k)| \geq 0$ , then  $q(j) = Q$

where  $A_{max}$  denotes the maximum amplitude of the received sequence, which is desired to be known a priori. The  $Q$  corresponds to a division number to classify the level of the amplitude of the received sequence. This value affects the performance of the equaliser. As the  $Q$  is increased, the performance is improved. However, a moderate selection for  $Q$  is enough to gain. The output is obtained by convolution between  $\mathbf{c}_a(k)$  and  $\mathbf{x}(k)$ . Thus the coefficient vector  $\mathbf{c}_a(k)$  is also updated

by the RLS algorithm, and then the updated coefficients  $\mathbf{c}_a(k+1)$  are inserted into the coefficient matrix  $\mathbf{C}_a(k+1)$ . For next iteration, a coefficient vector is again built up based on the elements of the input vector, and updated by the RLS algorithm. This adaptation is made in a non-linear operation, that is, *amplitude banding* of the received sequence. Thus we call the adaptation technique the amplitude banded technique. The above RLS-based algorithm provides the ABRLS algorithm for an  $M_f + M_b + 1$  length DFE. The ABRLS algorithm would update all the elements of the coefficient matrix  $\mathbf{C}_a(k)$ , because the input sequence is statistically distributed.

The ABRLS algorithm itself has the potential to track time variant channels more rapidly than the standard RLS algorithm. This is because the amplitude of the received sequence is directly associated with time variation of the channel, and based on the amplitude information, the equaliser coefficients are effectively selected and updated in the adaptation process. However, superior tracking of the ABRLS algorithm is not always guaranteed for all the adaptation process, due to the non-linearity which the ABRLS algorithm essentially has. Therefore, a parallel adaptation scheme is deployed here to achieve at least the performance of the conventional RLS DFE. Figure 1 depicts the whole con-

figuration of the ABRLS DFE to be proposed in this paper. In the structure of the ABRLS DFE, two DFEs, DFE(I) and DFE(II), are used to produce the equaliser output. The two DFEs are individually updated based on the error sequences  $e_{ak}$  and  $e_k$ , respectively. The comparator provides  $f_k = e_{ak}$  if  $(e_{ak})^2 \leq (e_k)^2$  and  $f_k = e_k$  otherwise. Based on the comparator output, the ABRLS DFE outputs  $y_{ak}$  when  $f_k = e_{ak}$ , and  $y_k$  when  $f_k = e_k$ .

### 3 SIMULATION RESULTS

Two channel models are used in our simulations. One is that the transfer function of which is given by

$$\text{Channel 1: } H_1(z) = 1 + \sin\left(\frac{2\pi}{T}k\right)z^{-1} \quad (2)$$

where  $T$  is the period to control the rate of time variation of the channel. The other is given by

$$\text{Channel 2: } H_2(z) = h_0(k) + h_1(k)z^{-1} + h_2(k)z^{-2} \quad (3)$$

where the time variant coefficients,  $h_0(k)$ ,  $h_1(k)$  and  $h_2(k)$  are generated by passing Gaussian white noise at 2400 sample/s through second order Butterworth filters with 3 dB bandwidths on the order of the fade rate. The input sequence of both channels is a uncorrelated, pseudo-random sequence with values of +1 or -1. Channel 2 is an HF channel model  $H_3(z)$  used in [3].

Figure 2 illustrates the convergence of the RLS LTE and ABRLS LTE for  $M_f = 3$ ,  $d = 0$ , and the forgetting factor  $\lambda = 0.92$  on channel 1 with the value of  $T = 3000$ . The RLS LTE and ABRLS LTE are DFEs without feedback, having the setting of  $M_b = 0$ . For the ABRLS LTE, two LTEs are used in a parallel form, the form of which is the same as that shown in Figure 1. The curves in Figure 2 are the average of 100 individual trials. The additive noise is -100 dB. The division number  $Q$  for the ABRLS algorithm is 5. It should be noted that among the factors causing channel distortion, time variation predominates over the additive noise and the delay in channel 1. Thus, by using channel 1, we can investigate the capability of the equalisers only against the time variation. Figure 2 visualises that the tracking done by the ABRLS algorithm aided by the RLS algorithm as depicted in Figure 1 is superior to that done by only the RLS algorithm. This would also validate the superior tracking capability which the ABRLS algorithm itself has.

Figure 3 illustrates the bit error rate (BER) performance against channel fade rates on channel 2 with a signal-to-noise ratio of 20 dB where the RLS DFE and the ABRLS DFE are compared. The equalisers have  $M_f = 4$  and  $M_b = 2$ . The forgetting factor has been optimised as  $\lambda = 0.94$  so that both equalisers provide the best performance. For the ABRLS algorithm,  $Q = 5$ .

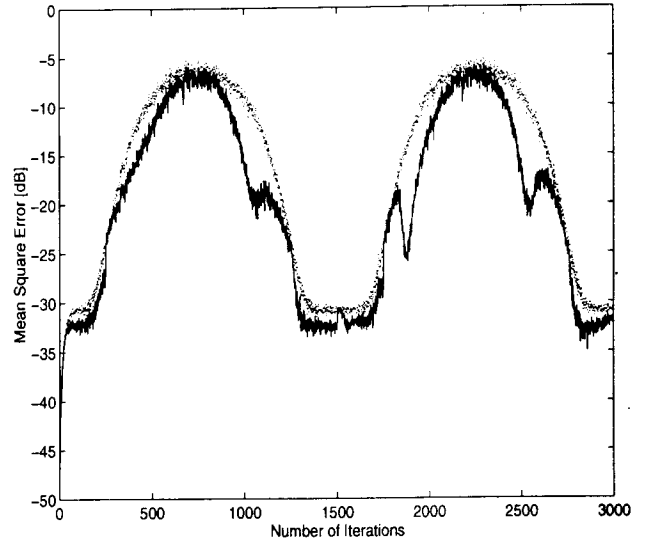


Figure 2: Convergence of the RLS LTE (dotted line) and the ABRLS LTE (solid line) on channel 1.

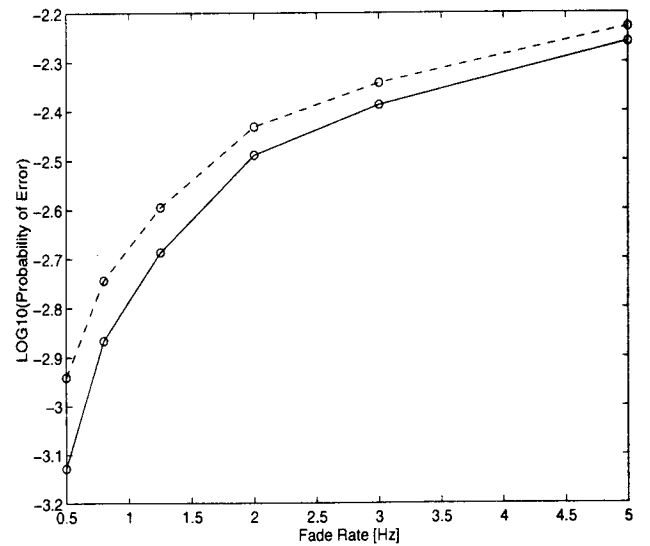


Figure 3: BER performance of the RLS DFE (dashed line) and the ABRLS DFE (solid line) against channel fade rates on channel 2.

From Figure 3, we observe that the ABRLS DFE provides better performance than the RLS DFE in a wide range of fade rates.

#### 4 CONCLUSION

This paper has proposed a novel DFE, the ABRLS DFE, which implements non-linear adaptation based on the amplitude level of the received sequence. Computer simulations have demonstrated that the ABRLS DFE provides better performance than the conventional RLS DFE in rapidly time variant environments. The ABRLS DFE, however, involves a parallel adaptation scheme and requires twofold the computational complexity of the RLS DFE.

#### References

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