Fourth-order cumulant-based algorithms for non-minimum phase MA system identification

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ABSTRACT

The purpose of this communication is the formulation of three new linear algorithms for blind non-minimum phase MA system identification. These methods have been derived starting from new equations involving system coefficients and q-slices of the $k-th$ order cumulant sequence of the ouput MA system. In particular, these new algorithms use only fourth-order cumulants, and thus, are specially useful when the driving system noise has symmetric probability density function and a possible additive Gaussian noisy process contaminates the system ouput.

1 INTRODUCTION

In the last decade more and more attention has been paid to the identification of non-minimum phase (NMP) systems using higher-order statistics. The methods developed are expected to find wide applications in diverse fields, such as sonar, radar, blind equalization, time-delay estimation, image and speech processing and seismology [1] [2]. So far, several methods have been proposed in the literature, among which linear algebra solutions [1], [2],[3], [4], [5], [6], [7] have attracted great interest due to their computational simplicity and because they provide good initial guesses to optimization-based solutions.

This paper presents three linear approaches based on a set of equations involving fourth-order output cumulants and their parameters to identify a (possibly) NMP linear system driven by a non-measurable independent and identically distributed non-Gaussian sequence from just only output measurements. Among the linear algebra-based methods, a vast majority of the literature exploits the third-order cumulants of the process in conjunction with data autocorrelations and/or fourth-order output cumulants [1], [2],[3], [4], [5], [6], and [7]. However, when the driving system noise has symmetric probability density funtion (pdf), its third-order cumulants are zero and fourth-order cumulants have to be used. There are many symmetric pdf non-Gaussian distributions of interest in seismic signal processing and data communications that have zero third-order cumulants but non-zero fourth-order cumulants. The proposed linear algorithms have been designed to be applied in these cases, and since they use only fourth-order cumulants, in principle, they are not sensitive to any additive Gaussian noise, regardless its power spectral density.

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2 NEW CUMULANTS EQUATIONS

Consider the following finite impulse response signal model:

$$x(n) = \sum_{i=0}^{q} b(i)w(n-i) \qquad b(0) := 1 \qquad (1)$$

where the observations of the signal $y(n)$ are contaminated by additive Gaussian noise ($y(n) = x(n) + v(n)$). The following assumptions are made in this paper:

- AS1) The driving noise sequence $w(n)$ is an unobservable, zero-mean, i.i.d. non-Gaussian process with symmetric pdf and at least cumulants of orders up to $2k$ finite, $k > 3$.
- AS2) The additive noise $v(n)$ is a i.i.d. zero-mean Gaussian process (white or coloured), independent of the input $w(n)$ and hence of the output $x(n)$.
- AS3) The LTI MA system is causal, possibly NMP, with $b(q) \neq 0$ and $b(0) = 1$. The last condition fixes the inherence scale ambiguity.
- AS4) The system order q is known. If it is not, it may be estimated using a similar scheme to that proposed in [8].

For the process (1), the Brillinger-Rosenblatt formula relates the output cumulants of the FIR system driven by non-Gaussian i.i.d. noise with its coefficients. Based on this formula, we are interested in looking for a relation between the q-slice of the fourth-order cumulants sequence $c_{4y}(\tau_1, \tau_2, q)$ and system coefficients. To accomplish that, we use the following result, which will be proved in the Conference:

Proposition 1. *Under the conditions AS1-AS3, the following relations involving system coefficients and the* $(\tau_2, \tau_3, ..., q)$ *and* $(\tau_3, \tau_4, ..., q)$ *slices of the k-th order cumulant sequence hold:*

$$\sum_{i=0}^{q} b^r(i) c_{ky}^{k-m}(\tau_1 + i, \tau_2, \ldots, q) = \frac{\gamma_{kw}^{k-m}}{\gamma_{(k-m+r)w}}$$
$$b^{k-m}(\tau_2) \ldots b^{k-m}(q) c_{(k-m+r)y}(\tau_1, \tau_1/0, \ldots, \tau_1/0) \tag{2}$$

$$\sum_{i=0}^{q} b^r(i) c_{ky}^{k-m}(\tau_1 + i, \tau_1 + i, \ldots, q) = \frac{\gamma_{kw}^{k-m}}{\gamma_{(2k-2m+r)w}}$$
$$b^{k-m}(\tau_3) \ldots b^{k-m}(q) c_{(2k-2m+r)y}(\tau_1, \tau_1/0, \ldots, \tau_1/0) \tag{3}$$

for $r = 1, \ldots, k-1, m = 0, \ldots, k-1,$ *and* $r \leq m$ *in Eqn. 2, and for* $r = 1, \ldots, k-1, m = 0, \ldots, k-1,$ *and* $r + k \leq 2m$ *in Eqn. 3, the argument of* c_{k-m+ry} *or* $c_{2k-2m+ry}$ *containing r-1 zeros.*

Corollary 1. *Under assumptions AS1-AS3, the following relationships involving the (multidimensional) q-slices of fourth-order ouput cumulants of* $MA(q)$ *system hold:*

$$\sum_{i=0}^{q} b(i) c_{4y}^3(\tau_1 + i, \tau_2, q) = \gamma_{4w}^2$$
$$b^3(\tau_2) b^3(q) c_{4y}(\tau_1, \tau_1, \tau_1) \tag{4}$$

$$\sum_{i=0}^{q} b^2(i) c_{4y}(\tau + i, \tau + i, q) = b(q) c_{4y}(\tau, \tau, 0) \tag{5}$$

$$\sum_{i=0}^{q} b^2(i) c_{4y}^2(\tau_1 + i, \tau_2, q) = \gamma_{4w}$$
$$b^2(\tau_2) b^2(q) c_{4y}(\tau_1, \tau_1, 0) \tag{6}$$

$$\sum_{i=0}^{q} b^3(i) c_{4y}(\tau_1 + i, \tau_2, q) = b(\tau_2) b(q) c_{4y}(\tau_1, 0, 0) \tag{7}$$

3 ALGORITHMS DESCRIPTION

The equations derived en previous section have been obtained after some manipulations from Proposition 1 with k=4. Based on the equations of the above Corollary, three algorithms for MA parameter estimation are proposed. These algorithms for the symmetric pdf case can be summarized as follows:

Algorithm 1 (LS1). *Consider equation (4). Using the relations*

$$c_{4y}(0, \tau_2, q) = \gamma_{4w} b(\tau_2) b(q) \tag{8}$$

and

$$\gamma_{4w} = \frac{c_{4y}^2(0, 0, q)}{c_{4y}(0, q, q)} \tag{9}$$

we get the set of equations

$$\sum_{i=0}^{q} b(i) c_{4y}^3(\tau_1 + i, \tau_2, q) =$$
$$= \frac{c_{4y}^3(0, \tau_2, q) c_{4y}(0, q, q)}{c_{4y}^2(0, 0, q)} c_{4y}(\tau_1, \tau_1, \tau_1) \tag{10}$$

Forming the system of equations that results for $\tau_1 = -q, \ldots, 0, \ldots, q$ *and* $\tau_2 = 0, \ldots, q-1$ *we obtain an overdetermined system of equations with* $q+1$ *unknowns* $(b(0), b(1), \ldots, b(q))$*. The least-squares solution is then found and the model parameters are obtained as* $(1, \frac{b(1)}{b(0)}, \ldots, \frac{b(q)}{b(0)})$*, provided that b(0) is near to the unity.*

Algorithm 2 (LS2). *Consider equation (5). Forming the system of equations that results concatenating (5) for* $\tau = -q, \ldots, q$*, we obtain* $2q+1$ *equations with* $q+1$ *unknowns* $(b(q), b^2(1), \ldots, b^2(q))$*. Taking the square root we obtain the absolute value of the MA coefficients. To elliminate the sign uncertainty we take the sign of the coefficients given by LS1.*

Algorithm 3 (LS3). *Another alternative for the calculation of the MA model parameters is to solve the equation (7). Using the fact*

$$c_{4y}(0, \tau_2, q) = \gamma_{4w} b(\tau_2) b(q), \tag{11}$$

(7) can be rewritten as

$$\sum_{i=0}^{q} b^3(i) c_{4y}(\tau_1 + i, \tau_2, q) = \frac{c_{4y}(0, \tau_2, q)}{\gamma_{4w}} c_{4y}(\tau_1, 0, 0) \tag{12}$$

If this equation is concatenated for $\tau_1 = -q, \ldots, q$ *and* $\tau_2 = 0, \ldots, l_1, (l_1 < q), l_1$ *being the last slice in the fourth-order cumulant space used to estimate the parameters. In this case we have* $(2q+1)(l_1+1)$ *equations and* $(q+1)$ *unknowns* $(\frac{1}{\gamma_{4w}}, b^3(1), \ldots, b^3(q))$ *which can be solved in the least-squares sense. Taking the cubic root we obtain the system parameters.*

The consistency (uniqueness) of the solution provided by these algorithms is guaranteed since coefficients matrix in either case can be proved to be full rank.

4 NUMERICAL RESULTS

In this section we apply the algorithms described previously to the NMP MA system identification problem from noisy output data. The driving system noise is a i.i.d. Laplacian random process with theoretical kurtosis $\gamma_{4w} = 12$. The additive noise is a coloured Gaussian process obtained by filtering the i.i.d. white Gaussian process through an ARMA system with AR coefficients $[1, -2.2, 1.77, -0.52]$ and MA coefficients $[1, -1.25]$. The performance of the three proposed algorithms is compared with the fourth-order version of the reformulated Giannakis-Mendel-Tugnait (GMT) method [2] [3]. Five hundred independent runs were performed in this study. Several MA models were tested, here we first show the results for the MA(5) model $[1, -2.33, 0.75, 0.5, 0.3, -1.4]$ used in [6]. Tables 1, 2, 3, and 4 show the mean and variance of the estimated parameters when the SNR was set up to 5dB, and Figures 1 and 2 plots the MSE of the estimated parameters over 500 realizations as a function of the SNR. Several conclusions can be drawn from these results:

- The proposed LS1 algorithm outperforms all the other methods in terms of bias, variance and MSE. This behaviour may be explained by the fact that the proposed algorithm makes use of more cumulant statistics than the other proposed methods. In addition, the goodness of the solution provided by this algorithm can be tested by comparing the estimated $b(0)$ with the unity, values far from it help to discard the solution.

- Estimations given by the GMT method are clearly biased since this method makes use of the signal autocorrelation sequence affected by the coloured gaussian noise.

- When the record of available data is short, the performance of all algorithms degrade due to the high variance in the cumulant estimates.

Finally, to corroborate these results, Tables 5 and 6, and Figure 3 shows the same than in the previous example, for the $MA(2)$ process used in [1] [4], with parameters [1, -2.0833, 1] in the same work conditions.

5 CONCLUSION

We have proposed three new algorithms for MA system identification based on the use of the system ouput fourth-order cumulant sequence. The first proposed method shows a good behaviour in terms of bias and MSE. In the coloured Gaussian noise case, the improvements of this algorithm over the GMT method justify its use for MA modelling of random processes with symmetric probability density function. A brief study of the influence of noise level and number of data has been presented in order to corroborate this conclusion.

Coefficients	LS1	LS2	LS3	GMT
-2.330	-2.283	-0.781	-0.744	-0.512
0.750	0.755	0.630	0.562	0.437
0.500	0.510	0.398	0.353	0.313
0.300	0.288	-0.060	-0.070	0.275
-1.400	-1.526	-0.750	-0.695	-0.493
MSE	0.083	0.465	0.461	0.480

Table 1: Estimated parameters (mean) for the process MA(5) contaminated by $5dB$ of coloured noise ($N = 8192$ and 500 Montecarlo runs).

Coefficients	LS1	LS2	LS3	GMT
-2.330	0.528	0.939	0.835	0.226
0.750	0.262	0.297	0.235	0.143
0.500	0.266	0.325	0.277	0.177
0.300	0.261	0.445	0.436	0.217
-1.400	0.524	0.297	0.270	0.216
MSE	0.218	0.455	0.407	0.118

Table 2: Standard deviation from the estimated parameters for the process MA(5) contaminated by $5dB$ of coloured noise ($N = 8192$ and 500 Montecarlo runs).

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Coefficients	LS1	LS2	LS3	GMT
-2.330	-2.280	-1.185	-1.140	-0.602
0.750	0.749	0.567	0.521	0.468
0.500	0.510	0.408	0.375	0.314
0.300	0.279	0.060	0.047	0.209
-1.400	-1.438	-0.870	-0.834	-0.563
MSE	0.029	0.268	0.276	0.424

Table 3: Estimated parameters (mean) for the process MA(5) contaminated by $5dB$ of coloured noise ($N = 16384$ and 500 Montecarlo runs).

Coefficients	LS1	LS2	LS3	GMT
-2.330	0.355	0.666	0.611	0.189
0.750	0.165	0.287	0.244	0.120
0.500	0.163	0.174	0.157	0.149
0.300	0.144	0.438	0.424	0.178
-1.400	0.251	0.241	0.237	0.169
MSE	0.029	0.294	0.268	0.086

Table 4: Standard deviation from the estimated parameters for the process MA(5) contaminated by $5dB$ of coloured noise ($N = 16384$ and 500 Montecarlo runs).

Coefficients	LS1	LS2	LS3	GMT
-2.0833	-2.209	-1.497	-1.324	-0.406
1.000	1.037	0.889	0.802	0.577
Standard deviation	LS1	LS2	LS3	GMT
-2.0833	0.624	1.200	0.971	0.079
1.000	0.319	0.508	0.372	0.089
MSE	0.08	0.32	0.26	0.47

Table 5: Estimated parameters (mean and standard deviation) for the process MA(2) contaminated by $5dB$ of coloured noise ($N = 8192$ and 500 Montecarlo runs).

Coefficients	LS1	LS2	LS3	GMT
-2.0833	-2.116	-1.844	-1.680	-0.410
1.000	0.990	0.922	0.877	0.567
Standard deviation	LS1	LS2	LS3	GMT
-2.0833	0.248	0.735	0.618	0.056
1.000	0.145	0.384	0.301	0.073
MSE	0.01	0.11	0.10	0.47

Table 6: Estimated parameters (mean and standard deviation) for the process MA(2) contaminated by $5dB$ of coloured noise ($N = 16384$ and 500 Montecarlo runs).

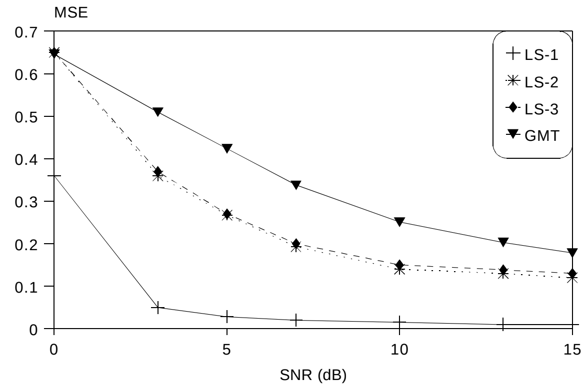

Figure 1: MSE as a function of the SNR for a the MA(5) model ($N = 16384$ and coloured noise).

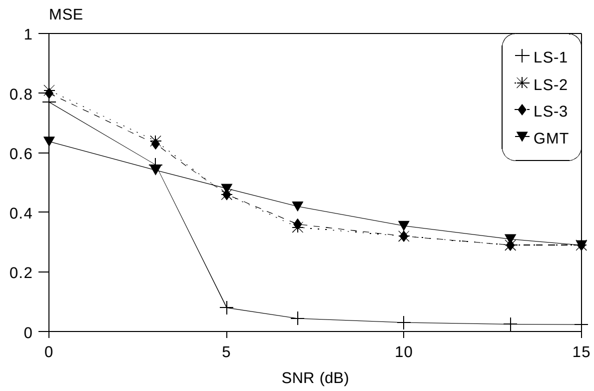

Figure 2: MSE as a function of the SNR for a the MA(5) model ($N = 8192$ and coloured noise).

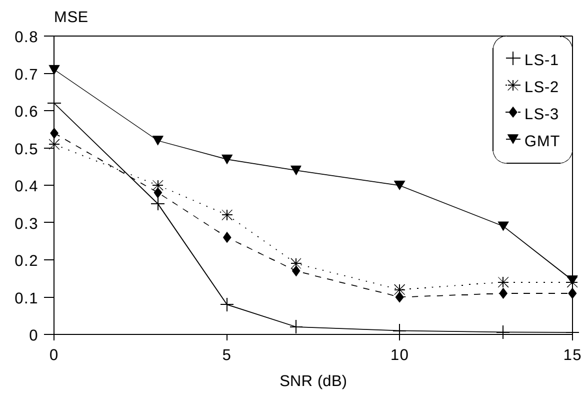

Figure 3: MSE as a function of the SNR for a the MA(2) model ($N = 16384$ and coloured noise).