# A METHOD OF DIRECTION FINDING OPERATING ON AN HETEROGENEOUS ARRAY

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# ABSTRACT

In a multipaths context, direction finding must resort to hight resolution algorithms. Classically, they operate on arrays of isotropic or identical sensors because they use the geometrical phase that the space diversity induces on the received signals. We propose to derive the standard MUSIC algorithm as the array is made up with sensor which are different from each other, assuming that their spatial complex responses are well know. An original computation is then described and some consequence advantages are underlined; for example, a new approach of the array ambiguity is presented, as the geometrical phase is no more the single relevant parameter attached to a direction of arrival. At last, some experimental results are listed in the field of H.F. radio direction finding, aiming to achieve a single site localization.

## **1** INTRODUCTION

The aim of this paper is to present an original method of direction finding using a heterogeneous array, i.e. made up with non identical sensors. The classical MUSIC algorithm with modified normalized steering-vectors is derived by integrating the directional sensors responses which are supposed to be well known. The array manifold doesn't only depend on the geometry but also on the directional functions of the sensors. It is demonstrated that it can improve the performance of the direction finding in the radio applications, especially when two types of polarizations are received (right or left polarization in the V.H.F. range, ordinary or extraordinary polarization in the H.F. range). The advantages of the method are outlined and experimental validations in the field of H.F. (3-30 MHz) radio direction finding are presented.

# 2 MUSIC ALGORITHM OPERATING ON AN ARRAY OF IDENTICAL SENSORS

The parametric methods aim to estimate parameters which are bound by the incident sources such as angles of arrival, power, and by the computation of a directional function from the steering vector of the array. For a source whose direction of arrival is denoted  $\theta$  (angle or couple of angles in a 3-D problem), this vector is written as :

$$a(\theta) = \begin{pmatrix} F(\theta) & F(\theta)e^{j\varphi_2(\theta)} & \cdot & F(\theta)e^{j\varphi_{NC}(\theta)} \end{pmatrix}^t \quad (1)$$

 $F(\theta)$  is the common complex directional response of the NC sensor (supposed to be identical) and  $\varphi_k(\theta)$  is the geometrical phase of sensor k (sensor 1 being chosen as a reference).

The MUSIC algorithm is based on the orthogonality of the incident steering-vectors and the noise subspace of the data covariance matrix : the pseudo-spectrum is computed from the norm of the projection of the normalized steering-vector  $b(\theta) = \frac{a(\theta)}{|a(\theta)|}$  in that subspace :

$$PSSP(\theta) = \frac{1}{\sum_{k=NSE+1}^{NC} \left| v_k^T \cdot b(\theta) \right|^2}$$
(2)

Where the set of vectors  $v_k$  define the noise subspace. The pseudo-spectrum  $PSSP(\theta)$  can be computed without the knowledge of the sensor responses  $F(\theta)$ :

$$\underline{b}(\theta) = \frac{\underline{a}(\theta)}{|F(\theta)|\sqrt{NC}} = \lambda(\theta)\underline{b_{iso}}(\theta)$$

Where  $\underline{b_{iso}}(\theta)$  is the normalized steering-vector for an array of isotropic sensors and  $\lambda(\theta) = \frac{F(\theta)}{|F(\theta)|} = e^{j \arg(F(\theta))}$  is a complex function whose norm equals one.

$$\left| \underline{v_k^T} \cdot \underline{b}(\theta) \right| = \left| \underline{v_k^T} \cdot \lambda \underline{b_{iso}}(\theta) \right|$$
$$= \left| \lambda \right| \cdot \left| \underline{v_k^T} \cdot \underline{b_{iso}}(\theta) \right| = \left| \underline{v_k^T} \cdot \underline{b_{iso}}(\theta) \right|$$
(3)

It can be noted that  $PSSP(\theta)$  is the same, whether we integrate the directional responses  $F(\theta)$  in the calculation or not.

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## 3 MUSIC ALGORITHM OPERATING ON A HETEROGENEOUS ARRAY

We propose an original method operating on an array that is made up of different sensors; their complex spatial responses, denoted  $F_k(\theta)$  are supposed to be well known [1]. The normalized steering-vector  $b(\theta)$  which appears in the calculation of the pseudo-spectrum is expressed as follows :

$$b(\theta) = \frac{1}{\sqrt{\sum_{k=1}^{NC} |F_k(\theta)|^2}} \begin{pmatrix} F_1(\theta) \\ F_2(\theta) e^{j\varphi_2(\theta)} \\ \vdots \\ F_{NC}(\theta) e^{j\varphi_{NC}(\theta)} \end{pmatrix}$$
(4)

Such a computation induces several improvements which are summarized in the following points.

#### 3.1 Non ambiguity of the geometrical phase

Seeking to estimate the D.O.A. in 3 dimensions, a problem of phase ambiguity occurs when a horizontal linear array is used : the geometrical phase is an ambiguous function of both azimuth Az and elevation El. For example, for an east-west oriented linear array,  $\varphi_k$  is expressed as  $\varphi_k(Az, El) = \frac{2\pi d_k}{\lambda} \sin(Az) \cos(El)$ . Using a heterogeneous linear array, the phase ambigu-

Using a heterogeneous linear array, the phase ambiguity does not affect the estimation of the angles of arrival because the set of sensors responses  $\{F_k(Az, El)\}$  can be identified separately from the seering-vectors b(Az, El).

#### 3.2 Polarization identification

If radiowaves are received with two possible types of polarization (circular right or left in the V/U.H.F. range or ordinary O and extraordinary X in the H.F. range), this method provides an identification of that information. The sensors have spatial responses which are different for each polarization type and can be identified as  $F_{nT}(\theta)$ , with index T for O or X in the H.F. example.

Two sets of steering-vectors :

$$b_{T}(\theta) = \frac{1}{\sqrt{\sum_{k=1}^{NC} |F_{kT}(\theta)|^{2}}} \begin{pmatrix} F_{1T}(\theta) \\ F_{2T}(\theta) e^{j\varphi_{2}(\theta)} \\ \vdots \\ F_{NCT}(\theta) e^{j\varphi_{NC}(\theta)} \end{pmatrix}$$
(5)

(T = O or X in the H.F. case) provide two pseudospectra  $PSSP_O$  and  $PSSP_X$  [2] defined as :

$$PSSP_T(\theta) = \frac{1}{\sum_{k=NSE+1}^{NC} |v_k^T \cdot b_T(\theta)|^2}$$
(6)

It allows the separation of the incident modes by their polarisation (also used in [5]) and achieves the direction finding even if the intermode correlation is strong (close to 1).

# 4 EXPERIMENTAL RESULTS

#### 4.1 Post processing

Experimental validations were obtained at the receiving site of Monterfil (University of Rennes 1, Latitude 48N, Longitude 1, 44'W), aiming at achieving radio direction finding in the H.F. range. European transmitters, over 700 Km far, were used. A 8 channels reception system, developed by D. Lemur [3] is connected to a linear array



Figure 1: Linear heterogeneous array of equi-spaced antennas

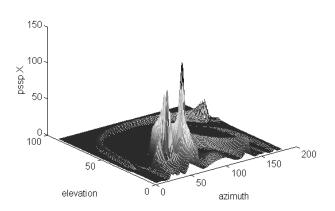


Figure 2: Pseudo-spectrum (X-mode) computed in the Horby experiment

of 8 equi-spaced loop antennas. The antennas are rotated about 30 degrees around a vertical axis every two positions within the array, so that the spatial responses are different each from another and the array becomes heterogeneous (figure 1).

The processed data were acquired from a transmitter located at Horby (Sweden). The carrier frequency is 13.69 MHz and each acquisition contains 6000 samples obtained at a sampling rate of 40 kHz. The bandwith being equal to 3 kHz and the SNR is equal to 30 dB. An example of  $PSSP_x(Az, El)$  function is plotted in figures 2 and 3, emphazing two extrema for Az = 47, El = 18and Az = 53, El = 10. The estimation azimuth can be compared to the geographical one (50) and the estimated elevations to the predicted ones (resorting to a propagation forecast program). These results are encouraging and they prove the possibility of achieving a 3D-direction finding with a heterogeneous linear array.

#### 4.2 Real time processing

Real time data processing, which achieves spectra calculation and extrema finding every 5 seconds was recently implemented. The experimental results are presented in figures 4 and 5. The German transmitter was located in Hamburg (carrier frequency : 10.101 MHz). A set of 11 data files were acquired. For each one, the data covariance matrix was estimed and the 2 pseudospectra were computed. They underline the presence of two complementary modes (one *O* and one *X* mode)

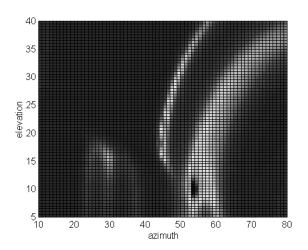


Figure 3: Zooming in around the azimuth 50

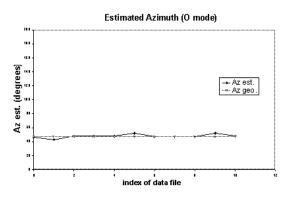


Figure 4: Estimated Azimuth in the Hamburg experiment (O mode)

whose azimuths of arrival were estimed and were compared with the expected (geographical) one (47). It can be seen that the measurements are well fitted in the O-wave and that a bias close to 5 degrees in the X-wave is showing in figure 5.

The corresponding estimations of the elevation, indicate that the O mode propagate in a 1E hop and the Xmode in a 1F2 mode. This suggest that within the F2layer a "tilt" effect (longitudinal slope of the electronic density) occured which makes the azimuth of arrival different from the geographical one.

### 5 CONCLUSION

An original method of radio direction finding which integrates the sensors responses, operating on an array of different sensors was proposed. Some additional studies will be carried out to test the usual assumption of spatially white noise. However, the MUSIC algorithm can be processed with a preliminary step of data whitening : which implies that the noise covariance is estimated at

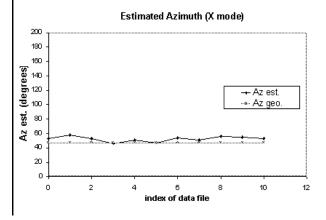


Figure 5: Estimated Azimuth in the Hamburg experiment  $(X \mod e)$ 

first. For example with a slight frequency shift from the carrier. This technique also operates on a sensor made up with collocated antennas. Some encouraging results have already been published [4]

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