WAVELET-PACKET BASIS SELECTION FOR ABRUPT CHANGES DETECTION IN MULTICOMPONENT SIGNALS

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ABSTRACT

We propose a "best basis" selection method, to detect abrupt changes in noisy multicomponent signals. The basis, selected from a wavelet-packet library with an energetic criterion, separates the different frequency components of the signal while keeping the best possible time localization. From the obtained basis, we reconstruct monocomponent signals in each band. The detector is based on the adequation of this set of signals with an abrupt change model. A performance analysis is realized from synthetic signals, in terms of selected basis and detection results.

1 INTRODUCTION

We present an abrupt change detection algorithm based on the decomposition of signals on a basis obtained from a wavelet-packet library. The families of considered signals are noisy multicomponent mixtures in which abrupt frequency changes happen.

This work follows a detection method [2] that consists of analyzing the time-frequency plane along the time axis according to successive cuts at fixed scales, through band-filtered signals obtained from the discrete wavelet transform (DWT). While the DWT is particularly well adapted to detection of transient signals (generally quick events, well localized in the time-scale plane) it is, on the other hand, more limited in a filter banks approach, because the bandwidth depends on the scale and then of the frequency. This problem does not exist any more with wavelet-packets, from which it is possible to choose a filter basis adapted to the signal.

Wickerhauser and Coifman have proposed an entropic criterion [1] to select the best basis for signal compression. This criterion is defined as a measure of distance between a signal and its decomposition, in terms of the Shannon entropy, and provides the basis where information is the mostly concentrated.

We search here the "best basis" in an other context : the wanted basis must distribute the various frequency components of the studied signal into different frequency bands and, on the other hand, provide the best possible time localization. One can represent the wavelet-packet decomposition as a binary tree, in which each node contains a set of coefficients coming from the signal filtering by a bandpass filter. The filter bandwidth depends on the node level in the tree. The two constraints of frequency separation and time localization become : to select the nodes corresponding to filters with only one frequency component, and with a maximal bandwidth under this constraint. After this segmentation of the time-frequency plane, we reconstruct a set of bandpass filtered signals from the selected basis. The detector is based on the comparison of these signals with an abrupt change model.

2 SIGNAL ADAPTED DECOMPOSITION

2.1 Wavelet-packet decomposition

A wavelet-packet decomposition [3] [5], is an extension of the DWT and can be obtained by a generalization of the fast pyramidal algorithm [4]. This decomposition is depicted in Figure 1, where h and g are the pair of quadratic miror filters (QMF) used in the DWT, associated to the scaling function ϕ and the wavelet function ψ. The wavelet packets $(C_{jm}(k))$, where j denotes the resolution level, m the spectral band and k the translation parameter, result from the signal decomposition on the basis of W_m functions such that :

$$C_{jm}(k) = \langle x(t), 2^{-j/2} W_m\left(2^{-j}t - k\right)\rangle$$

$$W_{2m}(t) = 2^{1/2} \sum_k h_k W_m\left(2t - k\right)$$

$$W_{2m+1}(t) = 2^{1/2} \sum_k g_k W_m\left(2t - k\right)$$

W_0 *corresponds to* ϕ
W_1 *corresponds to* ψ

As shown in Figure 1, wavelet-packet decomposition can be represented as a binary tree, in which each node corresponds to a wavelet-packet $(C_{jm}(k))$, j being the level in the tree.

In terms of time-frequency representation or, in an equivalent way, of filtering, one can associate at each of the successive steps of the decomposition (levels in the tree) a time-frequency sampling (Fig. 2). Through the

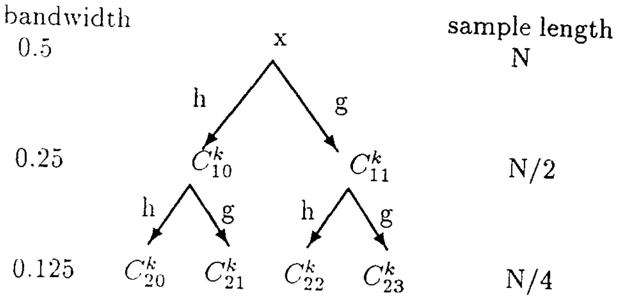

Figure 1: : Wavelet-packet decomposition represented as a binary tree. h and g correspond to the QMF filters of the DWT. For each level, the bandwidth and the length of the wavelet packet C^k_{jm} are indicated.

decomposition, the frequency resolution increases when the time resolution decreases.

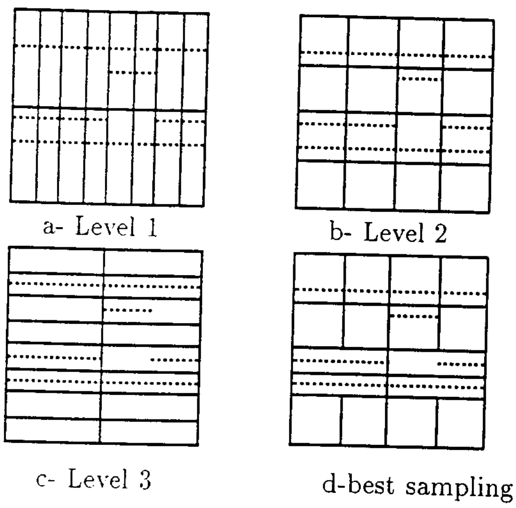

Figure 2: : Time-frequency pavement (solid lines) and ideal time-frequency representation (dot lines) of a 4-component signal (test signal used in this paper). a,b,c : different levels of the wavelet-packet decomposition. d : ideal sampling for the test signal.

2.2 Principles for the choice of a "best basis"

Wavelet-packet decomposition is a redundant representation of the signal, from which we must choose a "best basis" according to our objectives. Our goal is to find a basis allowing separation of the different components of the signal, and then to obtain monocomponent tracks having the best possible time resolution. Figure 2-d shows the time-frequency pavement corresponding to this requirement .

2.3 "Best basis" construction algorithm

The entropic criterion, well adapted for signal compression-reconstruction, is not suitable for our objectives. Effectively, applied to multicomponent signals, it results in a too precise frequency localization of the components, leading to low time resolution. We have chosen an energetic criterion allowing to decide if the signal has a significant activity in a given frequency band characterized by the j,m indices of the wavelet-packet. From this criterion, the principle of the basis selection algorithm is the following :

1. From the binary tree of wavelet-packets, we build a corresponding tree with the same structure, and where a given node takes the value 0 or 1 according to the value of the corresponding wavelet-packet variance :

$$\begin{cases} 1 \; if \; var \; [(C_{jm}(k))] > sv \\ 0 \; else \end{cases} \qquad (1)$$

   Taking again the example of the four-component signal used in Figure 2, we obtain the tree of Figure 3-a where each node at 1 corresponds to a packet having at least one component.

2. The inspection of the tree from the bottom gives the multi and mono component nodes. The nodes having more than one component are put to 2 (Fig. 3-b).

3. We select the nodes at 1 having a father at 2 (Fig. 3-c), that is to say monocomponent wavelet-packets whose father is multicomponent. This guarantees separation of components with maximal time resolution.

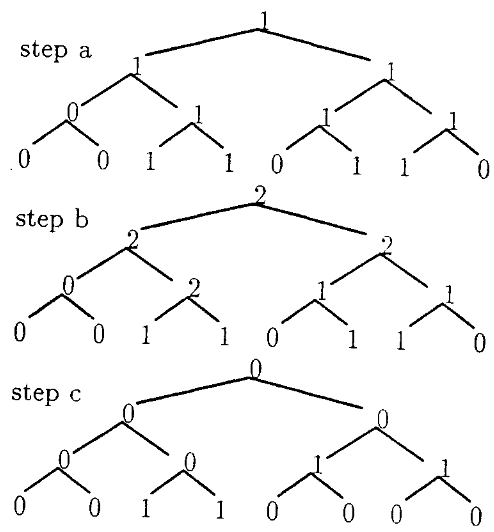

Figure 3: "Best basis" construction steps on the 4-component signal used in Fig.2 : a- nodes at 1 have at least one frequency component, b- mono(1) and multi(2) component nodes, c- "best basis" selection.

3 RECONSTRUCTION AND DETECTION

Given a tree with n nodes at 1 (indexed by $i = (j, m)$), we construct from the corresponding wavelet-packets C_i, n signals $(x_i(t))$ of the same length N, using the fast pyramidal algorithm, with $C_{i'}(k) = 0$ for $i' \neq i$ (Fig. 4).

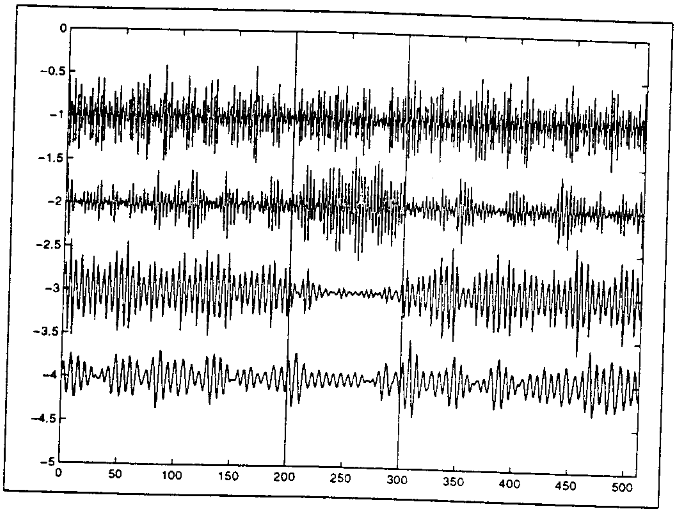

Figure 4: The set of monocomponent signals $x_i(t)$ reconstructed from the four wavelet-packets selected from a test signal. Vertical lines mark the position of the true changes.

To detect a shift from a frequency band to another one, an abrupt change is validated if one can observe, at the same time, in at least two tracks i and in opposite directions, a change in mean in the variance $(vi(t))$ of the signal $(x_i(t))$. For each level, a local index $(vr_i(k))$ is defined equal to the intercorrelation between $(v_i(k))$ and an abrupt change model defined as :

$$\begin{cases} m_r(t) = 1 \; si \; t < 0 \\ m_r(t) = -1 \; si \; t \geq 0 \end{cases}$$

$\{v_i(t)\}$ is estimated on a 20 points moving window centered on k :

$$\begin{aligned} v_i(t) &= E[(x_i(t))^2] \\ v_{ri}(t) &= E[v_i(n).m_r(n-t)]. \end{aligned}$$

Finally a global index $I(t)$ is defined from the local indices (figure 5):

$$I(t) = \max_{i \neq j} \{sqrt(|vr_i(t).vr_j(t)|)\} * \\ 1/2 \, |sign(vr_i(t)) - sign(vr_j(t))|$$

The first term is more important when the product concerns 2 levels affected by a shift. The second term allows to consider only opposite jumps in order to reduce noise sensibility. A change is detected in t if

$$I(t) > \beta \; and \; I(t) \; is \; a \; local \; maximum \quad (2)$$

where β is a threshold, whose different values will be used for construction of several ROC curves.

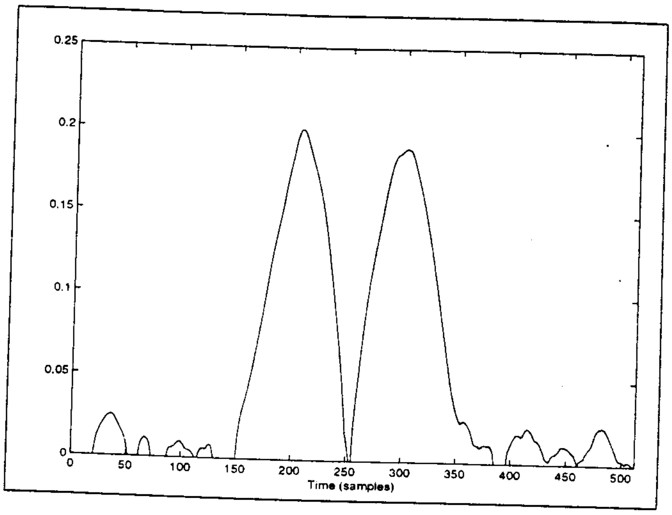

Figure 5: Index $I(t)$ calculated from the set of monocomponent signals x_i from Fig.4.

4 TEST SIGNALS AND RESULTS

All the simulated signals present two changes. Let us define a finite duration component : $s(f, a, [t1, t2])$, where f is the normalized frequency, a the amplitude and $[t1, t2]$ the time duration. The signal x consists in 4 components : $s(0.15, 0.5, [1, 512])$, $s(0.2, 1, [[1, 200] \cup [300, 512]])$, $s(0.35, 1, [200, 300])$, $s(0.4, 1, [1, 512])$. A white noise (SNR=5dB) is added to the signal and 100 records of the signal are generated by varying the noise draw. The wavelet used for the decomposition is a twenty samples Daubechies wavelet.

The performances are evaluated, on one hand, in terms of the selected basis and, on the other hand, through ROC curves showing final results of detection. Figure 6 shows the rates of selection of each node of the wavelet-packet tree, calculated on the 100 test signals. The threshold sv from Eq.(1) controls the selection of the basis. A small sv will detect more nodes having activity, i.e. having at least one frequency component. This will result in a too precise frequency localization leading to low time resolution. Conversely, a high sv may cause that one or several frequency components are missed. In our example, comparing with the four-packet ideal basis (Fig. 3-c) we can observe that 2 packets are selected at 100% and the two others at 96%.

Concerning the ROC curves (Fig 7), for a given threshold β (Eq.(2)), the true alarm rate (TP) is incremented every time a change is detected in tolerance windows centered around the true changes and the false alarm rate (FP) is incremented if at least a change is detected outside the tolerance windows. The percents of FP and TP are calculated on the 100 test signals (200 changes).

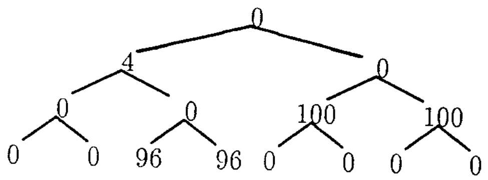

Figure 6: The sum of the "best basis" obtained from 100 test signals for a given threshold $sv = 0.4$ (the test signals x are normalized so that $\sigma_x = 1$). The ideal basis is plotted Fig.3-c.

5 CONCLUSION

While usually a best basis selection is motivated by signal compression, we have developed a decomposition method in a context of abrupt change detection. Our algorithm provides a basis that separates the different frequency components, with the largest bandwidth and then the best time resolution for a given dyadic decomposition.

We are currently working to improve the method in two directions : 1) defining a basis selection independent of any arbitrary threshold ; 2) improving time localization by selecting a basis independent of the binary representation of the wavelet-packet decomposition. Applications concerning musical signals are in progress.

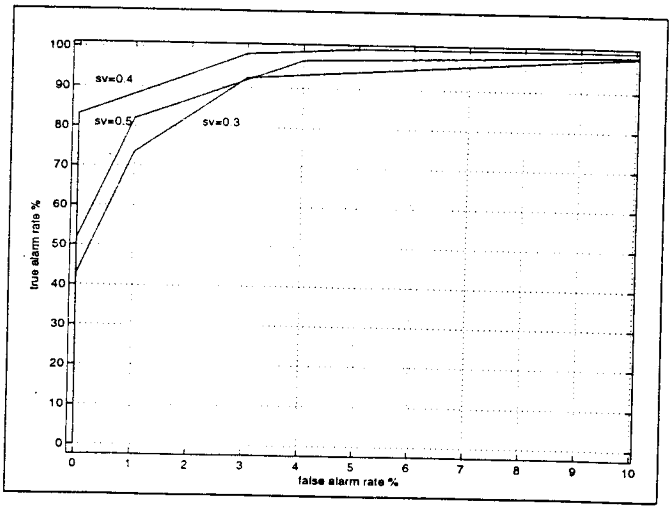

Figure 7: Roc curves plotted for 3 different basis selection thresholds sv. (The test signals x are normalized so that $\sigma_x = 1$.)

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