

LINEAR PREDICTION MODELING FOR SIGNAL SELECTIVE DOA ESTIMATION BASED ON HIGHER-ORDER STATISTICS

Hiroyuki Tsuji[†], Jingmin Xin[†], Yoshihiro Hase[†], and Blagovest Shishkov[‡]

[†]Communications Research Laboratory,
M.P.T. of Japan
3-4, Hikarino-oka,
Yokosuka, 239-0847, Japan
Tel: +81 468 47 5062
e-mail: tsuji@crl.go.jp

[‡]Technical University of Sofia
Inst. Of Appl. Math. & Informatics
1756 Sofia, Bulgaria

ABSTRACT

The direction-finding approach for impinging signals is one of the most important issues in array processing. By exploiting the cyclic statistics and higher-order temporal properties of communication signals, cyclic higher-order statistics (CHOS) direction-finding approaches have been proposed for narrow-band non-Gaussian signals. However, conventional cumulant-based algorithms become very complicated and are computationally intensive when a cumulant higher than the fourth-order is used. In this paper, by utilizing a linear prediction (LP) model of the sensor outputs, a new cyclic higher-order method is given to detect the signals of interest (SOI). The proposed method can not only reduce the computational load and completely exploit the CHOS temporal information, but can also correctly estimate the DOA of desired signals by suppressing undesired signals. We also show the effectiveness of the proposed method through simulation results.

1. INTRODUCTION

The direction-finding approach for impinging signals is one of the most important issues in array processing. Some types of modulated signals like QPSK and digital QAM can not be processed adequately by using second order statistics, which are appropriate only when the signals are Gaussian [1]. In recent years, some new signal-selective direction-finding algorithms such as cyclic MUSIC and ESPRIT have received much attention in communication systems for their ability to improve signal detection. These algorithms have led to the development of the theory of second-order cyclostationarity of signals. On the other hand, by exploiting the higher-order temporal properties of communication signals, many algorithms for direction-finding (DF) have been proposed for narrow-band non-Gaussian signals. However, there is a class of signals whose order is greater than two originating from, for example, second-order cyclostationary signals filtered by

channels whose bandwidth is less than their minimum cycle frequency. Moreover, cyclostationary-based approaches cannot detect signals which have the same cycle frequency. In general, dealing with higher-order cyclic statistics requires a large amount of data. Further, conventional cumulant-based algorithms become very complicated and are computationally intensive when a cumulant higher than the fourth-order is used [4]. Some studies [1][2][5][6][7] are based on the fundamental properties of the cyclostationarity concept and discuss the problem of using cyclic higher-order statistics (CHOS), where cyclic MUSIC was generalized by using fourth-order cyclic cumulants for one lag τ , to estimate the direction of arrivals (DOAs) of cyclostationary signals. In this paper, by utilizing a linear prediction (LP) model of the sensor outputs, a new cyclic higher-order method is given to detect the signals of interest (SOI). The proposed method can reduce the computational load and completely exploit CHOS temporal information multiple lags τ through the use of the LP model. It can also correctly estimate the DOA of desired signals by exploiting the cyclostationarity of the signals to suppress undesired signals. The proposed methods, appropriate for uniform linear arrays, employ CHOS of the array output and suppress additive Gaussian noise of unknown spectral content -- even when the noise shares common cycle frequencies with the non-Gaussian SOI. In addition, CHOS are tolerant of non-Gaussian interferences with cycle frequencies other than those of the desired signals and allow for the consistent estimation of the angles of arrival of signal sources whose number can be greater than the number of sensors.

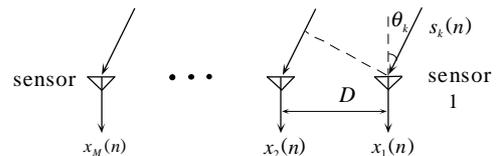


Fig. 1 The structure of an uniform linear array with M sensors in the base station.

2. SIGNAL MODEL AND CYCLIC CUMULANTS

If the uniform linear array consists of M sensors with separation distance D as shown in Fig.1, then the narrow-band signal model is given by

$$x_i(n) = \sum_{k=1}^P s_k(n) \exp(-j2\pi f_c(i-1)D \sin \theta_k / c) + v_i(n) \quad (1)$$

where $s_k(n)$ [$v_i(n)$] is the k -th source signal [sensor noise], θ_k is its DOA, P is the number of source signals, f_c is the carrier frequency, and c is the velocity of propagation. In this study, we will work under the following assumptions on the signal model:

[A1] $s_k(n)$'s are non-Gaussian, m -th order cyclostationary with a common cycle frequency and with absolutely summable cumulants $\forall m$ and nonzero cumulants of order m .

[A2] $v_i(n)$ in (1) is zero-mean, either stationary or Gaussian, and independent of the source signals or (non) Gaussian with different cyclostationarity to the source signals.

The above assumptions will be used all the following properties of cumulants [3]:

[P1] $\text{cum}\{\rho_1 x_1, \rho_2 x_2, \dots, \rho_n x_n\} = \rho_1 \rho_2 \dots \rho_n \text{cum}\{x_1, x_2, \dots, x_n\}$ for constant $\rho_1, \rho_2, \dots, \rho_n$

[P2] If the random variables $\{x_1, x_2, \dots, x_n\}$ and $\{y_1, y_2, \dots, y_n\}$ are independent, then

$$\begin{aligned} \text{cum}\{x_1 + y_1, x_2 + y_2, \dots, x_n + y_n\} \\ = \text{cum}\{x_1, x_2, \dots, x_n\} + \text{cum}\{y_1, y_2, \dots, y_n\}. \end{aligned}$$

[P3] For the set of random variables, $\{y_1, x_1, x_2, \dots, x_n\}$,

$$\text{cum}\{x_1 + y_1, x_2, \dots, x_n\} = \text{cum}\{y_1, x_2, \dots, x_n\} + \text{cum}\{x_1, x_2, \dots, x_n\}.$$

[P4] If any group of the x_i 's are independent of the remaining x_i 's then $\text{cum}\{x_1, x_2, \dots, x_n\} = 0$.

[P5] $\text{cum}\{x_1, x_2, \dots, x_n\}$ is symmetric in its arguments.

[P6] For Gaussian random variables $\{x_1, x_2, \dots, x_n\}$, $\text{cum}\{x_1, x_2, \dots, x_n\} = 0$ for $n \geq 3$.

By collecting the received signal $\{x_i(n)\}$ for $i=1, \dots, M$, then the received signal vector $\mathbf{x}(n)$ can be expressed as

$$\mathbf{x}(n) = \mathbf{A}(\theta) \mathbf{s}(n) + \mathbf{v}(n), \quad n=0, 1, \dots, N-1 \quad (2)$$

where

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1) \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_p)], \quad \mathbf{s}(n) = [s_1(n), s_2(n), \dots, s_p(n)]^T$$

and the steering vector is parameterized as follows

$$\begin{aligned} \mathbf{a}(\theta_k) = [1, \exp(-j2\pi f_c D \sin \theta_k / c), \\ \dots, \exp(-j2\pi f_c (M-1) D \sin \theta_k / c)]^T. \end{aligned} \quad (3)$$

Most communication signals are not only non-Gaussian but also exhibit cyclostationarity due to a modulation with carrier. Further, the corresponding discrete-time signals obtained by oversampling these continuous time signals are also cyclostationary. Therefore, a process $\{\mathbf{x}(n), n=0, 1, \dots\}$ is said to exhibit m -th order

cyclostationarity when its time-varying cumulants, up to order m , are (almost) periodic functions of time. The m -th order cyclic cumulant with cycle frequency α of $\mathbf{x}(n)$ is the Fourier series coefficient of its time-varying cumulant

$$C_{mx}(n; \tau_1, \dots, \tau_{m-1}) \equiv \text{cum}\{x(n), x^{(*)}(n + \tau_1), \dots, x^{(*)}(n + \tau_{m-1})\} \quad (4)$$

and is given by

$$C_{mx}(\alpha; \tau_1, \dots, \tau_{m-1}) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} C_{mx}(n; \tau_1, \dots, \tau_{m-1}) e^{-j2\pi \alpha n}, \quad (5)$$

where $(*)$ is either a conjugate manipulation or nothing, that is $(*)$ is an optional conjugation [3]. With the finite data and under absolute cumulant summability (i.e. mixing), the estimate

$$\hat{C}_{mx}(\alpha; \tau_1, \dots, \tau_{m-1}) = \frac{1}{N} \sum_{n=0}^{N-1} C_{mx}(n; \tau_1, \dots, \tau_{m-1}) e^{-j2\pi \alpha n} \quad (6)$$

is consistent and asymptotically normal. Specifically for $m=3, 4$ when $E\{\mathbf{x}(n)\} = \mathbf{0}$, we have, for example,

$$C_{3x}(\alpha; \tau_1, \tau_2) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} E\{x(n) x^*(n + \tau_1) x(n + \tau_2)\} e^{-j2\pi \alpha n} \quad (7)$$

$$\begin{aligned} C_{4x}(\alpha; \tau_1, \tau_2, \tau_3) \\ = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} [E\{x(n) x^*(n + \tau_1) x(n + \tau_2) x^*(n + \tau_3)\} \\ - E\{x(n) x^*(n + \tau_1)\} E\{x(n + \tau_2) x^*(n + \tau_3)\} \\ - E\{x(n) x(n + \tau_2)\} E\{x^*(n + \tau_1) x^*(n + \tau_3)\} \\ - E\{x(n) x^*(n + \tau_3)\} E\{x^*(n + \tau_1) x(n + \tau_2)\}] e^{-j2\pi \alpha n} \end{aligned} \quad (8)$$

With finite data, the estimate of the k th-order cyclic moment is given by

$$\hat{M}_{kx}(\alpha; \tau_1, \tau_2, \dots, \tau_{k-1}) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) x^*(n + \tau_1) \dots x^{(*)}(n + \tau_{k-1}) e^{-j2\pi \alpha n}$$

and sample cyclic k th-order cumulants are estimated via sample cyclic moments of order $< k$ [11].

For stationary $\mathbf{x}(n)$, the cyclic cumulant is time invariant, and hence $C_{mx}(\alpha; \tau_1, \dots, \tau_{m-1}) = 0, \forall m, \alpha \neq 0$, whereas for Gaussian (cyclostationary or not) $\mathbf{x}(n)$, $C_{mx}(\alpha; \tau_1, \dots, \tau_{m-1}) = 0, m \geq 3, \forall \alpha$. Consequently, higher-order cyclic statistics can distinguish between stationary / cyclostationary and Gaussian / non Gaussian processes.

3. CHOS DOA ESTIMATION EXPLOITING LINEAR PREDICTION MODEL

Supposing that the received data $x_M(n)$ is predicted as a linear combination of the remaining $(M-1)$ sensor outputs expressed by

$$x_M(n) + \sum_{i=1}^{M-1} a_i x_{M-i}(n) = \varepsilon_M(n) \quad (9)$$

where $\{a_i\}$ are the LP coefficients. By multiplying (9) by appropriate delayed versions of the random process $x_M(n)$ and taking expectations, it is not difficult to show

that the third- and fourth-order cumulants satisfy the difference equation (9).

$$C_{3x_M x_M x_M}(\alpha; \tau_1, \tau_2) + \sum_{i=1}^{M-1} a_i C_{3x_{M-i} x_M x_M}(\alpha; \tau_1, \tau_2) = 0 \quad (10)$$

$$C_{4x_M x_M x_M x_M}(\alpha; \tau_1, \tau_2, \tau_3) + \sum_{i=1}^{M-1} a_i C_{4x_{M-i} x_M x_M x_M}(\alpha; \tau_1, \tau_2, \tau_3) = 0 \quad (11)$$

Recall the narrow-band signal model (1) and consider the third-order and fourth-order cyclic cumulant of the M -th sensor output, which under assumptions [A1] and [A2] and properties of cumulant [P1-P6] are respectively

$$\begin{aligned} C_{3x_M x_M x_M}(\alpha; \tau_1, \tau_2) \\ = \sum_{k=1}^{P_\alpha} e^{-j2\pi f_c (M-i) D \sin \theta_k / c} C_{3s_k}(\alpha; \tau_1, \tau_2) \end{aligned} \quad (12)$$

and

$$\begin{aligned} C_{4x_M x_M x_M x_M}(\alpha; \tau_1, \tau_2, \tau_3) \\ = \sum_{k=1}^{P_\alpha} e^{-j2\pi f_c (-i) D \sin \theta_k / c} C_{4s_k}(\alpha; \tau_1, \tau_2, \tau_3) \end{aligned} \quad (13)$$

where P_α are the number of SOI. If the cumulants of the process are known, then by evaluating (10) and (11) for various lags τ_1, τ_2, τ_3 , it is possible to obtain linear equations to solve for coefficients $\{a_k\}$. For example, letting $\tau_1 = \tau_2 = \tau_3 = 0, 1, 2, \dots, L-1$ yields

$$\begin{bmatrix} C_{3x_{M-1} x_M x_M}(0,0) & \dots & C_{3x_1 x_M x_M}(0,0) \\ \vdots & \ddots & \vdots \\ C_{3x_{M-1} x_M x_M}(L-1, L-1) & \dots & C_{3x_1 x_M x_M}(L-1, L-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{M-1} \end{bmatrix} \quad (14)$$

$$= - \begin{bmatrix} C_{3x_M x_M x_M}(0,0) \\ \vdots \\ C_{3x_M x_M x_M}(L-1, L-1) \end{bmatrix}$$

$$\begin{bmatrix} C_{4x_{M-1} x_M x_M x_M}(\alpha; 0,0,0) & \dots & C_{4x_1 x_M x_M x_M}(\alpha; 0,0,0) \\ \vdots & \ddots & \vdots \\ C_{4x_{M-1} x_M x_M x_M}(\alpha; L-1, L-1) & \dots & C_{4x_1 x_M x_M x_M}(\alpha; L-1, L-1, L-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{M-1} \end{bmatrix} \quad (15)$$

$$= - \begin{bmatrix} C_{4x_M x_M x_M x_M}(\alpha; 0,0,0) \\ \vdots \\ C_{4x_M x_M x_M x_M}(\alpha; L-1, L-1, L-1) \end{bmatrix}$$

or

$$C\mathbf{a} = \mathbf{y} \quad (16)$$

Figure 2 depicts the block diagram for generating the third-order cyclic cumulants which follow the equation (14). As shown in Fig. 2, the structure is very simple and can reduce the computational load.

The rank of the cumulant matrix in (14) and (15) is equal to the number P_α of SOI with cycle frequency α .

Taking the singular-value decomposition (SVD) into C gives $C = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$, where $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_{M-1}]$, $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_{M-1}]$, and $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_{M-1})$. By using the SVD and the number of P_α , the estimate $\hat{\mathbf{a}}$ of the coefficients \mathbf{a} is obtained by

$$\hat{\mathbf{a}} = \sum_{i=1}^{P_\alpha} \frac{\mathbf{v}_i \mathbf{u}_i^H}{\lambda_i} \mathbf{y}. \quad (17)$$

After the parameters $\{a_i\}$ have been estimated their DOA can be found by searching the positions of the peaks of the power spectral density given by

$$P(\omega) = P(\theta) = \frac{1}{|1 + \hat{a}_1 z^{-1} + \dots + \hat{a}_{M-1} z^{-(M-1)}|^2} \quad (18)$$

where $z = \exp(j\omega)$. Therefore, to obtain the DOA of the desired signals, we choose the cycle frequency α which corresponds to the desired signals and solve equation (16), and we search for the positions of the peaks of (18).

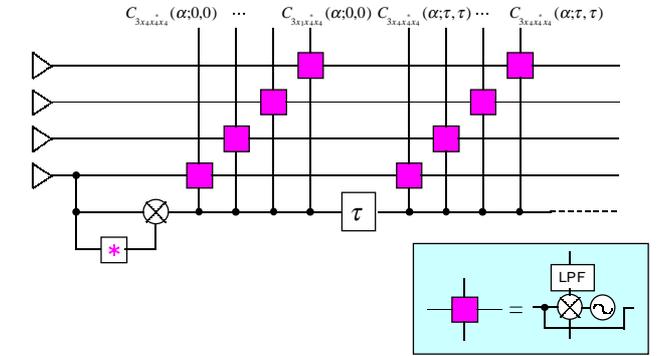


Fig. 2 Block diagram for the proposed methods (Third order)

4. NUMERICAL EXAMPLES

First of all, we show the effectiveness of the higher-order cyclic cumulant. Figures 3 and 4 show examples of the absolute values of second- and fourth-order cyclic cumulants at a varying cycle frequency α and lag parameter τ . The signal used in these examples contains one BPSK signal with $\alpha = 0.25$, one AM signal with $\alpha = 0.4$, and Gaussian background noise. The BPSK signal is filtered using a raised cosine filter with a 0.5 rolloff factor. The SNRs of the BPSK and AM signals (with respect to the noise) are 10 dB and 0 dB, respectively. The figure of the fourth-order cumulant shows significant improvement of the BPSK signal detection at cycle frequency $\alpha = 0.25$ compared to the second order.

Finally, we present the simulation results that show the effectiveness of the proposed method. We considered a uniform linear array having eight elements with half-wavelength spacing. Three signals impinge on the array. The SOIs are two BPSK signals with 0.25 baud rates ($\alpha = 0.25$) which arrive from 20° and 50° . The

interference is an AM signal arriving from -35° ($\alpha=0.6$). The length of the sample data is $N=512$, and the lag parameter is $L=16$. The signal-to-noise ratios (SNR) for each source is defined as the ratio of the power of each source to that of the background noise. In this example, we set the SNR of the SOI at 0 dB, and the SNR of the AM signal at 3 dB. Figure 5 shows the results of the proposed DOA estimation when we set $\alpha=0.25$, where conventional cyclic MUSIC and cyclic LS [8] methods are compared to the proposed method using fourth order cumulant. These simulations were performed under low SNR and strong interference conditions, which are advantageous for our method with respect to cyclic MUSIC and cyclic LS.

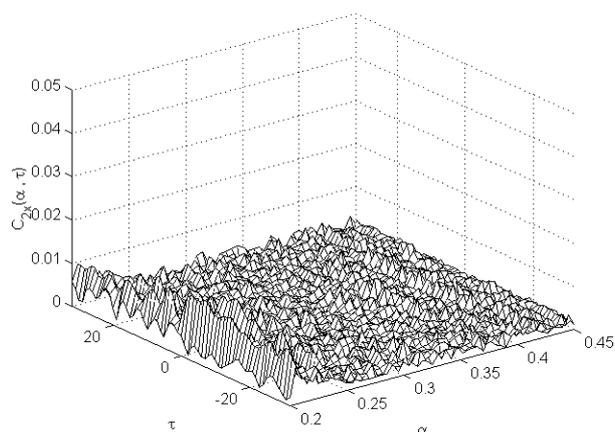


Fig. 3 Absolute value of second order cyclic cumulant

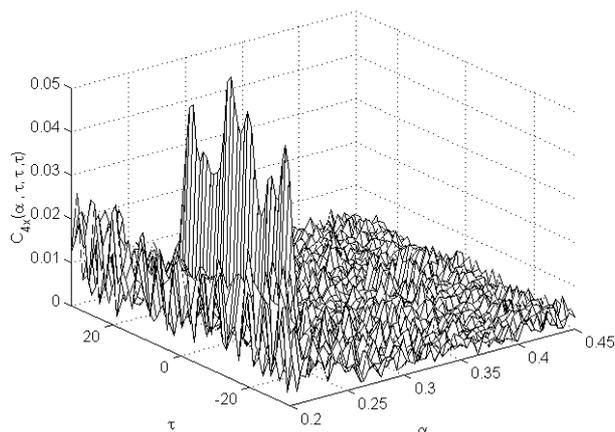


Fig. 4 Absolute value of fourth order cyclic cumulant

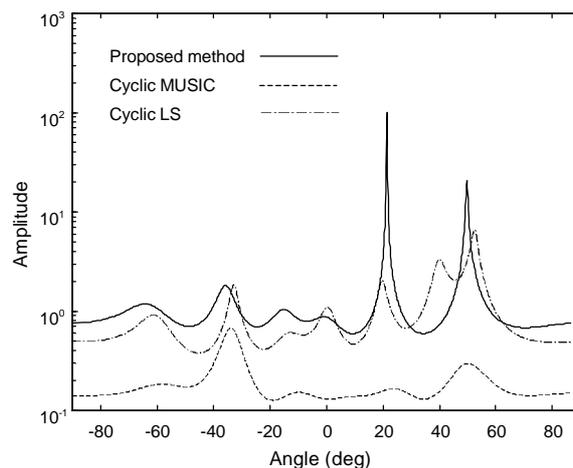


Fig. 5 Simulation results in DOA estimation

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