THE MIN-NORM BEAMFORMER : A NEW ESTIMATE OF THE PROPAGATION SPEED OF WAVES IN A CAR EXHAUST[⊥]

Gema Piñero, Luis Vergara

Dpto. Comunicaciones (UPV),
Camino de Vera, s/n, 46071 Valencia, Spain.
Tel : 34-96-3877304
Fax : 34-96-3877309
e-mail: gpinyero@dcom.upv.es

ABSTRACT

In this paper we present a new estimate of the propagation speed of acoustic waves propagating along a pipe based on array signal processing, the so-called Min-Norm Beamformer (MNB). In a previous contribution [1], the authors had already presented the formulation of this new estimate for narrowband signals, but the scenery of acoustic waves uses to be broadband. Therefore, we establish now the complete formulation of the MNB estimate for broadband waves for the two unique possible situations: a coherent or an incoherent processing of the data Sample Covariance Matrix (CSM), including the most efficient matrix transformations given in the literature [2][3]. Finally, we present several results for three experiments carried out in a real environment : a car exhaust with three different engines at work.

1. PROBLEM STATEMENT

There exist two low frequency broadband waves propagating along a car exhaust when the engine is working, the forward one generated by the engine, and the backward one originated from the misadjustment of acoustic impedances at the muffler input. Due to the large pressures of the waves, the acoustic model of propagation is not valid, and the propagation speed of the waves is the sum of the sound speed and the gas flow velocity. This last parameter depends on the instantaneous pressure of the wave and on the position where the wave samples are taken. This kind of waves are named finite waves and have a non-linear model of propagation [4].

As it is explained in [5], the separation of the forward and the backward waves propagating along the exhaust is useful in several applications, and a good estimate of the speed propagation is needed. On the other hand, this parameter is fundamental for obtaining a better knowledge of the characteristics of the engine when it is working. The estimation of the finite waves propagation speed is a new problem, although an approach has been tried for the narrowband case and the linear model of propagation [1].

In this paper we present a new estimation technique for the wideband case based on array signal processing, it makes use of the so-called Min-Norm Beamformer (MNB). This is a beamformer whose weight vector belongs to the noise subspace and its matrix equation is solved by means of the minimum norm method. The MNB is proposed for the two main types of Covariance Sample Matrix (CSM) for the broadband case: the coherent processing of the particular CSM in every frequency using some kind of transformation matrices, and the incoherent processing of the information by using the Discrete Fourier Transform (DFT).

Although in both formulations, coherent and incoherent CSM processing, the MNB is based on the acoustic model of propagation, some good results have been obtained in the estimation of the propagation speed of finite waves when the samples have been taken by an array of very small aperture because, in this case, it is possible to consider a linear propagation model.

2. MNB FOR NARROWBAND SIGNALS

In a uniform linear array the instantaneous sensor output is represented by the vector $\mathbf{x}(t)=\mathbf{A}(c_0)\mathbf{s}(t)+\mathbf{n}(t)$, being $\mathbf{A}(c_0)$ the steering matrix and c_0 the propagation speed of the sources, $\mathbf{s}(t)$ the source vector and $\mathbf{n}(t)$ the noise vector, where its components are zero-mean, i.i.d. circular gaussian variables.

The MNB propagation speed estimate for the narrowband case is then formulated as [1]

$$\hat{c}_0 = \arg\left\{\min_{c} P(c)\right\} = \arg\left\{\min_{c}\left(\mathbf{w}^H(c)\mathbf{R}_x\mathbf{w}(c)\right)\right\} \tag{1}$$

subject to

$$\mathbf{A}^H(c)\mathbf{w}(c) = \mathbf{0} \text{ and } \left\|\mathbf{w}(c)\right\| = 1 \tag{2}$$

The MNB calculates $\mathbf{w}$ in order to cancel the signal contribution ($\mathbf{A}^H(c)\mathbf{w}(c) = \mathbf{0}$) to the total output power, $P(c)$, but constraining the noise power to remain constant

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for every speed c, i.e., $P_n(c) = \mathbf{w}^H(c)\sigma^2\mathbf{I}\,\mathbf{w}(c) = \sigma^2\|\mathbf{w}(c)\| = \sigma^2$.

It can be demonstrated that for two sources propagating along a pipe, with the same frequency, speed, and DOAs of -90° (forward wave) and 90° (backward wave), the weight vector of the MNB has the following expression :

$$\mathbf{w}(c) = \frac{1}{\sqrt{\left(N^2-\beta_0^2\right)\left((N-1)^2-\beta_1^2\right)}}\cdot \begin{bmatrix} (N-1)^2-\beta_1^2 \\ 2\,\mathrm{Re}\left\{\left(\beta_1 e^{j2\pi f\frac{Nd}{c}}-(N-1)\right)\mathbf{a}_T(c)\right\} \end{bmatrix} \quad (3)$$

where N is the number of sensors and d the distance between two consecutive ones, $\mathbf{a}_T(c)$ is the 'truncated' steering vector of the progressive signal, i.e., the steering vector but where the first element has been eliminated, and β_0 and β_1 are defined as $\beta_0 \equiv \beta_0(c) = \left[\mathrm{sen}\left(2\pi fNd/c\right)\right]/\left[\mathrm{sen}\left(2\pi fd/c\right)\right]$ and $\beta_1 \equiv \beta_1(c) = \left[\mathrm{sen}\left(2\pi f(N-1)d/c\right)\right]/\left[\mathrm{sen}\left(2\pi fd/c\right)\right]$, respectively.

Consequently, the MNB has a closed formulation given by (3) for every speed propagation, which is an important advantage respect to the MUSIC and ML estimates. In any case, when the number of sensors is 3, the MNB produces ML estimates [1]. It must be also pointed out that the MNB can be applied in a coherent scenario without any type of previous decorrelation of the signals. Actually, this estimate takes advantage from the fact that the waves have the same frequency and propagation speed.

3. THE MNB FOR BROADBAND SIGNALS

The formulation of the MNB for the broadband coherent case depends on the transformation done to the CSM in every frequency in order to focusing all the information in a unique CSM at the frequency f_0. In this sense, there are two possible matrix transformations keeping the subspace structure of the original CSMs : the Rotational Signal Subspace Transformation (RSST) [2] and the Signal Subspace Transformation (SST) [3]. Both processing methods calculate a new covariance matrix, $\mathbf{R}_x(f_0)$, that can be respectively decomposed as the sum of a signal subspace at the focusing frequency f_0, and a noise subspace orthogonal to the signal subspace.

The CSM for the RSS Transformation is expressed as follows

$$\mathbf{R}_{RSST}\left(f_0\right) = \mathbf{A}(f_0)\left(\sum_{k=1}^{K}\mathbf{R}_x(f_k)\right)\mathbf{A}^H(f_0) + \sigma^2\mathbf{I} \quad (4)$$

where $\sigma^2 = \sum_{k=1}^{K}\sigma^2(f_k)$ is the sum of the noise power at every frequency f_k and $\mathbf{A}(f_0)$ is the constraint matrix at the focusing frequency f_0, meanwhile the Covariance Sample Matrix for the SST method can be written as

$$\mathbf{R}_{SST}\left(f_0\right) = \mathbf{Q}\left(f_0\right)\left(\sum_{k=1}^{K}\tilde{\mathbf{R}}_s(f_k)\right)\mathbf{Q}^H\left(f_0\right) + \sigma^2\mathbf{I} \quad (5)$$

where $\tilde{\mathbf{R}}_s(f_k) = \mathbf{Q}^H(f_k)\mathbf{A}(f_k)\mathbf{R}_s(f_k)\mathbf{A}^H(f_k)\mathbf{Q}(f_k)$ and $\mathbf{Q}(f_0)$ and $\mathbf{Q}(f_k)$ are arbitrary orthonormal matrices NxN whose two first columns form an orthonormal basis of the subspace generated by $\mathbf{A}(f_0)$ and $\mathbf{A}(f_k)$, respectively. Consequently, the MNB for the SST and the RSST coherent matrix transformations is formulated as in (1)-(2), but using the focused covariance sample matrix $\mathbf{R}_x(f_0)$.

In the broadband incoherent case, the MNB estimate of the speed propagation is calculated as

$$\hat{c}_0 = \arg\left\{\min_{c} P(c)\right\} = \arg\left\{\min_{c}\left\{\sum_{k=1}^{K} P\left(c, f_k\right)\right\}\right\} \quad (6)$$

where $P\left(c, f_k\right) = \mathbf{w}^H\left(c, f_k\right)\mathbf{R}_x\left(f_k\right)\mathbf{w}\left(c, f_k\right)$ subject to $\mathbf{A}^H\left(c, f_k\right)\mathbf{w}\left(c, f_k\right) = \mathbf{0}$ and $\left\|\mathbf{w}\left(c, f_k\right)\right\| = 1$.

This formulation of the MNB tries to minimise the arithmetic mean of the mean square error of the estimate obtained for every frequency, in an analogous way to what it is established in [6] for the broadband ML estimate.

The MNB for the incoherent preprocessing case suffers from a high computational cost because the weight vector must be calculated for every frequency bin and for every propagation speed. However, the minimum of the output power function, $P(c)$, can be obtained by means of a very efficient gradient descent method, the Newton's method, so the number of iterations needed is 4 or 5 at maximum and the computational time is severely decreased.

4. IMPROVEMENT OF THE BROADBAND INCOHERENT MNB ESTIMATE

The equation expressed in (6) supposes the signal and the noise to be uncorrelated. This is true if we consider the theoretic mean output power, but, in practice, there is a finite number of samples and we have to work with an estimate of the mean output power, $\hat{P}(c)$.

If a delay-and-sum beamformer is used to detect a signal, the output signal power is much larger than the cross-correlation term between signal and noise and the contribution of this last term is negligible. However, when we are trying to cancel the signal as the MNB does, the cross-correlation term is comparable to the noise power, so the minimum of $\hat{P}(c)$ is achieved in another c different from c_0 causing an error in the propagation speed estimate.

To show this effect, let's analyze the complete expression of the estimated mean output power of the Min Norm Beamformer, given for the propagation speed c and at a generic frequency by

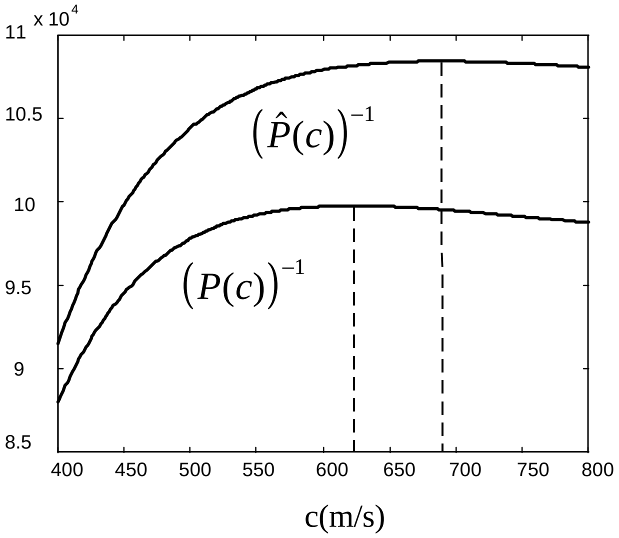

Fig. 1. Inverse of the estimated mean output power $\hat{P}(c)$ and inverse of the theoretic mean output power *P(c)* for $\alpha \in (0.025, 0.049)$ and SNR = 50 dB. The estimated value of *c* is 689 m/s and the true value is 623 m/s. Units are linear.

$$\hat{P}(c) = \mathbf{w}^H(c)\left\{\left(\mathbf{A}(c_0)\mathbf{s}+\mathbf{n}\right)\left(\mathbf{A}(c_0)\mathbf{s}+\mathbf{n}\right)^H\right\}\mathbf{w}(c) = P(c) + \mathbf{w}^H(c)\left(\mathbf{A}(c_0)\mathbf{s}\mathbf{n}^H + \mathbf{n}\mathbf{s}^H\mathbf{A}^H(c_0)\right)\mathbf{w}(c) \qquad (7)$$

It can be shown that the perturbation of $\hat{P}(c)$ generated by the cross-covariance between signal and noise is not negligible depending on the range of c and c_0 with respect to f and d, and on the signal-to-noise ratio (SNR) at the beamformer input. If we define the steering angle $\alpha = 2\pi fd/c$, we can demonstrate that when α is very small, the curve of the mean output power *P(c)* is nearly flat around c_0 and the cross-covariance term has a large influence as it can be seen in figure 1. On the other hand, an small signal-to-noise ratio implies the same behavior of the estimated mean output power, then causing the same kind of error on the MNB estimate.

Therefore, the broadband incoherent MNB estimate expressed in (6) must take into account the expression of the mean output power estimate at every frequency given in (7). It is necessary then to establish some criteria to eliminate those low frequencies whose steering angle α_i is very small, and those high frequencies where the SNR is not large enough. Doing this process previously to calculate (6), we can assure that $\hat{P}(c)$ is a good estimate of *P(c)* and, consequently, $\hat{c}_0$ is a good estimate of c.

5. EXPERIMENTS AND RESULTS

We present the experimental results obtained with two different engines, the cold-flow engine, whose main characteristic is that there is no gas flow in the propagation of the waves, so a linear model of propagation can be assumed, and the petrol car engine, which propagation model is highly non-linear and its sound speed varies with time and position. We have done two different experiments using this last engine, one with the engine rotating at 2000 r.p.m. and another rotating at 4000 r.p.m.

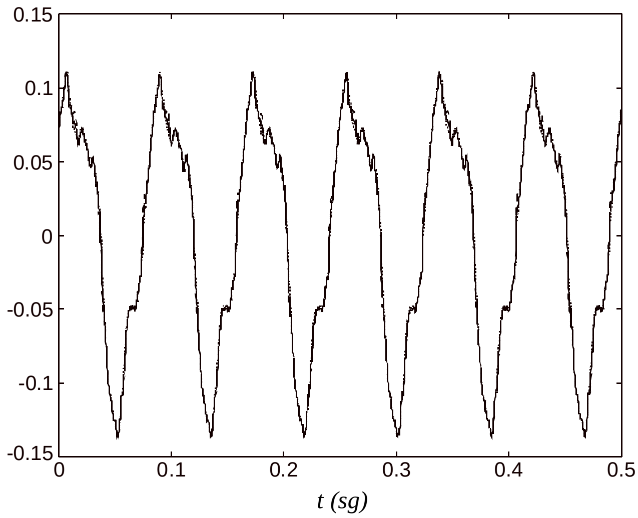

Fig. 2. Array output for the cold-flow engine.

Table I shows the main characteristics of the three experiments. The distance between sensors is 5 cm for all the experiments and the propagation speed is the theoretic propagation speed calculated as $c = \sqrt{\gamma RT}$, where γ is the ratio of specific heats, R is the gas constant per unit mass and T is the temperature [4]. Figures 2, 3 and 4 show the cold-flow engine waveform, and the 2000 and 4000 r.pm. petrol engine waveforms obtained at the array output, respectively.

The propagation speed estimates are shown in table II for six different kinds of estimators: the MUSIC estimate for the coherent (RSST and SST-MUSIC) and incoherent (I-MUSIC) case, and the MNB estimate for the coherent (RSST and SST-MNB) and incoherent case (I-MNB).

It can be seen from table II that the MNB used with the RSST and SST covariance matrices outperforms the MUSIC estimate applied to the same matrices, meanwhile both incoherent estimates have achieved similar results. Some simulation results have shown that coherent methods are very sensitive to the chosen value of the focusing frequency, so the propagation speed estimate is not certainly reliable.

On the other hand, the incoherent MUSIC and MNB methods have a simpler formulation, although the computational cost is much bigger than that of the coherent estimates. However, the incoherent MNB can be optimized using a Newton descent method as we have mentioned before.

Finally, another main advantage of the MNB estimate over the MUSIC in the incoherent case is that the analysis

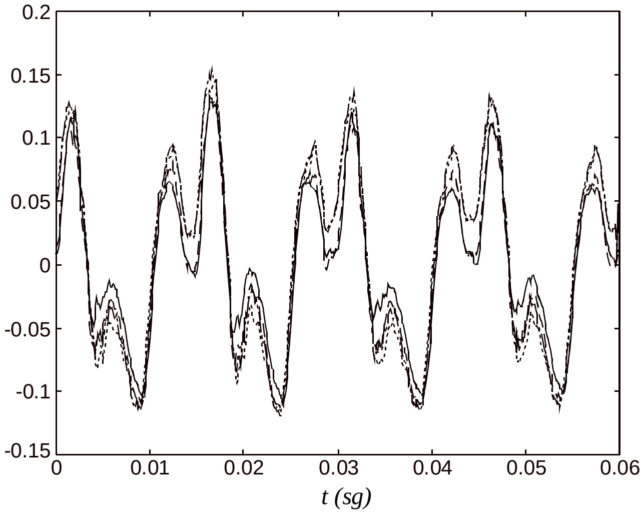

Fig. 3. Array output for the petrol engine rotating at 2000 r.p.m.

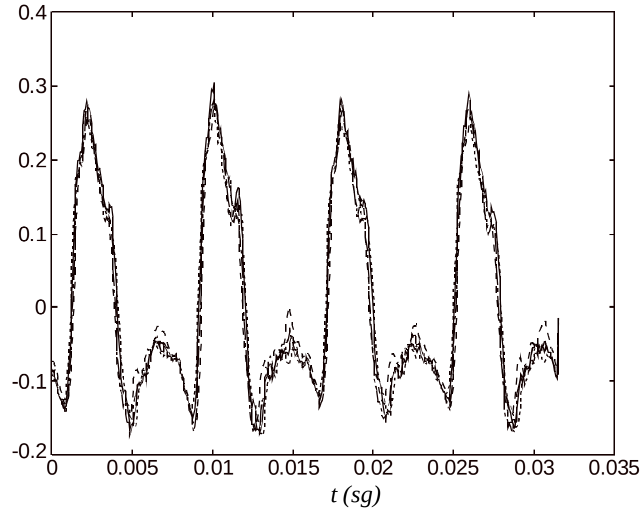

Fig. 4. Array output for the petrol engine rotating at 4000 r.p.m.

of its simple formulation allows to find the relation between the α and SNR parameters and the error produced for every frequency, so it can be easily optimized in bad conditions.

6. REFERENCES

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PARAMETER	***exp.1***	***exp.2***	***exp.3***
sound propagation speed (c_0)	342 m/s	623.3 m/s	671.8 m/s
noise power (σ^2)	-77 dB	-57.1 dB	-52.5 dB
signal bandwidth	1.5 KHz	2 KHz	2.5 KHz
number of sensors (N)	3	4	4

***Table I**. Main characteristics of the three experiments: number 1 corresponds to the cold-flow engine and number 2 and 3 correspond to the same car engine but with different number of r.p.m.*

EXP.	***RSST-MUSIC***	***SST-MUSIC***	***I-MUSIC***	***RSST-MNB***	***SST-MNB***	***I-MNB***
1	332.2 (*2.8*)	347(*1.5*)	344.7 *(0.8)*	346.5 (*1.3*)	345.8 (*1.1*)	346.1 *(1.2)*
2	633.1 (*1.6*)	700 (*12.3*)	613.6 *(1.5)*	633.1 (*1.6*)	624.7 (*0.2*)	588.7 *(5.5)*
3	730 (*8.7*)	696.3 (*3.6*)	699.6 *(4.1)*	670.4 (*0.2*)	625.9 (*6.8*)	656.6 *(2.3)*

***Table II**. Propagation speed estimates. The figure between parenthesis is the relative error (in %) between the estimate and the theoretic value.*