# A FUZZY LOGIC FILTER FOR COHERENT DETECTION IN MOBILE COMMUNICATION RECEIVERS.

A. Pérez-Neira, M. A. Lagunas, A. Jové, A. Artés

Dpt. of Signal Theory and Communications - Universitat Politècnica de Catalunya c/ Jordi Girona 1-3. Campus Nord UPC. Edifici D5 - 08034 Barcelona (SPAIN) Tel: +34 3 4016447; fax: +34 3 4016447; e-mail: {anuska}@gps.tsc.upc.es

## **ABSTRACT**<sup>1</sup>

This paper proposes and demonstrates the use of a fuzzy logic filter for carrier phase synchronization in digital mobile communication receivers. The use of *a priori* knowledge of the dynamics of the phase in mobile fading channels allows the fuzzy filter to achieve fast phase tracking over Rician and Rayleigh flat-fading channels, as well as achieving quick acquisition. Additionally, a fuzzy CEMAC (Cerebellar Model Arithmetic Computer) architecture is proposed to increase the processing speed. The low computational requirements and the satisfactory performance of the proposed system, comparable or better than the Kalman filter for low SNR and low sampling rate, presents the fuzzy filter as a good alternative to other fading compensation techniques.

## **1** INTRODUCTION

Until the mid-1980s, very few studies had been done on fading compensation techniques because differential detection or FM discriminator schemes were considered to be the most appropriate demodulation scheme for land mobile communication systems. However, since the mid-1980s, studies on coherent detection have been started because its optimal performance is superior to that of differential detection or frequency disciminator detection if we have some precise fading compensation techniques. For this purpose, a lot of systematic or adhoc studies [1] have been done on the improvement of coherent demodulators; i.e. fading compensation techniques for coherent demodulators.

Fuzzy systems have recently been used extensively and successfully in control problems [2]. The primary cause of their initial slow acceptance among the control community has been their simplicity. In mobile communications the applications of fuzzy systems are quite novel. For instance, in [3] an adaptive fuzzy power control (AGC) as a fading compensation technique for mobile radio systems is introduced, which achieves better system stability than conventional feedback power control algorithms. The present work focusses on the phase synchronization subsystem and proposes a robust fuzzy logic solution (i.e. less susceptible to system changes or noise). Informally, fuzzy modelling of a physical system consists in capturing its dynamic input-output behaviour using linguistic expressions. These are represented and handled using fuzzy

sets and systems [4]. Relatively few fuzzy applications have been made for estimation problems, which many times lack of an exact reference signal. The objective of this contribution is to evaluate and illustrate the employment of fuzzy relational for estimation. The performance and complexity of the designed fuzzy system in mobile scenarios is compared with those of an Extended Kalman Filter (EKF) [5]. This work shows that in spite of its reduce complexity, the fuzzy phase estimator outperforms the digital EKF in scenarios with shadowing and multipath fading at high vehicle speed and slow sampling rate.

#### 2 PROBLEM STATEMENT

Consider a discrete, time-invariant system given by

$$x_{k+1} = f(x_k) + v_k;$$
  $z_k = h(x_k) + n_k$  (1)

where  $x_k$  is the state vector,  $z_k$  is the measurement, and  $v_k$ and  $n_k$  are noise processes. The problem of finding an estimate  $\hat{x}_k$  for  $x_k$  based on measurements  $z_i$  ( $i \le k$ ) is known as the *a posteriori* filtering problem. One popular form of a recursive estimator is the predictor/corrector, which is given by

$$\hat{x}_{k} = \hat{f}(\hat{x}_{k-1}) + g(z_{k}, \hat{x}_{k-1})$$
(2)

where  $\hat{f}(.)$  is an estimate of the function which maps the state from one time step to the next, and g(.) is the correction function. Often the process model f(.) is already known or can be found using system identification methods. In that case, only the correction mapping g(.) needs to be determined. When using analytic methods for obtaining g(.) such as the Kalman filter, the unmodeled effects and measurement imperfections are described with additive "noise" processes. This is the case of the nonlinear phase estimation problem, where the Extended Kalman Filter is used [5]. Alternatively, the correction mapping g(.) could be implemented as a fuzzy function in order to obtain an estimator easier to design, cheaper to produce, and more robust (i.e., less susceptible to the conflicting requirement of tracking quick system changes while at the same time rejecting the noise).

Assuming a so-called non-frequency-selective, narrowband fading channel, the received signal r(t) at the channel output is

<sup>&</sup>lt;sup>1</sup> This work was supported by the National Plan of Spain, CICYT, TIC96-0500-C10-01.

$$r(t) = \alpha(t) e^{j\theta(t)} s(t) + w(t)$$
(3)

where  $\alpha(t)$  represents the envelope of the fading variation,  $\theta(t)$  is the phase of the channel, which is uniformly distributed over  $[-\pi,\pi)$ , s(t) is the transmitted modulated signal at carrier frequency fc, and n(t) is a Gaussian noise process. For coherent demodulation, the receiver not only needs to know the nominal frequency fc but has also to estimate  $\theta(t)$ , which is corrupted by the channel noise. We assume that the frequency offset between transmitter and receiver is small (i.e. an Automatic Frequency Control or AFC is assumed if necessary), so that only the time-varying carrier phase has to be acquired.

Figure 1 shows a carrier regeneration circuit for M-ary PSK with a *Mth* power non-linear operation.



Fig.1 Proposed carrier regeneration circuit for M-PSK.

As shown in fig. 1, the phase tracker processes further the one-shot phase estimate  $z_k$  and produces the phase estimates  $\hat{\theta}_k$ . Its structure is that of the predictor/corrector formulated in (2)

$$\hat{x}_{k} = \hat{f}\left(\hat{\theta}_{k-1}\right) + g\left(z_{k}, \hat{\theta}_{k-1}\right)$$
(4)

where the prediction term is

$$\hat{f}(\hat{\theta}_{k-1}) = \hat{\theta}_{k-1} + w_c T$$
 T: time between updates (5)

and the correction term  $g(z_k, \hat{\theta}_{k-1})$  is the one computed by the fuzzy filter.

#### **3** FUZZY ESTIMATION

Good overviews of fuzzy logic can be found in [6]. In this work the fuzzy correction mapping to compute has two inputs. One input is the error  $e_k$  and the other is the change in error  $\Delta e_k$  or frequency error

$$(input 1)_{k} = e_{k} = z_{k} - \hat{\theta}_{k-1}$$

$$(input 2)_{k} = \Delta e_{k} = (input 1)_{k} - (input 1)_{k-1}$$
(6)

The output of the correction mapping is a fuzzy variable which is determined by correlation-product inference. The fuzzy rule base for the mapping g(.) was chosen as shown in Table I. Gaussian input and output membership functions were chosen as shown in fig. 2. We note that if big phase variations have to be tracked, then the input and output ranges and also the number of membership

functions have to be increased if the performance is to be manteined.

e <sub>k-&gt;</sub>	LN	MN	SN	ZE	SP	MP	LP
$\Delta e_k$							
LN	LN	LN	MN	SN	ZE	SP	MP
MN	LN	LN	MN	SN	ZE	SP	MP
SN	LN	MN	SN	ZE	SP	MP	LP
ZE	LN	MN	SN	ZE	SP	MP	LP
SP	LN	MN	SN	ZE	SP	MP	LP
MP	MN	SN	ZE	SP	MP	LP	LP
LP	MN	SN	ZE	SP	MP	LP	LP

Table I. Fuzzy Rule Base

The initial rule base and membership function shapes were constructed on the imprecise basis of experience, and trial and error. An appropriate initial knowledge base is critical, because without an initial knowledge, we cannot proceed any further with any optimization schemes. In spite of its importance, the generation of initial knowledge remains as a difficult and ill-defined task in the construction of fuzzy logic systems.



Fig. 2. Membership functions.

In general, we denote the controid and half-width of the *ith* fuzzy membership function of the *jth* input by  $c_{ij}$  and  $b_{ij}$  respectively. So the degree of membership of a crisp input x in the *ith* category or fuzzy set of the *jth* input is given by (7)

$$f_{ij}(x) = \exp\left[-\frac{(x - c_{ij})^2}{2b_{ij}}\right] g(z_k, \hat{\theta}_{k-1}) = \frac{\sum_{j=1}^{n} m(y_j) y_j J_j}{\sum_{j=1}^{n} m(y_j) J_j}$$
(7) (8)

The fuzzy output is mapped into a crisp numerical value using the centroid defuzzification formulated in (8). In (8) yj and Jj is the centroid and area of the *jth* output fuzzy membership function and n is the number of fuzzy output fuzzy sets. The fuzzy output function m(y) is computed as

$$\sum_{j,k}^{n} m_{ik}(y)$$
, where  $m_{ik}(y)$  is the consequent fuzzy output

function when  $(e_k \in \text{fuzzy set i})$  and  $(\Delta e_k \in \text{fuzzy set k})$ . It is computed as  $m_{ik}(y) = w_{ik} m_{oik}(y)$ . The weight  $w_{ik}$ 

is the activation level of consequent when  $(e_k \in \text{fuzzy set i})$ and  $(\Delta e_k \in \text{fuzzy set k})$ . Under this situation  $m_{oik}(y)$  is the fuzzy function of consequent which is activated. The computation of the weigths  $w_{ik}$  is described in [6] and depends on the measured data noise. For low noise the socalled singleton fuzzification is used. For higher noise, the non-singleton fuzzification is more suitable. In this work both techniques have been considered. We note that in both cases, at each instant of time the fuzzy estimator computes parallely n multiplications of 3 operants each and just one division as shown in (8).

### **4 OPTIMIZATION**

The set of membership functions shown in fig. 2 have been defined on the basis of heuristics. Perhaps other membership functions would result in better performance by the fuzzy filter. In this work the gradient descent algorithm is used to optimize the membership functions.

Consider an error function given by

$$E = 1/2N \sum_{q=1}^{n} E_q^2 \qquad E_q \equiv \hat{\theta}_q - \theta_q \tag{9}$$

we optimize E by using the partial derivatives of E with respect to: a) the centroids of the input fuzzy membership functions  $c_{ij}$ ; b) the half-width of the input fuzzy membership functions  $b_{ij}$ ; and c) the centroids of the output fuzzy membership functions yj. We note that because one of the main goals in this work is to preserve simplicity just the main diagonal in table I is to be taken into account. Fig. 3 shows an example of the mapping surface implemented by the fuzzy system before and after training.



Fig. 3. Fuzzy mapping: before training -> after training

#### **5** IMPLEMENTATION

One of the goals in this work is to obtain a low complexity tracker suitable not only for software implementation but also for digital VLSI circuitry. The proposed fuzzy system does not require any matrix inversion as the EKF in [5] does. Additionally, as other fuzzy systems, it is very suitable for parallel implementation [6]. In this work the implementation of the proposed fuzzy phase tracker has been done considering two different ways. The first one is by means of a decision table relating quantized measurements to crisp control actions. As in [3] this table can be generated off-line using control rules in order to shorten the running time. The second procedure resorts to the CMAC network of Albus [7-8]. The proposed fuzzy CMAC architecture is an excellent universal approximator that is able to learn an arbitrary function to any degree of accuracy. Its prominent feature lies in the extremely fast learning rate (at least 10 time faster) in comparison with the other Neural Networks. The CMAC is a perceptron-like

associative memory that performs a nonlinear function mapping over a particular region of a function space as those in fig. 3.

Next, in the simulations two fuzzy CMAC possibilities have been implemented resulting little degradation in comparison with the non-digitized fuzzy filter. This two options are: GCMAC1 and GCMAC2. GCMAC 1 approximates the fuzzy surface after each training of the fuzzy sytem. GCMAC 2 approximates the fuzzy surface only at the beginning and it is trained periodically afterwards resulting a faster approach.

#### 6 SIMULATIONS

To illustrate the performance of the proposed system we have considered a BPSK signal of a carrier frequency of 900 MHz. The sampling frequency is 4 times the symbol rate. This symbol rate has been chosen as  $(50 \ \mu s)^{-1}$ , which is slower than the  $(3.6 \ \mu s)^{-1}$  used in the GSM estándar. In this way we want to bring out the low sampling rate requirements of the proposed system. The mobile speed is considered to be: 100 Km/h, 150 Km/h or higher, resulting closeness in the value between the doppler (i.e. inverse of the fade period) and the symbol rate.

In fig.4 and 5 a receiver with the proposed pilot-aided fuzzy synchronizer and without AGC and AFC is compared with an Extended Kalman Filter that also acts as an AGC and an AFC because i estimates not only phase but also amplitude and frequency. A Rice flat-fading channel the diffuse signal direct-path to with ratio K=0.91/0.205=4.44 (GSM specification) has been considered. The diffuse multipath component is modeled with Rayleigh distributed envelope and uniform distributed phase. From the results that we have obtained in other simulations, we note that although the fuzzy system is also able to cope with Rayleigh channels, its performance can only then compete with that of a phase only EKF.

The comparison has been made in terms of the normalized phase square error. The fuzzy filter has been trained with: 50 samples and a SNR of 20 dB. In the EKF, the state noise variance matrix Q is designed to follow quick channel variations (i.e.  $Q=diag(0.1\ 0.1\ 0.001)$ ). Fig. 4 compares the instantaneous phase error. Fig.5 compares the mean square error for different SNR after 10 Monte Carlo runs of 2000 samples each.

From fig.4 we note the ability of the fuzzy system to compensate deep fadings. This is the reason why in fig.5 the fuzzy system presents better performance than the EKF, specially for low SNR. Additionally, as expected, the fuzzy non-singleton system is more suitable for noisy environments and performs as the singleton one for low noise.

Next, fig.6 plots the phase error variance against the SNR for K=10, which is a typical value for land-mobile stellite environments. All curves approach the asymptotic value  $\pi^2/3$  as the SNR tends to 0, while at high SNR's, they tend to decrease linearly with a fixed slope. For comparison, the

fig. also shows the Cramer Rao Bound (CRB) for the estimation of a nonrandom phase over the Gaussin channel, plotted as a straight line. We remark that the behaviour of the pilot aided fuzzy filter is close to the CRB for high SNR. For low SNR, the CRB is no longer valid for a Rician channel as it is shown in [9].



**Fig. 4.** a) Fading envelope variation in dB for a mobile speed of 100 Km/h, K=4.44; b) Phase error for and EKF with; c) Phase error for the fuzzy system after training.



Fig. 5. Mean square error comparison (v=100 Km/h).

In the same scenario as in fig.6, fig.7 shows the fuzzy filter performance without digitizing and with two digitized implementations with GCEMAC.

We note that the presented EKF simulations depend on the Q matrix. After studying different possibilities, the one chosen presents the best compromise for the different SNR scenario. In contrast, the fuzzy system does not present this trade off between tracking capability and noise rejection.

Finally, scenario with a double doppler rate have been also simulated. If the sampling frequency is not augmented, the pilot-aided fuzzy system keeps on giving better results than the EKF

#### 7 CONCLUSIONS

A pilot-aided fuzzy estimator has been proposed and applied to phase synchronization in mobile receivers. This approach offers the benefits of fuzzy logic: simplicity and robustness (i.e. due to the proposed linguistic modeling the system is able to track quick phase changes while at the same time rejects the noise). This performance gives prospects for applications to other fading compensation sub-systems for mobile demodulators as for instance the automatic frequency control stage.



**Fig. 6.** Performance of the Fuzzy system and the EKF Q=diag(0.1 0.1 0.001); mobile speed= 150Km/h; K=10; 10 Monte Carlo runs of 2000 samples each.



Fig. 7. Performance of the Fuzzy system and its digitized implementations with GCMAC; mobile speed= 150 Km/h; K=10.

#### 8 REFERENCES

[1] H.Meyr et al., "Digital communication receivers," J.Wiley&Sons, Inc. 1998

[2] E.Mamdani, "Twenty years of fuzzy control: Experiences gained and lessons learnt," in IEEE Int. Conf.Fuzzy Syst., San Francisco, Mar.1993, pp.339-344

[3] P.Chang, B.Wang, "Adaptive fuzzy power control for CDMA mobile radio systems," IEEE Trans. on VT, vol.45, May 1996.

[4] L.A. Zadeh, "Toward a theory of fuzzy systems; aspects on Networks and Systems Theory," Kalman R.E. et al., New York.

[5] P.J.Parker, B.D.O.Anderson, "Frequency Tracking of nonsinusoidal periodic signals in noise," Signal Processing, vol.20, no.2, June 1990, pp.127-152.

[6] J.Mendel, "Fuzzy Logic Systems for Engineering: A Tutorial," Proceedings of the IEEE, vol.83, no.3, March 1995.

[7] J.Albus, "A new approach to manipulator control: the cerebellar model articulation controller," Journal on Dynamic Systems, Measurements and control, vol. 63, n°.3, pag. 220-227, September 1975.

[8] F. González, "Predistorsión en comunicaciones digitales mediante la arquitectura CMAC," PHD Thesis, Universidad de Vigo (SPAIN) 1997.

[9] R.Reggiannini, "A fundamental lower bound to the performance of phase estimators over Rician-fading channels," IEEE Trans. on Comm., vol. 45, no.7, July 1997.