OPTIMIZED FRAME DESIGN FOR A MATCHING PURSUIT BASED COMPRESSION SCHEME

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ABSTRACT

A technique for designing frames to use with vector selection algorithms, for example Matching Pursuits (MP), is presented along with a novel compression scheme using these optimized frames. The frame design algorithm is iterative and requires a training set. We apply the frame design algorithm and the complete multi-frame compression scheme to electrocardiogram (ECG) signals. Complete coding experiments using the optimized frames in the novel compression scheme are compared with coding experiments from transform based compression schemes like the Discrete Cosine Transform (DCT) with run-length entropy coding. The experiments demonstrate improved rate-distortion performance for the multi-frame compression scheme compared to DCT at low bit-rates.

1 INTRODUCTION

Traditional transform based compression schemes use an orthogonal basis, and the goal is to represent as much signal information with as few transform coefficients as possible. The optimal transform for a signal depends on the statistics of the stochastic process that produced the signal, but the signal is often nonstationary, and thereby no fixed transform is optimal in all signal regions. One way to try to overcome this problem is to use an overcomplete set of vectors. For a finite dimensional space, any finite overcomplete set of vectors which span the space form a frame [1]. The basic idea when using a frame instead of an orthogonal transform is that we have more vectors and thus a better chance of finding a small number of vectors that match the signal vector well. Since a linear dependent set of vectors is used, an expansion is no longer unique. Let $\mathbf{F}$ denote an $N \times K$ matrix whose columns, $\{\mathbf{f}_j\}$, $j = 0, 1, \ldots, K-1$, constitute a frame. Let $\mathbf{x}_i$ be a real signal vector, $\mathbf{x}_i \in R^N$. $\mathbf{x}_i$ can then be represented or approximated as

$$\tilde{\mathbf{x}}_\mathbf{i} = \sum_j w_i(j)\mathbf{f}_j, \tag{1}$$

where $w_i(j)$ is the coefficient corresponding to vector $\mathbf{f}_j$. Typically, many of the $w_i(j)$'s are zero. The corresponding error energy is:

$$\|\mathbf{r}_i\|^2 = \|\mathbf{x}_i - \tilde{\mathbf{x}}_i\|^2 \tag{2}$$

where $\|\cdot\|$ denotes the Euclidian norm in R^N. For a set of M signal vectors, the mean square error (MSE) can be calculated:

$$MSE = \frac{1}{NM}\sum_{i=0}^{M-1} \|\mathbf{r}_i\|^2. \tag{3}$$

In a compression scheme the goal is to use as few vectors as possible to obtain a good approximation of each signal vector. Finding the optimal vectors to use in an approximation is a NP-complete problem and thus requires extensive calculation. Orthogonal Matching Pursuit (OMP) or Matching Pursuit (MP) [2] are greedy algorithms for selecting vectors from a frame. They are suboptimal and have relatively low computational complexity. The use of frames in compression schemes have been given some attention [3, 1, 4, 5, 6] whereas the problem of *frame design* in this context is largely unexplored. In this paper our aim is to present an algorithm for optimizing frames using training sets, and use these frames in a novel compression scheme for ECG signals.

Section 2 presents the algorithm for frame design. Section 3 describes the novel multi-frame compression scheme using trained frames. Experiments on frame design and compression using the multi-frame compression scheme are presented, and results are shown in Section 4. Section 5 contains some concluding remarks.

2 ALGORITHM FOR FRAME DESIGN

We first introduced this algorithm in [7], where more details can be found. The algorithm begins with an initial frame, and approximates each training vector according to Equation 1 using a vector selection algorithm. We use OMP as vector selection algorithm. The frame performance is improved by modifying each frame vector according to the residuals from the training vectors that used the actual frame vector in their approximations. Consider a scheme where two frame vectors are selected for approximating each training vector i.e.:

$$\tilde{\mathbf{x}}_i = w_i(1)\mathbf{f}_{i_1} + w_i(2)\mathbf{f}_{i_2} \tag{4}$$

$$\mathbf{r}_i = \mathbf{x}_i - \tilde{\mathbf{x}}_i, \tag{5}$$

where $\mathbf{x}_i$ is a training vector, $\tilde{\mathbf{x}}_i$ it's approximation and $\mathbf{r}_i$ the residual of $\mathbf{x}_i$. The frame vector j is adjusted:

$$\tilde{\mathbf{f}}_j = \mathbf{f}_j + \delta \sum_{k \in T_j} \mathbf{r}_k, \tag{6}$$

where T_j is the set of all training vectors that used $\mathbf{f}_j$ in their approximations. If only one of the frame vectors is adjusted, the new residual for a training vector using this frame vector is:

$$\mathbf{r}'_i = \mathbf{r}_i - w_i(j_i)\delta \sum_{k \in T_j} \mathbf{r}_k, \tag{7}$$

where $w_i(j_i)$ is the coefficient corresponding to the adjusted vector, $\mathbf{f}_j$, for the approximation of training vector $\mathbf{x}_i$ before the adjustment of $\mathbf{f}_j$. The total residual of all the training vectors using the frame vector that has been adjusted is:

$$\begin{aligned} \sum_{l \in T_j} \mathbf{r}'_l &= \sum_{l \in T_j} \mathbf{r}_l - \delta \sum_{l \in T_j} w_l(j_l) \sum_{m \in T_j} \mathbf{r}_m \\ &= \sum_{l \in T_j} \mathbf{r}_l - \delta C \sum_{l \in T_j} \mathbf{r}_l \\ &= (1 - \delta C) \sum_{l \in T_j} \mathbf{r}_l \end{aligned} \tag{8}$$

where C is the sum of all the coefficient used with frame vector $\mathbf{f}_j$, before the adjustment:

$$C = \sum_{l \in T_j} w_l(j_l). \tag{9}$$

The residuals for the rest of the training vectors are not influenced by adjustment of the frame vector. From equation 8 it is seen that if $\| \sum_i \mathbf{r}'_i \| \leq \| \sum_i \mathbf{r}_i \|$ then $0 \leq \delta C \leq 2$. This means that:

$$\text{sign}(\delta) = \text{sign}(C). \tag{10}$$

Thus for each frame vector in each iteration $\text{sign}(\delta)$ is set according to Equation 10.

Selection algorithms for frames are not optimal, so there is no way to guarantee a better frame when using a practical selection algorithm like OMP. In addition, to make the iterations faster we adjust *all* the frame vectors in each iteration, and we normalize the vectors to unit length. In [7] it is shown that the performance of the optimized frames are significantly better in terms of MSE than performance using frames designed by ad-hoc techniques. In Section 4, Figure 2, some training plots using this iterative training algorithm for frame design are shown.

3 COMPRESSION SCHEME DESCRIPTION

A transform based compression scheme can be as follows: A transform, e.g. the DCT, is used to find the transform coefficients for a signal vector. The coefficients are thresholded, that is all the coefficients with values $w \in [-T, T]$ are set to zero. Then the coefficients are quantized with an uniform quantizer, and a run-length coder is used to indicate the position of the coefficients. A quantized coefficient and the associated run are combined into one symbol, and the symbols are coded with an entropy coder. An end of block (EOB) symbol is used after the last nonzero transform coefficient for each signal vector. When using a compression scheme like this, the number of coefficients *not* quantized to zero will vary for different signal vectors. In the context of frame expansions this means that for a given quantizer step the number of vectors needed in the approximation will vary for different signal vectors if the approximation quality is to be constant. A frame optimized with the algorithm presented in the previous section is designed for use with *a specific number of signal vectors in each approximation.* This problem is easily solved by designing several frames, one frame is optimized for using just one vector in the approximation, one is optimized for using two, and so forth. In the compression scheme the desired approximation quality will decide the number of vectors to be used in an approximation. The frame used for a signal vector will be the one optimized for the number of vectors required in the approximation.

If only one frame is used in conjunction with run-length entropy coding we need an EOB symbol between each signal block. If different frames are used when using different numbers of vectors in an approximation, an EOB symbol is not enough. A start of block (SOB) symbol is needed to tell which frame is used when approximating the next signal vector. The EOB symbol is no longer needed because the SOB tells us all we need to know. The SOB tells which frame is used, and thereby how many frame vectors we use to approximate the signal vector. This means that we can use run-length coding and entropy coding where each frame has its own entropy coder because the number of symbols transmitted before the next SOB is given from the previous SOB. A symbol consists of an amplitude and a run, and these are entropy coded. A SOB symbol has to consume more bits than the EOB symbol used in traditional transform coding because it needs to tell which one of a limited number of frames are used in the next block, not only that a new block is starting.

Figure 1 illustrates the compression scheme. Let $\mathbf{F}_i$, $i = 1, 2, \ldots, L$, where L is the maximum number of vectors allowed in an approximation, be a frame of size $N \times K$ optimized for using i vectors in each approximation. For low bit rates the probability of using $\mathbf{F}_1$ or $\mathbf{F}_2$ is much larger than using $\mathbf{F}_i$, $i = 3, 4, \ldots, L$. Thus the entropy for the SOB symbols is low. If the SOB symbols are Huffman coded, the extra side information when using these SOB instead of EOB symbols is typically less than 0.03 bit/sample in our experiments.

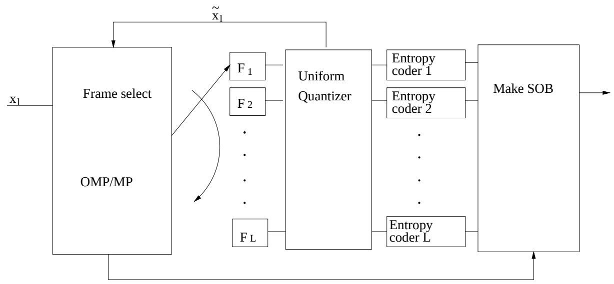

Figure 1: Illustration of the compression scheme.

The basic idea of the compression scheme is that for a signal vector, the compression scheme using the optimized variable size frames starts by using $\mathbf{F}_1$ to approximate the signal vector, and the residual is calculated. The residual is compared to a target MSE. If the approximation is good enough $\mathbf{F}_1$ is used, if not $\mathbf{F}_2$ is tried and so forth. For frame $\mathbf{F}_i$ a signal vector, $\mathbf{x}_l$, is approximated:

$$\tilde{\mathbf{x}}_\mathbf{l} = \sum_{j=1}^{i} w_l(j)\mathbf{f}_{l_j}, \tag{11}$$

where $w_l(j)$ is the coefficient corresponding to vector $\mathbf{f}_{l_j}$, and $l_j \in [0, 1 \ldots K-1]$. An uniform quantizer with quantizing step, Δ, is used to quantize these coefficients.

The best choice of the target MSE is related to Δ. For a signal vector approximated with frame $\mathbf{F}_i$ it is possible that the requirement on the target MSE is not satisfied, and still one or more of the i coefficients are quantized to zero. In this situation it is not always the best to increment i, and try with the next frame. If that solution were to be chosen the scheme would use the frames with the maximum number of vectors allowed in an approximation often, which increases the bit-rate. If one coefficient is quantized to zero when using frame $\mathbf{F}_i$, only $i-1$ frame vectors are used in the approximation, and it may be better to use the frame $\mathbf{F}_{i-1}$ which is designed for using $i-1$ vectors in an approximation. An idea is therefore to decrement i and go back to the previous frame when this situation occurs. This way the compression scheme always tries to use as few vectors as possible in each approximation. This resulted in good rate-distortion plots, but for some signal vectors the residual became very large. To avoid this, a compromise solution was adopted: when the above described situation occurs for frame $\mathbf{F}_i$, calculate the residual when using frame $\mathbf{F}_{i-1}$. If this residual is less than $factor \times MSE_{target}$, where MSE_{target} is the target MSE, the frame $\mathbf{F}_{i-1}$ is used in the approximation of the signal vector. If not, i is incremented and the approximation using frame $\mathbf{F}_{i+1}$ is calculated.

The novel multi-frame compression scheme algorithm can be described as follows:

1. A desired approximation quality, MSE_{target}, is chosen in terms of a target MSE for a signal vector. Assign counter variable $i = 1$.
2. MP or OMP is used to find the approximation when using $\mathbf{F}_i$.
3. The coefficients are quantized with uniform quantizer with thresholding. The residual after quantization is calculated, and the MSE_i is compared to MSE_{target}.
4. If $i = L$ go to 8.
5. If $MSE_i < MSE_{target}$ go to 8.
6. If none of the coefficients are quantized to zero, $i = i + 1$ and go to 2.
7. If $MSE_{i-1} < factor \times MSE_{target}$, $i = i - 1$ and go to 8. Else $i = i + 1$ and go to 2.
8. $\mathbf{F}_i$ is used when approximating the signal vector. The approximation is entropy coded. Each frame has its own entropy coder.
9. A SOB symbol telling which frame used is entropy coded.

4 EXPERIMENTS AND RESULTS

The ECG signals used in the experiments are signals from the MIT arrhythmia database [8]. The records are represented with 12 bit per sample, and the sampling frequency is 360 Hz. The two signals used are normal sinus rhythm, MIT100, and a ventricular arrhythmia, MIT207. The training signals are MIT100, 0:00 to 5:00 minutes, and MIT207, 6:00 to 11:00 minutes. The test signals are MIT100, 5:30 to 10:30 minutes, and MIT207, 12:00 to 17:00 minutes.

Frames of size $N \times K$ are trained for using $1, 2, \ldots 12$ vectors in each approximation using the training signals for both MIT100 and MIT207. The signal vector size, N, is 32. The number of frame vectors in each frame, K, is $2N = 64$. The initial frame vectors are normalized versions of the first K signal vectors in the training set, and the frames are trained using the algorithm described in Section 2. Figure 2 shows training plots for some of the frames used in the compression experiments.

Complete compression experiments were done using the test signals for both MIT100 and MIT207 and the compression scheme explained in this paper. The $factor$ in step 7 of the algorithm was experimently set to 5. For different values of the desired approximation quality, MSE_{target}, the quantizing step, Δ was varied. A thresholding $T = \Delta$ was used in all the experiments, i.e. if a coefficient $w \in [-\Delta, \Delta]$, it was set to zero. Experiments on the same test signals were also done using ordinary DCT with uniform quantizing, thresholding at $T = \Delta$, run-length and entropy coding. The results are compared in Figure 3. Small examples from the test signals are showed in Figure 4 together with reconstructed signals with bit-rate at 0.4 bit/sample.

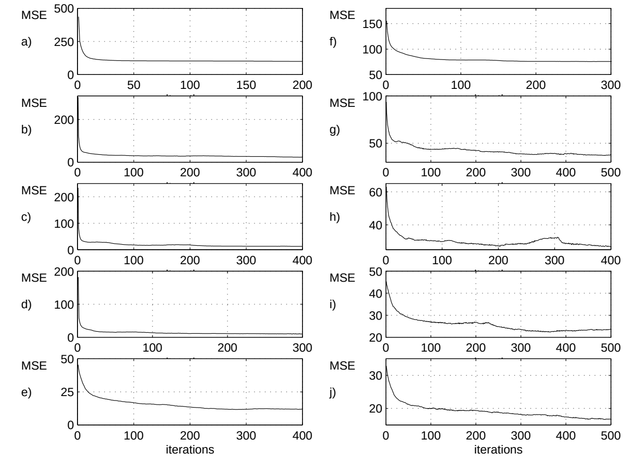

Figure 2: MSE is plotted as a function of training iterations where 1,2,3,4, and 5 frame vectors are used in each approximation: a), b), c), d), and e) MIT100. f), g), h), i), and j) MIT207.

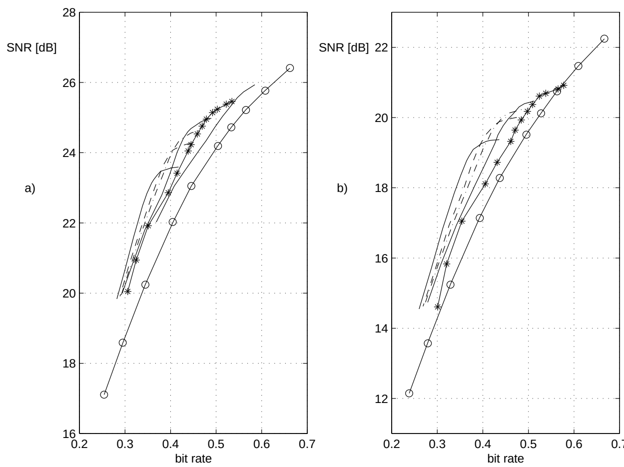

Figure 3: Rate-distortion plots for DCT (solid with o's) and multi-frame compression scheme with different MSE_{target}. a) MIT207 b) MIT100

5 CONCLUSION

The proposed compression scheme, using frames optimized for a particular class of signals, is shown to work well. In terms of rate-distortion it is better than traditional transform based techniques like the DCT for low bit rates. Figure 3 shows that when using this novel multi-frame compression scheme, the SNR reaches an almost constant level when the bit rate increases. The major reason for this is the specification of a desired approximation quality, MSE_{target}. There is a strong connection between the quantizing step, Δ, and MSE_{target}. For a target bit-rate there is an optimal combination of MSE_{target} and Δ. For a given MSE_{target}, if Δ is reduced to be less than the optimal Δ, the improvement in SNR will be very small, especially for large MSE_{target}.

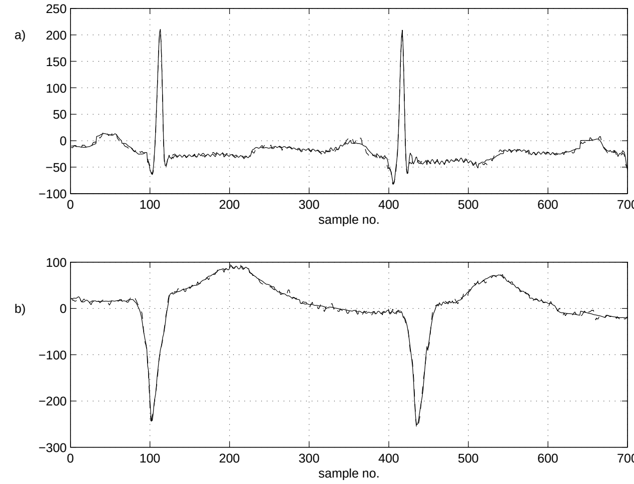

Figure 4: The first 700 samples of the test signal (dashed) and reconstructed signal with bit-rate at 0.4 bit/sample (solid) a) MIT100, b) MIT207.

The increase of the bit-rate will also be small. Different MSE_{target} has to be used for different target bit-rates. In our experiments we have been concentrating on very low bit rates. We have restricted the coding scheme never to use more than $L = 12$ vectors in an approximation. For higher target bit-rates we would increase L.

Future work will deal with the use of frames of different sizes in the proposed compression scheme. We will also focus on the connection between the desired approximation quality, MSE_{target}, and the quantizing step, Δ.

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