STATISTICAL MODELLING OF FULL FRAME DCT COEFFICIENTS

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ABSTRACT

Because of the increasing influence it has in image watermarking applications, the estimation of the distribution shape of full frame DCT coefficients is here addressed. Based on previous analyses on block-DCT, the coefficients are first assumed to follow a Generalized Gaussian distribution. The shape parameter ν is then evaluated according to the maximum likelihood criterion applied to a set of 170 natural images. The analysis has been further validated by using the χ^2 test-of-fit criterion. In contrast to the block-DCT case, experimental results prove that full frame DCT coefficients can be modeled without appreciable loss of performance by a Laplacian density function having variance decreasing with frequency.

1 INTRODUCTION

Due to the major role it plays in still images and video compression algorithms, block-DCT has been thoroughly studied. In particular, several approaches have been proposed to model block-DCT coefficients through analytically manageable probability density functions (pdf) [3, 4, 5, 7, 8]. More recently, increasing attention has been given to full frame DCT, especially for image watermarking applications, where the use of the full frame transform better suits the spread spectrum characteristics of the codemarks to be hidden in the image [1, 2, 6]. To design a robust and reliable watermarking system, however, the pdf of full frame DCT coefficients has to be known, or, at least, a reasonably simple model should hold. In this paper, some results of goodness-of-fit tests regarding the modelling of full frame DCT coefficients are presented. The maximum likelihood (ML) criterion is applied to a set of 170 gray level natural images to estimate the shape parameter ν and the standard deviation σ of the Generalized Gaussian distribution (GGD) of a subset of 1276 coefficients appropriately spread all over the frequency spectrum. Unlike the behaviour of block-DCT coefficients, for which the estimated shape parameter of the GGD is usually much lower than one [5], witnessing a significant deviation from the Laplacian assumption, the pdf of full frame DCT coefficients proves to be closer to a Laplacian distribution, especially when low-medium frequency portions of the spectrum are considered.

Results obtained through the maximum likelihood criterion are further validated by means of the χ^2 test-of-fit: in this way the effectiveness of the Laplacian model is verified again, thus proving that when the full frame transform is taken into account, virtually no improvement is achieved by adopting the more complex GGD model.

Results also point out that the variance of coefficients rapidly decreases with frequency and that coefficients corresponding to horizontal and vertical frequencies have a higher energy, i.e. they are characterized by a larger variance.

2 GENERALIZED GAUSSIAN DISTRIBUTION

By relying on the central limit theorem, Pratt [7] conjectures that the AC DCT coefficient follows a Gaussian distribution; Reininger and Gibson [8] use the Kolmogorov-Smirnov (KS) test to verify that the AC DCT coefficient has a Laplacian distribution; Müller [5] shows that by modelling the DCT coefficients with a Generalized Gaussian density function a significantly smaller test statistic χ^2 is obtained than that achieved by assuming a Laplacian pdf. Joshi and Fischer [4] model the DCT coefficients with a Generalized Gaussian density function, and use the χ^2 test of fit to validate the results reported in [5].

Following the above works, the probability density function of block-DCT coefficients can then be assumed to be a zero mean Generalized Gaussian distribution, defined as

$$f(x) = \frac{\nu\alpha(\nu)}{2\sigma\Gamma(1/\nu)} e^{-\left[\alpha(\nu)\left|\frac{x}{\nu}\right|\right]^{\nu}}, \tag{1}$$

where

$$\alpha(\nu) = \sqrt{\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}}, \tag{2}$$

$\Gamma(\cdot)$ denotes the usual gamma function, ν and σ are real valued positive constants, and a zero mean has been assumed. More specifically, the parameter ν is the shape parameter of the pdf describing the exponential rate of decay, with a Laplacian corresponding to a GGD with $\nu = 1$, a Gaussian to $\nu = 2$, and smaller values of ν denoting more peaked distributions; σ equals the standard deviation of the distribution.

Techniques based on both the ML criterion and the χ^2 test-of-fit lead to the conclusion that block-DCT coefficients are better modeled by using a GGD, even if the coding gain achievable through this more accurate model is fairly small [4]. We now want to extend the analysis carried out for the block-DCT case to the full frame transform.

3 ML ESTIMATE OF SHAPE PARAMETER ν AND STANDARD DEVIATION σ

Having assumed a GGD, the shape parameter ν and the standard deviation σ have to be estimated. In particular, aiming at applying the statistical analysis to image watermarking [1], we are mainly interested in the estimation of ν, since a wrong estimation on the distribution shape may have a severe impact on watermark detection effectiveness. The shape parameter ν is estimated through the maximum likelihood criterion, by using the same formulas given in [3].

Du [3] proposed the following equations for the ML estimation of the GGD parameters ν and σ:

$$\frac{\psi(1/\nu+1)+\log(\nu)}{\nu^2} + \frac{1}{\nu^2}\log\left(\frac{1}{n}\sum_{i=1}^{n}|x_i|^{\nu}\right) - \frac{\sum_{i=1}^{n}|x_i|^{\nu}\log|x_i|}{\nu\sum_{i=1}^{n}|x_i|^{\nu}} = 0 \quad (3)$$

where:

$$\psi(\tau) = -\gamma + \int_0^1 (1-t^{\tau-1})(1-t)^{-1}dt, \quad (4)$$

x_i denotes the i-th observed DCT coefficient, n is the number of observed samples, and γ is the Euler constant. Eq. (3) gives the ML estimate $\hat{\nu}$ of the shape parameter ν. When the ML estimate $\hat{\nu}$ is known, the ML estimate $\hat{\sigma}$ of the standard deviation is given by:

$$\hat{\sigma} = \left[\frac{\hat{\nu}\alpha(\hat{\nu})^{\hat{\nu}}\sum_{i=1}^{n}|x_i|^{\hat{\nu}}}{n}\right]^{1/\hat{\nu}}. \quad (5)$$

As opposed to works dealing with 8×8 block-DCT coders, in which the pdf of DCT coefficients are estimated by averaging over all the 8×8 blocks of a single image, when considering the full frame DCT, the coefficient densities must be averaged over a set of many images. To this aim, a database of 170 256×256 images has been built. Since the estimation of all the 65536 DCT coefficients would be too computationally expensive, the estimation is made on a selected group of coefficients; the choice of the subset of coefficients being made so that the extension of the results to the other elements is possible. As shown in figure (1), five groups of coefficients (a, b, c, d, and e) have been considered; by denoting with F the DCT coefficients matrix of the image I, the five groups of coefficients are:

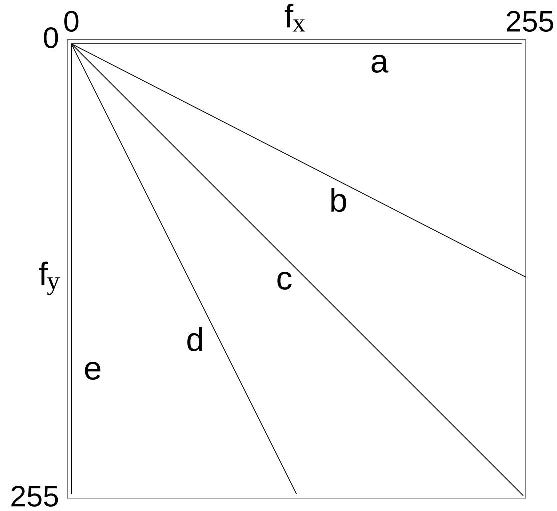

Figure 1: The five subsets of DCT coefficients.

$$\begin{aligned} a &= \{F(0,0), F(0,1), \ldots, F(0,255)\} \\ &= \{F(k,l)\}_{k=0,l=0,\ldots,255}, \end{aligned} \quad (6)$$

i.e. the vector of the elements $F(k,l)$ belonging to the first row of the transformed matrix F;

$$\begin{aligned} b = \{&F(0,0), F(0,1), F(0,2), \\ &F(1,3), F(1,4), \ldots, F(106,255)\}, \end{aligned} \quad (7)$$

i.e. the vector of the elements $F(k,l)$ of the transformed matrix F such that $k = int[l \cdot tg(22.5°)]$ $\forall l = 0, 1, \ldots, 255$;

$$\begin{aligned} c &= \{F(0,0), F(1,1), \ldots, F(255,255)\} \\ &= \{F(k,k)\}_{k=0,\ldots,255} \end{aligned} \quad (8)$$

i.e. the vector of the elements $F(k,l)$ belonging to the main diagonal of the transformed matrix F;

$$\begin{aligned} d = \{&F(0,0), F(1,0), F(2,0), \\ &F(3,1), \ldots, F(255,106)\}, \end{aligned} \quad (9)$$

i.e. the vector of the elements $F(k,l)$ of the transformed matrix F such that $l = int[k \cdot tg(22.5°)]$ $\forall k = 0, 1, \ldots, 255$;

$$\begin{aligned} e &= \{F(0,0), F(1,0), \ldots, F(255,0)\} \\ &= \{F(k,l)\}_{k=0,\ldots,255,l=0} \end{aligned} \quad (10)$$

i.e. the vector of the elements $F(k,l)$ belonging to the first column of the transformed matrix F.

The elements of these 5 groups take on a radial distribution in the matrix; this sampling geometry allows to

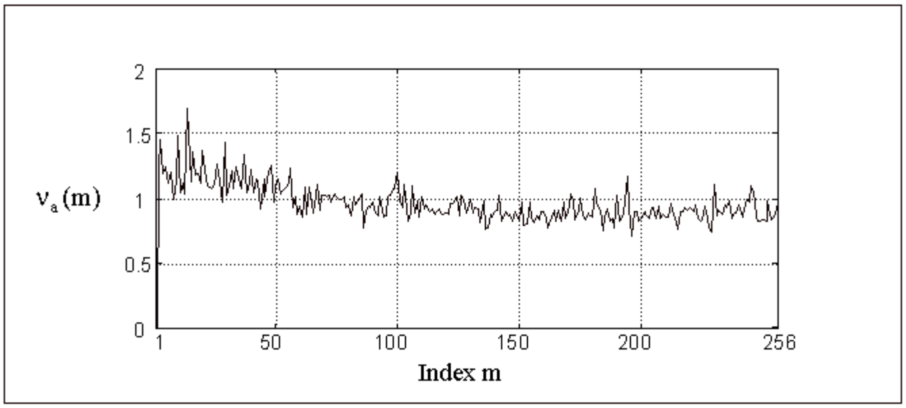

Figure 2: Estimated ν for the coefficients belonging to the first DCT row (class a).

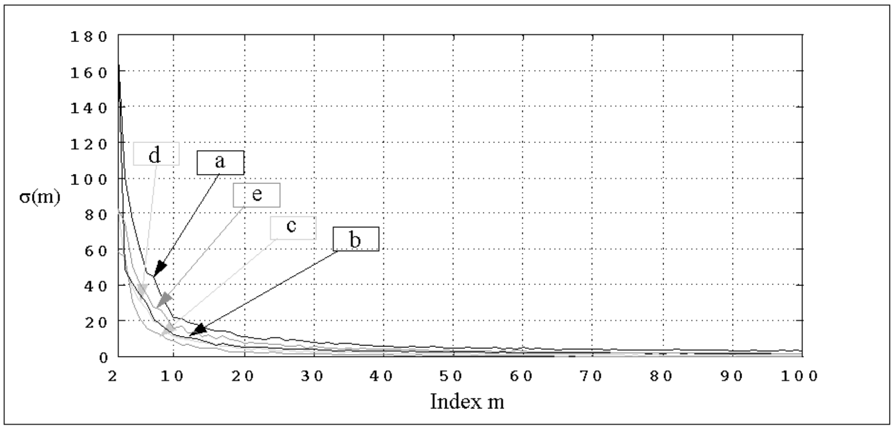

Figure 3: Estimated variance of the DCT coefficients belonging to the low-medium portion of the frequency spectrum.

estimate the pdf of the remaining coefficients by means of interpolation. As a result, 1276 sets of 170 samples have been analysed, under the hypothesis that the DCT coefficients follow a Generalized Gaussian distribution with zero mean, shape parameter ν and standard deviation σ. The maximum likelihood criterion has been applied to estimate the shape parameter ν and the standard deviation σ of each of the 1276 coefficients, thus obtaining the values $\hat{\nu}_h(m)$ and $\hat{\sigma}_h(m)$, where $h = \{a, b, c, d, e\}$ refers to the subsets in figure (1), and $m = \{0, 1, 2, \ldots, 255\}$ indicates the coefficient position inside the subsets. By observing the results we obtained with regard to $\hat{\nu}_a(m)$ (figure (2)), and by considering the mean value of the sequence $\hat{\nu}_a(m), m = 2, \ldots, 256$ (the DC term has been neglected), which is equal to 0.9580, a somewhat interesting conclusion can be drawn. As opposed to the behaviour of block-DCT coefficients, for which the estimated shape parameter is usually significantly smaller than 1 [4], full frame DCT coefficients can be assumed to follow a Laplacian function, especially when low-medium frequency portions of the spectrum are considered. In addition, the estimated standard deviation $\hat{\sigma}$ of the DCT coefficients (figure (3)) is rapidly decreasing with frequency; by looking at figure (3), it can also be noted that for any given frequency (i.e. m = constant) $\hat{\sigma}$ takes bigger values for coefficients belonging to the first row and to the first column, whereas it takes smaller values in the subset c, thus exhibiting a symmetry with respect to the main diagonal.

4 THE χ^2 TEST

To validate the above analysis, the maximum likelihood criterion was applied again to estimate the standard deviation σ of each coefficient, under the hypothesis that the shape parameter ν is equal to 1 (i.e. the pdf is approximated to a Laplacian function) obtaining the values $\sigma_h^L(m)$; then the same procedure has been applied by letting $\nu = 2$ (i.e. the pdf is assumed to be a Gaussian function) obtaining the values $\sigma_h^G(m)$.

The Chi-square test has finally been used to verify which of the above hypotheses (Laplacian function, Gaussian function, or GGD with the estimated param-

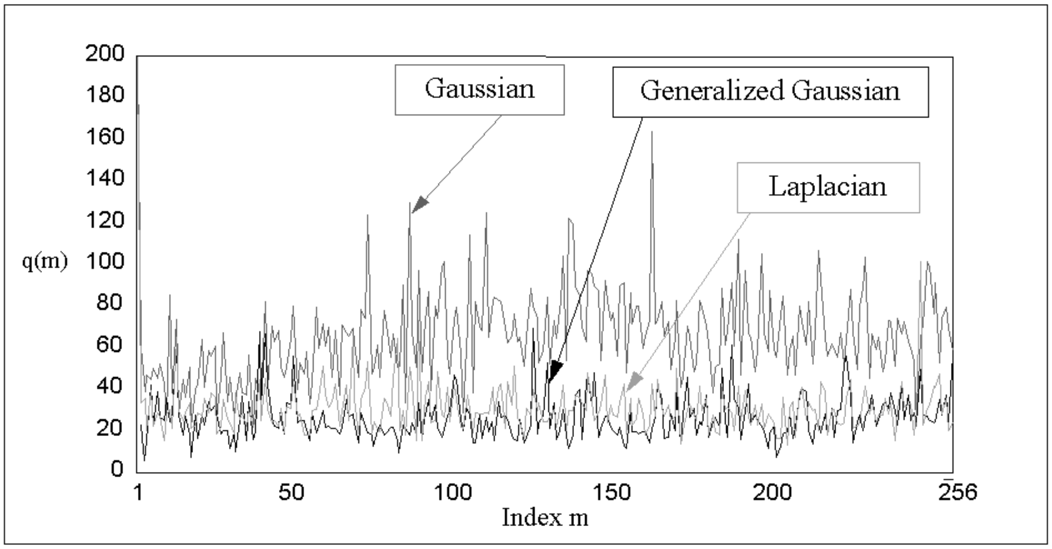

Figure 4: Results of χ^2 test under different pdf approximations (the higher the q the worse the fit).

eters $\hat{\nu}$ and $\hat{\sigma}$) best fits the data samples. The results of the test are displayed in figure (4), where the goodness-of-fit parameter q is reported for the coefficients belonging to the first row of full frame DCT (i.e. the coefficients belonging to the set a in figure (1)). Of course, the best fit is obtained by assuming a Generalized Gaussian pdf, however, when the approximation to a Laplacian distribution is made, only a slight loss of performance results, whereas considerably higher q's (the higher the q the worse the model fits data) are obtained under the Gaussian approximation.

5 CONCLUSIONS

In this paper the estimation of the distribution shape of full frame DCT coefficients has been addressed. According to previous analyses on block-DCT coefficients, the probability density function of full frame DCT coefficients has first been assumed to follow a zero mean Generalized Gaussian distribution. In contrast to the behaviour of block-DCT coefficients, experimental results demonstrate that full frame DCT coefficients can be effectively modeled by a Laplacian density function, especially when low-medium frequency portions of the spectrum are considered.

6 ACKNOWLEDGEMENTS

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