ROBUST IMAGE RESTORATION MATCHED WITH ADAPTIVE APERTURE FORMATION IN RADAR IMAGING SYSTEMS WITH SPARSE ANTENNA ARRAYS

Ivan Prudyus, Sviatoslav Voloshynovskiy, Taras Holotyak
Radio Engineering Faculty, State University "Lvivska polytechnika",
S. Bandery Str., 12, Lviv, 290646, UKRAINE

 $\hbox{$E$-mails: \{iprudyus, svolos, t.holotyak}$@polynet.lviv.ua}\\$

ABSTRACT

The paper presents a complex approach to radar imaging system development. The approach consists of two main stages. The first stage is adaptive aperture formation and the second one is robust adaptive image restoration. The use of adaptive aperture formation strategy makes possible to estimate the principle image components in spatial frequency domain and increase the reliability of received data. The robust adaptive image restoration allows to compensate the blurring effect of sparse aperture in the presence of mixed noise (i.e. Gaussian and impulse). The efficiency of the proposed approach is investigated on numerous test examples.

1 INTRODUCTION

The last decade in radar system development is characterized by increasing interest to imaging systems as efficient informational means of current situation presentation. The further development of such kind of systems requires the novel approaches both in the principles of image formation and in the methods of the following image processing. One of the main parameter that defines the image quality is the spatial resolution of imaging system. In the systems without further information processing potential spatial resolution is defined by the ratio of wavelength to physical antenna aperture size. In many cases, its improving via physical antenna size increasing is impossible because of constructive constrains that are conditioned by particularities of system implementation, i.e. airborne or mobile systems. It causes the replacement of continue aperture by the sparse antenna array with the number of antenna array elements as the limiting parameter. One of the advantages of sparse antenna array is a simplicity of aperture geometry and field distribution formation that opens the possibility of adaptive geometry array creation. Features of adaptive geometry sparse array design are closely related with particularities of radar image spatial spectra [1].

Investigation of the image spatial spectrum calculated by means of Fourier transform from image intensity demonstrates certain harmonics localization. As a rule, they place along the two main image frequency coordinate axes and along the several other axes shifted on the certain angles [2].

To form image without degradation connected with the finity of antenna aperture we should complete all plain in frequency domain. However, aperture restrictions of real radar imaging systems do not permit to use the infinite number of antenna elements that limits the obtained spatial image spectrum. To receive the maximum information about spatial structure of the investigated image in condition of aperture and time restrictions we should solve the problem of selective aperture formation. Therefore, the main goal of adaptive aperture formation is to form antenna array systems with variable geometry matched with particularities of spatial image spectrum. This problem is closely related with sensor planning in computer vision applications.

Obviously, in this case antenna system degrades image due to aperture sparseness. To compensate this degradation influence, methods based on inverse problem solution are used. A lot of methods to solve the above problem in the least square sense are known. Some of them use the additional *a priori* information on the smoothness of the solution expressed by the regularization functional (Tikhonov regularization), correlation matrices of the initial image and noise (Wiener filtering) or nonnegativity which is mathematically performed in diverse ways. Linear methods like Tikhonov regularization and Kalman filtering restore the image high frequency components only in the range of cut-off frequency of the spatial spectrum. Nonlinear methods make possible to extrapolate spatial spectrum over the cut-off frequency.

We propose to use the robust adaptive iterative image restoration approach [3, 4, 5] which has a lot of advantages over the rest of known methods due to its ability to incorporate *a priori* information on the solution, simplicity of programming, existence of strict conditions of iteration convergence and ability to take into account local image structure. The proposed method is based on the theory of M- and L-estimators in application to the image restoration problems [3, 5]. As a result, nonlinear adaptive-parametric

method is developed that incorporates the scheme of functional minimization according to the robust Mestimation strategy, adaptive image prefiltering based on the modified L-estimator for Gaussian and impulse noise suppression, adaptive regularization [6] that plays the role of soft constrain on the solution and hard parametric constrains in spatial coordinate and frequency domains to solve simultaneously the problems of data interpolation and extrapolation, and preserve fine details and edges in the restored image that could be damaged by soft constrain [4, 5]. In this sense, the soft constrain limits the upper bound of the solution in frequency domain and hard constrains limit lower one.

The problem formulation of adaptive aperture formation is presented in Section 2. The restoration algorithm is considered in Section 3. In Section 4, experimental results are presented and Section 5 concludes the paper.

2 PROBLEM FORMULATION

The general structure of the proposed approach could be shown according to Fig. 1 and consists of 3 main stages:

- 1) Image spectrum is estimated by small size antenna array with square geometry A_1 to determine the shift angles $\hat{\Theta}_i$ (Fig. 2). This antenna alone could not guarantee high quality image g_1 due to its low-pass structure in spatial frequency domain. However, its structure makes possible to uniquely determine shift angles.
- 2) Antenna array A_2 performs the spatial analysis of image g_2 spectrum in estimated directions changing its geometry according to the above shift angles. However, the spatial spectrum of this array is characterized by high level of ambiguity due to the array sparseness.
- 3) Restoration algorithm recovers the initial image on the base of 2 images $(g_1 \text{ and } g_2)$ obtained by means of the above arrays.

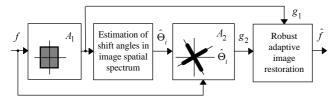


Fig. 1. Block diagram of the proposed approach to imaging system development.

Mathematically this problem could be written according to the next equation

$$g = g_1 + g_2 = H_1 f + H_2 f = (H_1 + H_2) f = H_e f$$
, (1)

where g is resulted image obtained by antenna A_1 with directional antenna pattern which forms the integral operator H_1 during beam scanning and matched antenna A_2 with corresponded antenna pattern H_2 , f is original image and H_e is equalent integral operator $H_e = H_1 + H_2$.

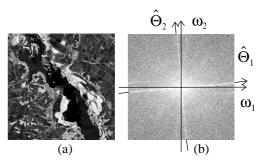


Fig. 2. Real radar image (a) and its spatial spectrum (b).

The generalized model of image formation additionally includes image distortion conditioned by Gaussian and impulse (mixed) noise

$$g = \begin{cases} H_e f + n, & \text{with propability } 1 - p, \\ n_{imp}, & \text{with probability } p. \end{cases}$$
 (2)

where n represent the Gaussian noise component n_{imp} is impulse noise which occurs with

$$n_{imp} = \begin{cases} g_{\min}, & with \ probability \ 1-q, \\ g_{\max}, & with \ probability \ q. \end{cases}$$
 (3)

where g_{\min} and g_{\max} are minimum and maximum values of the dynamic image range and q denotes the corresponded probability.

3 PROBLEM SOLUTION

The solution of restoration problem consists in the estimation or retrieval of original image based on the degraded image using available *a priori* information about model of observation (2), knowledge of blurring function and statistics of noise and original image given in the form of image smoothness, solution constrains and parametric models. According to the chosen approach this problem could be formulated as the minimization of compound functional

$$\min \Phi[\hat{f}] = \rho(\hat{g} - H_e \hat{f}) + \alpha \|C\hat{f}\|_{S}^{2}, \tag{4}$$

where $\|\cdot\|$ is weighted matrix norm, α is regularization parameter and $\rho(\cdot)$ is robust objective estimation function, C is high-pass filter, \hat{g} is pre-filtered image. The application of steepest descent method to the solution of minimization problem (4) results in the next iterative scheme that additionally includes constrains on the solution

$$\hat{f}^{k+1} = C_S \Re \left[\hat{f}^k \right] + \beta H_e^T \psi \left(\hat{g} - H_e \Re \left[\hat{f}^k \right] \right), \quad (5)$$

where \hat{f}^{k+1} is the estimation of f on k+1 iteration, $C_S = I - \alpha \beta C^T S C$ represent adaptive low-pass filter [6] that smoothes the restored image and it is soft constrain on smoothness, β is relaxation parameter, $\psi(r) = \frac{d\rho(r)}{dr}$ is soft limiter or indicator function and $r = \hat{g} - H_e \hat{f}$ that is chosen

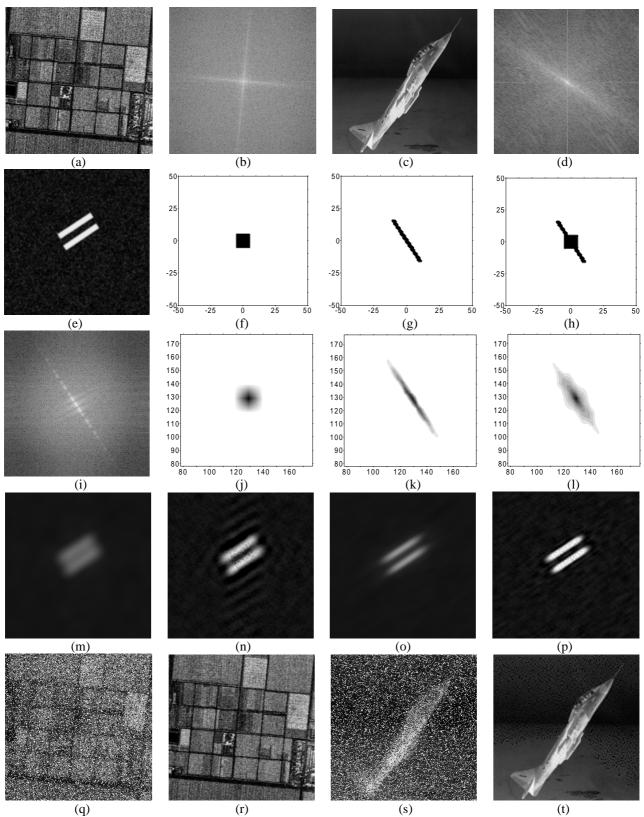


Fig. 3. Test images "Fields" (a), "Harrier" (c) and synthetic image (e) and their spatial spectra (b), (d) and (i), respectively. Small-size antenna (f) used for angles $\hat{\Theta}_i$ estimation and its spatial spectrum (j). Matched antenna (g) synthesized according to the angles $\hat{\Theta}_i$ estimated in spatial spectrum of synthetic image (i) and its spatial spectrum (k), and resulted antenna geometry (h) with spatial spectrum (l). Image (e) obtained by antenna (f) is shown in (m) and restored image in (n). Degraded and noisy images (q) and (s) and restored ones (r) and (t), respectively.

to restrict the influence of outliers on the estimation procedure [3], "T" denotes matrix transpose, \hat{g} is pre-filtered image on the base of modified L-estimator [5] that includes identification strategy, i.e. "detection-removal", and $\Re[\cdot]$ is the operator of constrains on the solution:

$$\Re = C_N C_M, \tag{6}$$

where C_N is local constrain on the field of possible solutions in coordinate domain [4] (i.e. constrain on nonnegativity is partial case of it) and C_M is constrain on module of the solution given according to the parametric model of image spectrum that takes into account spatial anisotropy of real image spectra [3, 5]. The upper bound of the restored image module is determined by soft constrain on smoothness and the above constrain enables to determine the lower bound that preserves edges and fine details in the restored image.

4 RESULT OF COMPUTER MODELING

This section demonstrates the main properties of the proposed approach. The comparison measure is defined according to *SNR* between original image f, degraded image \hat{g} and restored image \hat{f}^k

$$SNR = 10 \log_{10} \frac{1}{\delta_{\hat{f}}^2}, dB, \text{ where } \delta_{\hat{f}}^2 = \frac{\|f - \gamma\|}{\|f\|},$$
 (7)

where $\gamma = g$ for the direct observation and $\gamma = \hat{f}^k$ for the restored image.

The real radar images "Fields" (Fig. 3(a)) and "Harrier" (Fig. 3(c)) and test image (Fig. 3(e)) were chosen to investigate the potential possibilities of the proposed approach. The corresponded spatial spectra are shown in Figs. 3(b, d, i) for Figs. 3(a, c, e), respectively. First, investigation is performed for the test image. The square small-size aperture A_1 is depicted in Fig. 3(f). The matched sparse antenna array A_2 with image spectrum from Fig. 3(i) is shown in Fig. 3(g) and resulted aperture in Fig. 3(h). The corresponded spatial spectra for antenna A_1 is shown in Fig. 3(j), array A_2 in Fig. 3(k), and aperture from Fig. 3(g) is depicted in Fig. 3(1). The image g_1 formed using antenna A_1 is shown in Fig. 3(m) and restored image by means of the algorithm (5) for 30 iterations in Fig. 3(n). The error of direct image formation is 56.98% and restored image is 38.73% according to (7). The use of matched antenna array satisfies image formation error 48.34% and restoration error 32.93%. Image formed by means of combined (square and matched) aperture (Fig. 3(h)) and restored image are shown in Fig. 3(o, p) with error 45.85% and 25.68%, respectively. Comparing images from Fig. 3(n) and Fig. 3(p) we can conclude that in the second case the restored image has higher quality both qualitatively and quantitatively according to the chosen image quality norm.

The robust features of the proposed approach are studied on the real images. The images Fig. 3(a) and Fig. 3(c) were obtained using the corresponded matched arrays adding Gaussian noise to obtain SNR equals 35 dB and impulse noise with p = 0.3. The resulted images are shown in Fig. 3(q) and Fig. 3(s) with SNR 1.82 dB and -1.86 dB, respectively. The restored images for 30 iterations and $\alpha = 0.01$ are depicted in Figs. 3(r, t) with resulted SNR 17.2 dB and 15.41 dB, respectively. The performed analysis demonstrates high efficiency of proposed robust restoration and adaptive aperture formation strategy.

5 CONCLUSIONS

In this paper the new complex approach to image formation is proposed. The key features of the proposed approach consist in the adaptive aperture formation in accordance with particularities of image spatial spectrum and robust image restoration based on the concept of M-estimation, adaptive image pre-filtering using modified L-estimator, adaptive regularization and parametric constrains on the solution. The comparative analysis is performed and approach efficiency is studied on the examples of computer modeling.

REFERENCE

- 1. Z. Grytskiv, S. Voloshynovskiy, "Spatial resolution enhancement of passive imaging system with unfilled array," *Proceedings of XI International Microwave Conference of "MIKON'96"*, May 27-30, 1996, Warsaw, Poland, pp.505-508.
- 2. I. Prudyus, S. Voloshynovskiy, T. Holotyak, "Adaptive aperture formation matched with radiometry image spatial spectrum," *Proceedings of XII International Microwave and Radar Conference "MIKON"98"*, May 20-22, 1998, Krakow, Poland (in print).
- 3. S. Voloshynovskiy, "Robust image restoration based on the M-estimation concept and parametric model of image spectrum," *Proceedings of 5th IEEE, IEE, EURASIP Workshop on Systems, Signals and Image Processing "IWSSIP'98"*, June 3-5, 1998, Zagreb, Croatia (in print).
- 4. I. Prudyus, Z. Grytskiv, S. Voloshynovskiy, "Radiometry image processing based on nonlinear iterative methods," *Workshop on Design Methodologies for Signal Processing*, August 29-30, Zakopane, Poland, pp.21-27.
- 5. Z.Grytskiv, S.Voloshynovskiy, "Nonlinear adaptive-parametric image restoration," 5th International Conference on Computer Graphics and Image Processing "GKPO'98", May 18-22, Warsaw, Poland (in print).
- 6. S.N. Efstatiadis, A.K. Katsaggelos, "Adaptive iterative image restoration with reduced computational load," *Optical Engineering*, vol.29, No12, 1990, pp.1458-1468.