INTERFERENCE SUPPRESSION IN SS SYSTEMS: A COMPARISON BETWEEN PTV P/S-TYPE AND W-TYPE FILTERS

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ABSTRACT

The recently proposed LCL-PTV structures for narrowband interference suppression in DS/SS systems are examined and compared in terms of output SIR. New LCL-PTV prediction/subtraction filters are proposed, which are shown to provide the same SIR performances of the LCL-PTV whitening structures, with the additional advantage of being amenable to blind adaptive implementation. Simulation results are provided to support and expand the theoretical claims.

1 INTRODUCTION

Narrowband-interference (NBI) suppression is widely employed in direct-sequence spread-spectrum (DS/SS) overlay systems, especially with reference to code-division multiple-access (CDMA) wireless information networks. Most of the linear NBI suppression structures proposed in the literature fall into the category of *prediction/subtraction-type* (P/S-type) or *whitening-type* (W-type) filters. The former attempt to reject the interference by subtracting from the received signal a linear time-invariant (LTI) prediction of NBI, whereas the latter perform the same task by attempting to extract from the received signal the white component (ideally, the DS/SS signal plus noise). Both types of structures admit implementations of reasonable complexity, even though particular attention has been devoted to P/S-type filters [1], because they can be made adaptive without requiring any training signal.

Recently, a new approach to NBI interference suppression has been considered [2], which takes advantage of the cyclostationary (CS) [3] nature of the interfering signals. The proposed structures can be still regarded as belonging to the P/S-type and W-type filter categories, but replace LTI filtering with linear conjugate-linear (LCL) polyperiodically time-varying (PTV) filtering, which, among all linear filters, can be shown to be optimal for CS signals according to a minimum time-averaged mean-square-error (TA-MSE) criterion. These new suppression structures, which require only the knowledge or estimation of the hidden periodicities of the signal to be predicted (i.e, the *cycle frequencies*), largely outperform [2] conventional reception structures.

In this paper, the LCL-PTV P/S-type and W-type structures are compared, in order to recognize whether and to what extent they can be considered equivalent. On the basis of the analysis, we propose new LCL-PTV P/S-type structures, which not only assure the same performance level of the W-type structures for a fixed complexity burden, but also have the advantage of a full adaptive implementation.

2 LCL-PTV FILTERING STRUCTURES FOR NBI SUPPRESSION

Let us consider a single-user DS/SS system, where the complex envelope of the the received signal, after chip-matched filtering and chip-rate sampling, is given by

$$r(n) = s(n) + w(n) + i(n) \,, \tag{1}$$

where $s(n)$ are samples of the DS/SS signal, $w(n)$ are samples of the noise, modeled as a complex white circular Gaussian random process, and $i(n)$ are samples of the NBI. The signals $s(n)$, $w(n)$, and $i(n)$ will be assumed to be mutually independent. Our theoretical analysis will rely on the simplifying assumption that $s(n)$ can be modeled as a white process, that is, its time-averaged autocorrelation function can be approximated as being impulsive. Therefore, denoting with $q(n) = s(n) + w(n)$ the white (non-predictable) component of the received signals, the following simplified model will be considered:

$$r(n) = q(n) + i(n) \,. \tag{2}$$

In [2], the effects of NBI are mitigated by processing $r(n)$ via a FIR-based (causal) LCL-PTV filter [4], whose output is:

$$\begin{aligned} y(n) &= \sum_{p=1}^{P} \sum_{k=0}^{L_p-1} h^{\alpha_p}(k)\, r(n-k) e^{j2\pi\alpha_p(n-k)} \\ &+ \sum_{q=1}^{Q} \sum_{k=0}^{M_q-1} h^{\beta_q}(k)\, r^*(n-k) e^{j2\pi\beta_q(n-k)} \,, \end{aligned} \tag{3}$$

where L_p and M_q denote the lengths of the FIR filters $h^{\alpha_p}(k)$ and $h^{\beta_q}(k)$, respectively. The input-output relation (3) can be re-expressed, in a more compact form, as

$$y(n) = \mathbf{h}_1^H \mathbf{r}(n) , \tag{4}$$

wherein the superscript H denotes Hermitian (conjugate transpose) operation and the column vectors $\mathbf{h}_1$ and $\mathbf{r}(n)$ are built by orderly stacking all the FIR coefficients corresponding to all α_p and β_q values and the (frequency-shifted and possibly conjugated) *past and present* samples of the received signals, respectively. The frequency shifts α_p and β_q are chosen [2] among the NBI cycle frequencies and sums and differences of some of these.

Such a filtering is referred to as a whitening-type (W-type) filter when the weight vector $\mathbf{h}_1$ is singled out by minimizing the TA-MSE between the output of the filter and the white component $q(n)$:

$$\mathbf{h}_1 = \arg\min_{\mathbf{h}} E[|q(n) - \mathbf{h}^H \mathbf{r}(n)|^2] , \tag{5}$$

with $E[\cdot]$ and $\langle\cdot\rangle$ denoting ensemble and infinite-time averaging, respectively. Solution of (5) is obtained straightforwardly as

$$\mathbf{h}_1 = \mathbf{R}_{\mathbf{rr}}^{-1} \mathbf{R}_{\mathbf{r}q} , \tag{6}$$

with

$$\begin{aligned} \mathbf{R}_{\mathbf{rr}} &= \langle E[\mathbf{r}(n)\mathbf{r}^H(n)]\rangle , & (7) \\ \mathbf{R}_{\mathbf{r}q} &= \langle E[\mathbf{r}(n)q^*(n)]\rangle . & (8) \end{aligned}$$

An alternative, but conceptually equivalent, approach to suppress NBI via LCL-PTV filtering is to predict the NBI and, then, subtract it from the received signal. The output of such a prediction/subtraction (P/S-type) filter can be defined, in analogy to the LTI case, as follows:

$$y(n) = r(n) - \mathbf{h}_2^H \mathbf{u}(n) \tag{9}$$

where $\mathbf{u}(n)$ collects the (frequency-shifted and possibly conjugated) *past* samples of the received signal and $\mathbf{h}_2^H \mathbf{u}(n)$ represents the LCL-PTV prediction of the NBI. The weight vector $\mathbf{h}_2$ can be determined by direct minimization of the TA-MSE between the current sample of the received signal and the NBI predicted value, i.e.,

$$\mathbf{h}_2 = \arg\min_{\mathbf{h}} E[|r(n) - \mathbf{h}^H \mathbf{u}(n)|^2] . \tag{10}$$

Note explicitly that whereas the LCL-PTV W-type filter defined by (4)-(6) performs minimum TA-MSE estimation of the white component $q(n)$, the LCL-PTV P/S-type filter defined by (9) and (10) can be easily show to perform minimum TA-MSE estimation of the NBI, followed by subtraction of the estimate from the current sample. The key difference between minimizations involved in (5) and (10) is that to adaptively implement (5) a training signal is needed, whereas (10) can be carried out *blindly*, i.e., without the need for a training signal.

The adoption of a minimum TA-MSE criterion in the synthesis of the filters allows one to single out the weights of the filters in a straightforward manner. Nevertheless, the final task of the receiver is to recover the digital information data stream associated to the DS/SS signal. Then, since the test statistic following NBI suppression is invariant to scaling of the signal at the output of the suppression stage, a more sensible measure of performance is the signal-to-interference ratio (SIR) at the output of the suppression stage, which can be expressed as

$$\mathrm{SIR} = \frac{|\rho|^2}{1 - |\rho|^2} , \tag{11}$$

where

$$\rho \triangleq \frac{\langle E[y(n)q^*(n)]\rangle}{\sqrt{P_y \, P_q}} , \tag{12}$$

denotes the correlation coefficients between $q(n)$ and the output of the filter $y(n)$, with $P_y \triangleq \langle E[|y(n)|^2]\rangle$ and $P_q \triangleq \langle E[|q(n)|^2]\rangle$. It is worthwhile to note that the SIR is a monotonically increasing function of $|\rho|^2$, which, therefore, can be assumed as an equivalent performance measure.

When considering the LTI counterparts of the W-type (4) and P/S-type (9) filters (that is, by considering only the $\alpha = 0$ branch for each structure), they turn out to exhibit the same SIR performance. The same equivalence, however, does not hold for LCL-PTV filter. In particular, for comparable number of branches and weights, the P/S-type LCL-PTV filter (9) exhibits poorer SIR performance than the W-type one (4). Such a performance degradation can be overcome observing that, unlike the LTI case, the LCL-PTV prediction stage can be made more effective if NBI prediction is carried out not only on the basis of the *past* samples, but also of the current one, since such a frequency-shifted (and possibly conjugated) current sample is uncorrelated with the white component $q(n)$ contained in $r(n)$. This intuition is behind the approach pursued in the next Section to derive new LCL-PTV P/S-type structures.

3 THE PROPOSED LCL-PTV P/S-TYPE STRUCTURE

Since P/S-type structures are generally preferred to W-type ones owing to their amenability to blind implementation, we propose here a LCL-PTV P/S-type that incurs in no loss of SIR performance compared with the W-type filter (4)-(6). The proposed structure is given by

$$y(n) = r(n) - \mathbf{h}_3^H \mathbf{v}(n) , \tag{13}$$

where $\mathbf{v}(n)$ is chosen so as to satisfy

$$\mathbf{r}(n) = \begin{bmatrix} r(n) \\ \mathbf{v}(n) \end{bmatrix} \tag{14}$$

and $\mathbf{h}_3$ is given by

$$\mathbf{h}_3 = \arg\min_{\mathbf{h}} E[|r(n) - \mathbf{h}^H \mathbf{v}(n)|^2] \,. \tag{15}$$

Indeed, we will show that (14) assures that the proposed LCL-PTV P/S-type filter exhibits the same output SIR as the LCL-PTV W-type filter (4)-(6). In order to prove the assertion, note that it suffices to show that the two considered filters exhibit the same values of $|\rho|^2$, or, equivalently, the same values of

$$\xi \stackrel{\triangle}{=} |\rho|^2 P_q = \frac{|\langle E[y(n)q^*(n)]\rangle|^2}{P_y} \,. \tag{16}$$

Then, let us consider first the general form of a filter for estimation of $q(n)$, given by:

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \,, \tag{17}$$

for which, substituting (17) into (16), it results:

$$\xi = \frac{|\mathbf{w}^H \mathbf{R}_{\mathbf{x}q}|^2}{\mathbf{w}^H \mathbf{R}_{\mathbf{xx}} \mathbf{w}} \,. \tag{18}$$

The W-type filter given by (4)-(8) is already in the form (17), with $\mathbf{x}(n) = \mathbf{r}(n)$ and $\mathbf{w} = \mathbf{h}_1$. Thus, substitution of (6) into (18) yields:

$$\xi_{\mathrm{W}} = \mathbf{R}_{\mathbf{r}q}^H \mathbf{R}_{\mathbf{rr}}^{-1} \mathbf{R}_{\mathbf{r}q} \,. \tag{19}$$

Observe that, accounting for the assumption that $q(n)$ and $w(n)$ are independent and $q(n)$ is white, one has:

$$\mathbf{R}_{\mathbf{r}q} = \mathbf{R}_{\mathbf{q}q} = P_q \,\mathbf{1} \,, \tag{20}$$

where $\mathbf{1} \stackrel{\triangle}{=} (1, 0, \ldots 0)^T$, with T denoting transpose operation. Therefore, (19) can be rewritten as

$$\xi_{\mathrm{W}} = P_q^2 \mathbf{1}^H \mathbf{R}_{\mathbf{rr}}^{-1} \mathbf{1} = P_q^2 \,\{\mathbf{R}_{\mathbf{rr}}^{-1}\}_{11} \,, \tag{21}$$

wherein $\{\cdot\}_{ij}$ denotes the ij-the element of the matrix in brackets.

The P/S-type filter defined by (13)-(15) can be put in the standard form (17) by letting $\mathbf{w}^H = [1, -\mathbf{h}_3^H]$ and $\mathbf{x}(n) = [r(n), \mathbf{v}(n)]^T = \mathbf{r}(n)$. If, moreover, one observes that $\mathbf{h}_3$, being solution of (15), can be expressed as

$$\mathbf{h}_3 = \mathbf{R}_{\mathbf{vv}}^{-1} \mathbf{R}_{\mathbf{v}r} \,, \tag{22}$$

then, substituting (22) in (18) and performing straightforward algebraic manipulations, one has:

$$\xi_{\mathrm{P/S}} = \frac{P_q^2}{P_r - \mathbf{R}_{\mathbf{v}r}^H \mathbf{R}_{\mathbf{vv}}^{-1} \mathbf{R}_{\mathbf{v}r}} \,. \tag{23}$$

Finally, (21) and (23) can be easily shown to coincide, since accounting for (14), $\mathbf{R}_{\mathbf{rr}}$ can be partitioned as follows:

$$\mathbf{R}_{\mathbf{rr}} = \begin{bmatrix} P_r & \mathbf{R}_{\mathbf{v}r}^H \\ \mathbf{R}_{\mathbf{v}r} & \mathbf{R}_{\mathbf{vv}} \end{bmatrix} , \tag{24}$$

and, hence, exploiting the inversion lemma for partitioned matrices one has:

$$\{\mathbf{R}_{\mathbf{rr}}^{-1}\}_{11} = \frac{1}{P_r - \mathbf{R}_{\mathbf{v}r}^H \mathbf{R}_{\mathbf{vv}}^{-1} \mathbf{R}_{\mathbf{v}r}} \,. \tag{25}$$

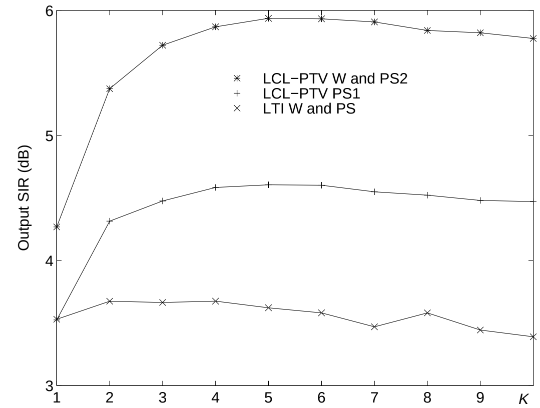

Figure 1: Output SIR versus K for a white signal plus NBI.

4 NUMERICAL RESULTS

In this section, we present numerical results aimed at quantifying the improvement that can be achieved by resorting to the new LCL-PTV P/S-type filter, as well as the advantage that LCL-PTV filters generally assure over LTI ones.

In all the experiments, the NBI is modeled as a BPSK signal with baud-rate 1/10 and frequency offset 1/3 (with respect to the center frequency of the band), and the following suppression structures are considered:

1. the PTV P/S-type filter defined by (9)-(10) (referred to as LCL-PTV PS1);

2. the LCL-PTV W-type and LCL-PTV P/S-type filters defined by (4)-(6) and (13)-(15), respectively (referred to as LCL-PTV W and LCL-PTV PS2);

3. the LTI W-type and LTI P/S-type filters (referred to as LTI W and PS).

The LCL-PTV P/S-type filters [see (3)] are composed by three ($P = 2$ and $Q = 1$) and equal-length ($L_1 = L_2 = M_1 \stackrel{\triangle}{=} K$) branches, corresponding to the cycle frequencies $\alpha_1 = 0$, $\alpha_2 = 1/10$ and $\beta_1 = 2/3$. The LCL-PTV W-type filters have the same number of branches and weights as the P/S ones, except for the $\alpha = 0$ branch, which has $K+1$ weights. Finally, the LTI P/S-type filters are characterized by $3K$ weights, whereas the LTI W-type filters have an overall number of $3K+1$ weights. Note that such a choice of the lengths is needed to obtain a fair comparison between the different suppression structures, i.e., for the same number of overall weights.

In the first experiment, $q(n)$ is modeled as a white Gaussian process, the input SIR is fixed at 0dB and the output SIR is evaluated and plotted in Fig. 1 for different values of K ranging from 1 to 10. First, it should be noted that all the LCL-PTV filters outperform the LTI ones. Moreover, as claimed, the new LCL-PTV P/S-type filter (LCL-PTV PS2) exhibits the same performance of LCL-PTV W and outperforms LCL-PTV

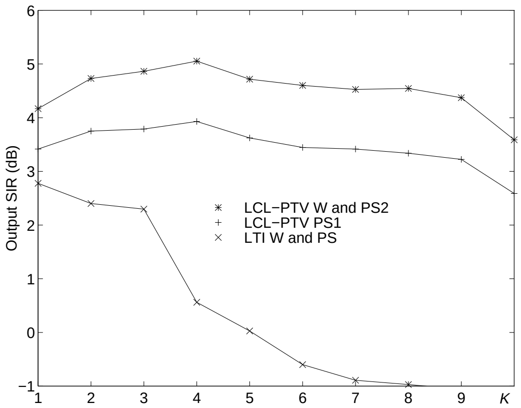

Figure 2: Output SIR versus K for a DS/SS signal ($N = 31$) plus NBI.

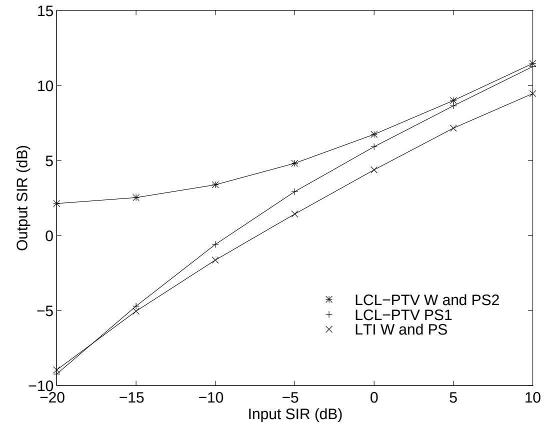

Figure 3: Output SIR versus input SIR for a DS/SS signal ($N = 31$) plus noise and NBI.

PS1 by a maximum of more than 1.5dB, without any increase in complexity. Finally, observe that the maximum SIR performance of the LCL-PTV structures is reached for values of K around $K = 5$, whereas increasing K beyond such value generally does not improve performances, while entailing additional computational burden.

In the second experiment, we examined the more interesting case where $q(n)$ is modeled as a BPSK DS/SS signal, with processing gain $N = 31$ and a random signature vector. The input SIR is again fixed at 0dB. Figure 2, which reports the output SIR versus K, confirms that all the LCL-PTV filters outperform the LTI ones. Moreover, compared with Fig. 1, it reveals also that the SIR performance in this case is generally less favorable, due to the departure from the whiteness assumption for $q(n)$. The advantage of LCL-PTV W and PS2 over LCL-PTV PS1 is less marked (around 1dB) in this case. Finally, the LCL-PTV filters exhibit a more pronounced performance degradation with increasing values of K, whose optimal value is around $K = 4$.

In the third experiment, we modeled $q(n)$ as the sum of a unitary-power BPSK DS/SS signal with processing gain $N = 31$ (random signature vector) and unitary-power white Gaussian noise, whereas the NBI power is varied so as to obtain values of the input SIR ranging from -20dB to 10dB. Figure 3, which reports the output SIR versus the input SIR for $K = 4$, shows that the advantage of the LCL-PTV W and PS2 filters over both the LCL-PTV PS1 and the LTI ones is especially significant when the input SIR is low, which is a typical situation in low-power overlay application. Finally, to show the bit-error-rate (BER) improvement that can be obtained by resorting to the new LCL-PTV suppression structure, in Tab.1 we report the BER as a function of the input SIR in the same operative conditions considered in Fig. 3, except for the processing gain which is now fixed at $N = 7$. The results show that the receiving structures employing the proposed LCL-PTV W and PS2 suppression stage assure a remarkable performance advantage in terms of BER against LCL-PTV PS1 and LTI structures, especially in correspondence of the lowest values of SIR. Moreover, their BER performance exhibit good robustness features with respect to SIR variability.

SIR	No filter	LCL-PTV W and PS2	LCL-PTV PS1	LTI W and PS
-20	0.324	0.007	0.169	0.148
-15	0.141	0.014	0.084	0.091
-10	0.063	0.007	0.035	0.035
-5	0.028	0.007	0.014	0.014

Table 1: BER versus SIR for a DS/SS signal ($N = 7$) plus noise and NBI.

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