An Approach to Power Allocation in MIMO Radar with Sparse Modeling for Coherence Minimization

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Abstract—**In this paper, we aim at improving the estimation performance of the direction of arrival (DOA) in a colocated MIMO radar through power allocation under the sparsity constraints. Specifically, by considering the sparse recovery techniques, we try to minimize the coherence of associated sensing matrix by optimally distributing the power among transmit antennas. To determine the optimal power distribution, we reformulate the coherence minimization problem and derive a convex optimization constrained by the total power budget. This helps us to efficiently evaluate and simulate the optimal power distribution policy. Simulation results confirm superiority of the proposed method compared to the existing techniques.**

I. Introduction

In a MIMO radar system, due to the existence of several transmit-receive antennas and the extent of the target space (e.g. the range-Doppler domain or the azimuth domain in case of DOA estimation), the amount of data and the search area are both huge. However, the number of existing targets is relatively small, which promote the use of sparse recovery methods. Moreover, the sparse nature of the targets allow for their detection and estimation even with much less received data (for example with fewer transmit/receive antennas). The application of sparse modeling to MIMO radars has been addressed both for colocated MIMO radars [1] (e.g. in [2]–[5]) and widely separated MIMO radars [6] (e.g. in [7]–[9]). In both cases, by assuming the targets to be sparsely spread over the target space, sparse recovery methods studied in the field of compressed sensing (CS) are used to detect and estimate the targets. In this paper, we consider the DOA estimation problem in a colocated MIMO (CL-MIMO) radar under sparse modeling. More precisely, we intend to improve the performance of the sparse recovery process (and thereby improving the performance of DOA estimation) by means of power allocation.

The study of power allocation problem in MIMO radars has been traditionally focused on minimizing the Cramer-Rao lower bound (CRLB) on the estimation of targets [10]–[12]. Due to the complexity of evaluating the CRLB under sparse models, the power allocation problem is treated differently in this context. Examples of such studies include [8], [13]. In [8] widely separated MIMO radars are considered, and an adaptive power allocation scheme is proposed. In this method, after obtaining an estimate of the targets, the powers of the next set of transmitting pulses are determined so as to maximize the minimum target return. In [13], the power allocation problem is addressed for both colocated and widely separated cases. In this work, for the colocated case the configuration of [3] is adopted, where the transmit (TX) and receive (RX) antennas of the MIMO radar are considered as the nodes of a small scale network that are randomly located in a disk of a small radius. Target space in [13] is assumed to be the azimuth space and the DOA estimation problem is considered. Power allocation in [13] is carried out in a way to improve the sparse recovery performance by making the gram matrix of the sensing matrix (i. e. $\mathbf{\Psi}^H\mathbf{\Psi}$ where $\mathbf{\Psi}$ is the sensing matrix) as close as possible to an identity matrix. This will reduce the coherence of the sensing matrix which can improve the recovery performance. Reducing the coherence of the sensing matrix has been also studied in a few other works in the context of compressive sensing based radars such as in [14], [15], to improve the sparse recovery process. Similiar to [13] the coherence is not minimized directly in these works.

In this paper, to conduct the power allocation, we adopt a direct approach to reduce the coherence of the sensing matrix. That is, considering the same model as in [13], we choose the coherence of the sensing matrix itself as the cost function to minimize and derive a convex optimization problem to obtain optimal powers.

The rest of the paper is organized as follows. A brief review on CS formulation and conditions for recovery will be given in section II. Then, the radar Signal model and CS formulation for received signals will be discussed in section III. The proposed power allocation scheme will be presented in section IV and simulation results will be provided in section V. Finally, we conclude the paper in section VI.

II. Compressed Sensing and Sparse Recovery

The theory of compressed sensing states that a K-sparse vector $\mathbf{x}_{N\times 1}$ (with at most K nonzero elements where $k \ll N$) can be recovered from M noisy linear measurements $\mathbf{y}_{M\times 1} = \mathbf{\Psi}\mathbf{x} + \mathbf{n}$ through solving the l_1 minimization problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad s.t. \quad \|\mathbf{\Psi}\mathbf{x} - \mathbf{y}\|^2 \leq \epsilon^2 \tag{1}$$

if 1) the sensing matrix $\mathbf{\Psi}_{M\times N}$ holds the following restricted isometry property (RIP) [16] for any $2K$-sparse vectors with a restricted isometry constant (RIC) $\delta_K < \sqrt{2} - 1$:

$$(1 - \delta_K)\|\mathbf{x}\|_2^2 \leq \|\mathbf{\Psi}\mathbf{x}\|_2^2 \leq (1 + \delta_K)\|\mathbf{x}\|_2^2 \tag{2}$$

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and 2) the noise process is bounded by $\|\mathbf{n}\|_2 \leq \epsilon$. Then, the reconstruction error for the solution $\hat{\mathbf{x}}$ to (1) will be bounded by:

$$\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \leq C\epsilon \tag{3}$$

where C is a small constant. The problem with RIP is that its testing for a given matrix is an NP-hard problem which makes its use intractable when M is large. A sufficient condition on RIP can be provided by coherence of the matrix which is easy to tract. The coherence of a matrix $\mathbf{\Psi}$ with $\mathbf{u}_i$s as its columns is computed by:

$$\boldsymbol{\mu}(\mathbf{\Psi}) = \max_{l \neq l'} \frac{\left|\mathbf{u}_{l'}^H \mathbf{u}_l\right|}{\|\mathbf{u}_{l'}\|\|\mathbf{u}_l\|}. \tag{4}$$

The advantage of the coherence over RIP is its capability to be formulated in a straightforward manner and computed in polynomial time. RIC is conservatively bounded by $\delta_K \leq (K-1)\mu(\mathbf{\Psi})$. So, it is important to keep $\mu(\mathbf{\Psi})$ as small as possible. Here we seek minimizing the coherence using power allocation.

III. Signal Model

Let us consider MIMO radar with the same configuration as in [3] and [13]. It is assumed in this configuration that M TX antennas and N RX antennas of the radar are randomly located on a small area. TX and RX antennas are assumed to be placed at (r_i^t, α_i^t) and (r_i^r, α_i^r) in polar coordinates, respectively. TX and RX antennas are colocated; that is they are close enough together to see a specified target in the radar far-field with the same radar cross section (RCS). Let us denote by $\mathbf{X}_{L\times M}$ the matrix of the transmitted waveforms in baseband. The i-th column of $\mathbf{X}$ is the baseband waveform transmitted by the i-th transmitter. Under far-field assumption we can define transmit steering vector at azimuth angle θ (normalized with respect to the origin) denoted by $\mathbf{a}_{M\times 1}(\theta)$ as:

$$\mathbf{a} = [e^{j\frac{2\pi}{\lambda}d_1^t(\theta)}, e^{j\frac{2\pi}{\lambda}d_2^t(\theta)}, \ldots, e^{j\frac{2\pi}{\lambda}d_M^t(\theta)}]^T \tag{5}$$

where $d_i^t(\theta) = r_i^t \cos(\theta - \alpha_i^t)$. If there exist K targets in the far-field of the radar which are all at the same range cell and at azimuth angles θ_ks, considering narrowband and far-field assumptions, the baseband signal received at the i-th receiver can be approximated by:

$$\mathbf{r}_i = \sum_{k=1}^{K} \beta_k e^{j\frac{2\pi}{\lambda}d_i^r(\theta_k)} \mathbf{X}\mathbf{a}(\theta_k) + \mathbf{n}_i \tag{6}$$

where β_ks are complex coefficients proportional to RCSs of the targets; $d_i^r(\theta) = r_i^r \cos(\theta - \alpha_i^t)$; and $\mathbf{n}_i$ is the noise vector at the i-th receiver which is modeled by a circularly symmetric complex Gaussain random vector. The complex reflection coefficients β_ks are modeled by a zero-mean complex Gaussain variable which is the case in the traditional swerling case I model which yeilds to exponentially distributed RCS values [17].

If we constitute a grid with linearly spaced azimuth angles $\gamma_1, \gamma_2, \ldots, \gamma_{N_g}$ and set the spacing between grid angles so close that each target azimuth angle will be approximately equal to one of the grid points, we can write (6) using an sparse representation $(N_g \gg K)$ in the matrix form as:

$$\mathbf{r}_i = \mathbf{\Psi}_i \mathbf{s} + \mathbf{n}_i \tag{7}$$

where $\mathbf{\Psi}_i = [e^{j\frac{2\pi}{\lambda}d_i^r(\gamma_1)}\mathbf{X}\mathbf{a}(\gamma_1), \ldots, e^{j\frac{2\pi}{\lambda}d_i^r(\gamma_{N_g})}\mathbf{X}\mathbf{a}(\gamma_{N_g})]$ which is regarded as sensing matrix in i-th RX; $\mathbf{s} = [s_1, s_2, \ldots, s_{N_g}]$ is a sparse vector with s_n being zero if there is no target at the corresponding grid angle, otherwise being nonzero and equal to the reflection coefficient of the target at that grid angle.

If we stack all received vectors from all RX antennas, the total measurement vector $\mathbf{r}$ can be written as:

$$\mathbf{r} = [\mathbf{r}_1^T, \ldots, \mathbf{r}_N^T]^T = \underbrace{[\mathbf{\Psi}_1^T, \ldots, \mathbf{\Psi}_N^T]^T}_{\mathbf{\Psi}} \mathbf{s} + \underbrace{[\mathbf{n}_1^T, \ldots, \mathbf{n}_N^T]^T}_{\mathbf{n}} \tag{8}$$

So, a simple CS formulation is obtaind in which $\mathbf{\Psi}$ is regarded as the sensing matrix and $\mathbf{n}$ is the total noise vector. Provided that $\mathbf{s}$ is sparse enough, it can be estimated using a variety of sparse recovery methods such as [18]–[21]. We use NESTA method proposed in [21] as our method for recovery of sparse vectors which is known to be well-performed in the case of complex vectors and matrices. After recovery of sparse vector $\mathbf{s}$, to estimate the DOAs of the targets, it is enough to obtain the support of $\mathbf{s}$.

IV. Power Allocation

As noted in section II, It is of great importance that the coherence of the sensing matrix be as small as possible to guarantee the recovery of larger number of sparse signals. In [13], [15], the difference between the gram matrix of the sensing matrix $\mathbf{G} \triangleq \mathbf{\Psi}^H\mathbf{\Psi}$ and an identity matrix is minimized to reduce the coherence. Here we seek minimizing the coherence itself using power allocation.

Note that in (6), there is no unit-norm restriction on the columns of the transmit signal matrix $\mathbf{X}$. If we extract powers from $\mathbf{X}$ and restrict it to have unit-norm columns, the k-th column of the sensing matrix can be stated in terms of TX powers as:

$$\mathbf{u}_l = \mathbf{b}(\gamma_l) \otimes (\mathbf{X}\mathbf{A}(\gamma_l)\hat{\mathbf{p}}) \tag{9}$$

where $\mathbf{b}(\gamma_l) = \left[e^{j\frac{2\pi}{\lambda}d_1^r(\gamma_l)}, \ldots, e^{j\frac{2\pi}{\lambda}d_N^r(\gamma_l)}\right]^T$ is the receive steering vector at azimuth angle γ_l, $\hat{\mathbf{p}} = \sqrt{\mathbf{p}}$ where $\mathbf{p} = [p_1, \ldots, p_M]^T$ is the power vector with p_i being the transmitted power at i-th TX and $\mathbf{A}(\gamma_l) = diag\{\mathbf{a}(\gamma_l)\}$. Note that $\mathbf{X}$ in (9) has unit-norm columns. Here the transmitted waveforms are known in advance ($\mathbf{X}$ is given) and we are just going to obtain TX powers contained in $\mathbf{p}$. It can be shown that when the transmitted waveforms are orthogonal which means $\mathbf{X}^H\mathbf{X} = \mathbf{I}$, the squared inner product between l-th and l'-th columns of the sensing matrix $\mathbf{\Psi}$ can be written as a quadratic form of $\mathbf{p}$ [13]:

$$|\mathbf{u}_{l'}^H \mathbf{u}_l|^2 = \mathbf{p}^T \left(b_{ll'} \mathbf{c}_{ll'} \mathbf{c}_{ll'}^H\right) \mathbf{p} \tag{10}$$

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where:

$$b_{ll'} = |\mathbf{b}^H(\gamma_{l'})\mathbf{b}(\gamma_l)|^2 = \left|\sum_{k=1}^{N} e^{j\frac{2\pi}{\lambda}\left(d_k^r(\gamma_l) - d_k^r(\gamma_{l'})\right)}\right|^2 \quad (11)$$

and $\mathbf{c}_{ll'}$ is the diagonal vector of $\mathbf{C}_{ll'} = \mathbf{A}^H(\gamma_{l'})\mathbf{A}(\gamma_l)$. It is obvious that the matrix in the above quadratic form is positive semi-definite and so (10) is a convex function. The norm of the l-th column of $\mathbf{\Psi}$ can also be computed using (10):

$$\|\mathbf{u}_l\| = N\|\hat{\mathbf{p}}\|^2 \quad \forall l = 1, \ldots, N_g \quad (12)$$

which is independent of l. Now, the coherence of the sensing matrix can be computed using (10) as:

$$\boldsymbol{\mu}(\mathbf{\Psi}) = \max_{l \neq l'} \frac{|\mathbf{u}_{l'}^H \mathbf{u}_l|}{\|\mathbf{u}_{l'}\|\|\mathbf{u}_l\|} = \max_{l \neq l'} \frac{\sqrt{\mathbf{p}^T\left(b_{ll'}\mathbf{c}_{ll'}\mathbf{c}_{ll'}^H\right)\mathbf{p}}}{N^2\|\hat{\mathbf{p}}\|^4} \quad (13)$$

We are going to minimize (13) with the constraint that the sum of TX powers will be equal to a pre-defined total power value P_t. The denominator of the fraction in (13) is positive and independent of l and so can be neglected. We can also neglect the square root and formulate the power allocation problem as:

$$\begin{aligned} &\min_{\mathbf{p}} \ \max_{l \neq l'} \ \mathbf{p}^T\left(b_{ll'}\mathbf{c}_{ll'}\mathbf{c}_{ll'}^H\right)\mathbf{p} \\ &s.t. \quad \mathbf{1}_{M\times 1}^T\mathbf{p} = P_t, \quad \mathbf{p} \geq \mathbf{0} \end{aligned} \quad (14)$$

The cost function in (14) is the maximum of several convex functions. So it's also convex. The above problem can be reformulated by exploiting a dummy variable t as:

$$\begin{aligned} &\min_{\mathbf{p},t} \ t \\ &s.t. \quad \mathbf{1}_{M\times 1}^T\mathbf{p} = P_t; \quad \mathbf{p} \geq \mathbf{0}, \quad \mathbf{p}^T\left(b_{ll'}\mathbf{c}_{ll'}\mathbf{c}_{ll'}^H\right)\mathbf{p} \leq t \end{aligned} \quad (15)$$

The goal and the constraint functions of (15) satisfy the conditions of a convex problem which can be efficiently solved using different packages developed for solving convex programs such as CVX [22].

V. Simulation

Consider the CL-MIMO configuration presented at section III. Let $M = 10$ TX and $N = 12$ RX nodes be randomly located on a small disk of radius 10m following a uniform distribution for their ranges and their angles. Orthogonal Hadamard sequences with $L = 32$ are used as the transmitted waveforms. We consider a azimuth angle grid as $-7° : 0.05° : 7°$. Note that a small range of azimuth angles is considered here to keep the computational burden of the simulation low. $K = 3$ targets are assumed to be present in the angle grid at $-1°$, $0°$, and $2°$. Targets has been considered close to each other in the angle grid to make more difficult the recovery of the scene. The optimtal TX powers are obtained by solving (15) using CVX. The total power P_t is set to the number of transmiters M. We also consider uniform power allocation and power allocation scheme proposed in [13] for comparison. The proposed method in [13] uses the squared norm of the difference between the gram matrix of the sensing matrix and an identity matrix as the cost function. This cost in the case of orthogonal waveforms takes the form of the sum of squared inner products between cross columns of the sensing matrix. DOA estimation error is taken as evaluation criterion. We consider two different scenarios with different antenna locations (antenna locations are random). The antenna locations in these scenarios has been shown in Fig. 1. We conduct 1500 Monte Carlo trials for each scenario to obtain the estimation error of the angles of the targets. In each trial the complex reflection coefficients of targets β_ks are independently generated using a complex Gaussian distribution with zero mean and with covariance matrix $\Sigma_\beta = \frac{1}{2}\mathbf{I}_{2\times 2}$ which means $E\{|\beta_k|^2\} = 1$ (swerling case I model, as discussed in section III). The corresponding plots for root mean square error (RMSE) of DOA estimation versus SNR has been shown for the first and the second scenarios in Fig. 2 and Fig. 3, respectively. The RMSE is obtained by: $RMSE = \frac{1}{k}\sum_{k=1}^{K}\sqrt{\frac{1}{1500}\sum_{n=1}^{1500}(\hat{\theta}_{k,n} - \theta_k)}$ where $\hat{\theta}_{k,n}$ is the estimate of θ_k in nth trial. These estimates are obtained in each trial by selecting 3 dominant peaks of the recovered signal. Finally, SNR is defined as $1/\sigma^2$ where σ^2 is the power of complex Gaussian noise at the receivers. As can be seen form the curves, our proposed method for power allocation outperforms the uniform power allocation scheme and the method proposed in [13].

We also obtained the receiver operating characteristics (ROC) curves of the angle estimates for the first scenario for SNR = 10dB shown in Fig. 4. ROC curves have been obtained according to 5000 independent runs. Here P_d is defined as the probability that all three targets are detected (at the grid points they really exist) and P_{fa} is defined as the probability of target declaration in grid points in which there are actually no targets. It is also indicated by Fig. 4 that our method has better DOA estimation performance compared to two other methods.

VI. Conclusion

In this paper, we proposed a power allocation scheme in a colocated MIMO radar configuration employing sparse modeling, to improve the DOA estimation performance. To obtain optimal powers, a convex optimization problem was derived which minimizes the coherence of the sensing matrix with a total power constraint. Simulation results was shown that our proposed method have better estimation accuracy compared to existing methods.

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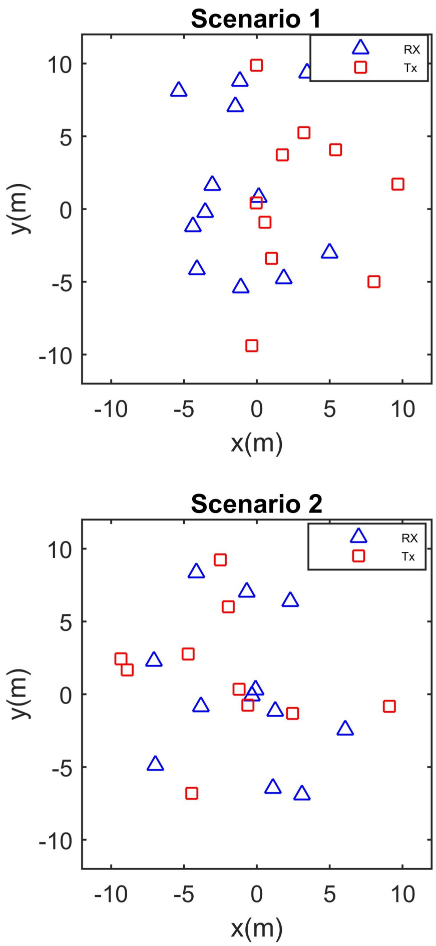

Fig. 1: Antenna locations in scenario 1 and scenario 2

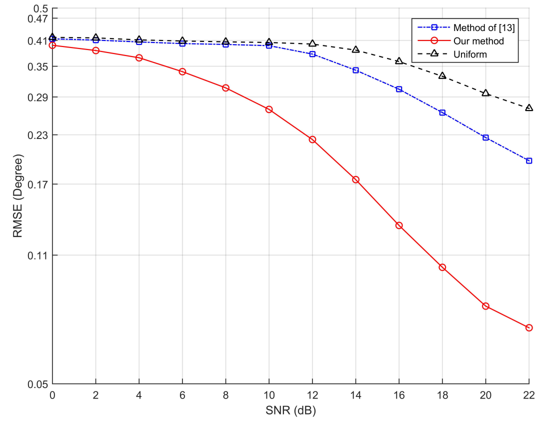

Fig. 3: RMSE of DOA estimation (Scenario 2)

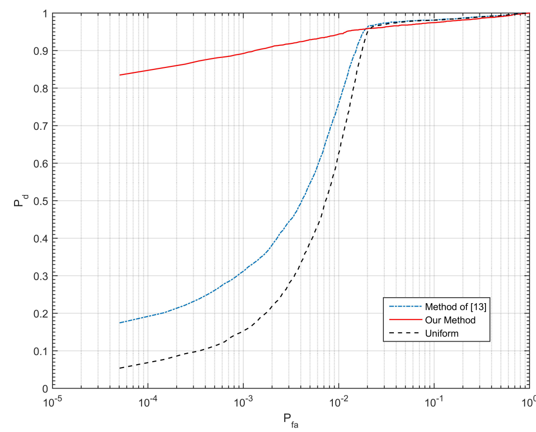

Fig. 4: ROC of angle estimates for the first scenario (SNR = 10dB

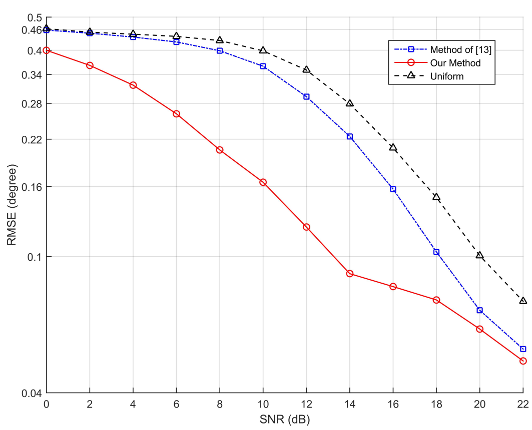

Fig. 2: RMSE of DOA estimation (Scenario 1)

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