Compressed Sensing Technique for Synchronization and Channel Estimation in OFDMA Uplink Transmissions

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***Abstract*—In this study joint estimation of the symbol timing offset (STO), and channel impulse response (CIR) of each active user under the presence of the carrier frequency offset (CFO) is considered for the uplink of an orthogonal frequency-division multiple access (OFDMA) system. A new method based on the compressed sensing (CS) framework using pilot symbols is proposed for the joint estimation of the STO and the CIR of each active user. Sparsity is achieved through incorporating the STO, the cyclic prefix (CP) samples, and the CIR coefficients into a new signal model. The proposed method does not require CIR coefficients to be sparse. Numerical results of the performance of the proposed method using the orthogonal matching pursuit (OMP) algorithm is presented. The presence of CTO is also considered as a perturbation to the CS dictionary of pilot symbols.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a favored multicarrier transmission technique over frequency selective multipath fading channels in broadband wireless communication networks due to reducing interblock interference (IBI) and providing high data rate transmission [1], [2]. Although OFDM allocates all subcarriers for the transmission of the symbols of a single user, it can be adapted for a multi user system by applying frequency division multiple access (FDMA) techniques. OFDMA (or OFDM-FDMA) allows a user to change its assigned subset of carriers to a different subset of carriers which may have better channel conditions [3]. However, OFDMA like OFDM is susceptible to both carrier frequency offsets (CFOs) and symbol timing offsets (STOs) making frequency and timing synchronization critical issues. CFO is almost always present since the oscillator frequencies of the receiver and the transmitter can never be perfectly aligned and also the receiver can be mobile causing Doppler spreading. The orthogonality amongst the subcarriers can no longer be maintained and so intercarrier interference (ICI) occurs [2], [3]. STOs result in IBI between adjacent OFDMA blocks. CFO and STO also degrades the quality of the channel estimates. Errors in timing, frequency and channel estimation ensue significant losses in the effective signal-to-noise ratio (SNR) at the receiver increasing the error probability of the system [4]. Time and frequency synchronization along with channel estimation must be performed for every user in the OFDMA system since the CFO, STO, and CIR of each user is unique.

There exits several solutions for frequency and timing estimation [5]–[8]. The cyclic prefix (CP) symbols are used in [6] in order to estimate both CFO and STO. Although this method provides accurate estimates, a filter bank is required at the base station (BS) for the separation of each user. Thus, this method can only be applied to subband based carrier assignment scheme (CAS) where a group of adjacent subcarriers is assigned to each user. [7] proposes a method that can be implemented in generalized CAS (GCAS), where there is no restriction in the selection of the subcarriers. The method in [7] estimates the CFO and the STO of a new user entering the OFDMA system but assumes that all existing users have already been synchronized. The maximum-likelihood (ML) estimation proposed in [8] can be used in GCAS. An alternating projection algorithm is used to reduce the multidimensional search of the exact ML solution into simple one-dimensional searches. The computational complexity of this method is higher when compared to other methods in [6].

This paper considers the estimation of the STO and the CIR of each active user in the uplink communication of an OFDMA system where the CFO is also present. The proposed method uses the compressive sensing (CS) framework. CS framework coined in [9], [10] aims to recover unknown signals observed from underdetermined system of linear equations. It is shown via CS that signals can be perfectly reconstructed using far fewer measurements than that of Nyquist theory's when they can be represented as sparse, only a few entries of the signal are nonzero, in a basis. The CS based sparse signal recovery methods have already been applied to estimate the CIR of the OFDM systems [11]–[13]. However it is assumed by these works either that the channel has a long delay spread and most of the coefficients of the CIR are zero or that the nonzero CIR coefficients occur at the same entries for several channel instantiantions. In our method the sparsity is exploited through the expansion of the CIR of each user with the STO and the CP

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symbols and so does not rely on the sparseness of the channel coefficients of the users. A training block of pilot symbols is required to create the CS dictionary and any sparse recovery algorithm can be used to recover both the STO and the CIR of the active users.

II. Signal Model for OFDMA Uplink

It is assumed that there are K active users communicating simultaneously with the BS in the uplink of an OFDMA network. The kth user sends a block of N symbols denoted as $X_k[m]$. Since the total number of subcarriers available to each user is N, the mth entry of $X_k[m]$ is nonzero if and only if the mth subcarrier is modulated by the kth user. Inverse discrete Fourier transform (IDFT) is performed on each user block

$$x_k[m] = \frac{1}{N}\sum_{m=0}^{N-1} X_k[m]e^{j2\pi m(n-N_g)/N}, n = N_g, ..., N_t - 1, \tag{1}$$

where N_t is the total number samples including the CP samples, N_g, added to the front of $x_k[m]$ to eliminate IBI. Then the time domain samples given in (1) are transmitted over the channel. The discrete time composite CIR of each user which includes the transmit/receive filters and the transmission medium can be written as $\mathbf{h}_k = [h_k[0], h_k[1], ..., h_k[L_k-1]]^T$, where $(\cdot)^T$ is the transpose operator and L_k is the channel length of the kth user. Since L_k cannot be known in practice, $\mathbf{h}_k$ is replaced by a $L_h \times 1$ dimensional vector $\mathbf{h}'_k = [\mathbf{h}_k^T \ \mathbf{0}^T_{(L_h-L_k)\times 1}]^T$ where $L_h \geq \max_k\{L_k\}$ is a parameter that depends on the duration of the transmit/receive filters and the maximum expected channel delay spread. The waveform arriving at the BS is given by the superposition of the signals from all active users. The received signal after passing through the channel can be written as

$$y[n] = \sum_{k=1}^{K}\{c_k[n]\left(h'_k[n] * x_k[n-\mu_k]\right)\} + v[n], \tag{2}$$

where the convolution operation denoted by $*$ is, $h'_k[n]*x_k[n-\mu_k] = \sum_{l=0}^{L_h-1} h'_k[l]x_k[n-l-\mu_k]$, and μ_k is the STO of the kth user expressed in sampling intervals, and the additive noise, $v[n]$, is complex Gaussian with zero mean and variance σ_v^2, and $c_k[n] = e^{j2\pi\epsilon_k n/N}$ where ϵ_k denotes the normalized CFO for the kth user. In order to see the effects of CFO, discrete Fourier transform (DFT) can be performed on the received signal (2)

$$Y[m] = \sum_{k=1}^{K}\{C_k[m] * (H'_k[m]X_k[m])\} + V[m], \tag{3}$$

where $C_k[m]$, $H'_k[m]$, $X_k[m]$ and $V[m]$ are DFT of $c_k[n]$, $h'_k[n]$, $x_k[n]$, $v[n]$ respectively. $C_k[m]$ is expressed as

$$C_k[m] = \frac{\sin(\pi(\epsilon_k - m))}{N\sin(\pi(\epsilon_k - m)/N)}e^{j\pi(\epsilon_k-m)(N-1)/N} e^{j(2\pi\epsilon_k N_g/N)}. \tag{4}$$

When the normalized CFO is greater than, $\epsilon_k \geq 0.5$, it comprises of two parts: an integer and a fractional part. The integer part of the normalized CFO introduces a cyclic shift of $Y[m]$ [4]. In this paper the normalized CFO for each user is assumed to consist of only the fractional part and so $|\epsilon_k| < 0.5$. (3) can be rewritten as

$$Y[m] = \sum_{k=1}^{K}\{C_k[0]H'_k[m]X_k[m] + I_k[m]\} + V[m], \tag{5}$$

where $I_k[m]$ is given as

$$I_k[m] = \sum_{r=1}^{N-1} C_k[r]H'_k[m-r]X_k[m-r]. \tag{6}$$

As it is seen from (5), the magnitude and phase of the term, $H_k[m]X_k[m]$, is affected due to CFO, $C_k[0]$. CFO also generates the ICI term in (6) from the other symbols. It is also assumed that a synchronization channel is used in the downwlink by each user to acquire timing before the uplink transmission can begin [7]. This assumption guarantees that the line-of-sight propagation delay is the only cause of the timing errors which cannot exceed $\mu_{\max}$.

The received samples given in (2) can be grouped into adjacent segments of length N_t each corresponding to an OFDMA block in the BS time reference. Then the CP samples are discarded and the remaining samples are collected into N dimensional vector $\mathbf{y}$ as

$$\mathbf{y} = \sum_{k=1}^{K}\mathbf{\Gamma}_k\mathbf{D}_k\mathbf{h}'_k + \mathbf{v}, \tag{7}$$

where the following quantities are defined as

$$\mathbf{\Gamma}_k = \mathrm{diag}(e^{j2\pi\epsilon_k N_g/N}, ..., e^{j2\pi\epsilon_k(N_g+N-1)/N}) \tag{8}$$

$$[\mathbf{D}_k]_{p,q} = [\mathbf{x}_k]_{|p-q-\mu_k|_N}, \quad 1 \leq p \leq N, 1 \leq q \leq L_h. \tag{9}$$

In (9), $[\mathbf{x}_k]_l$ denotes the lth entry of the vector $\mathbf{x}_k$ for $0 \leq l \leq N-1$ and the modulo-N operation $|p-q-\mu_k|_N$ means that $p-q-\mu_k$ is reduced to the interval $[0, N-1]$. The joint estimation of $\boldsymbol{\epsilon} = [\epsilon_1, ..., \epsilon_K]^T$, $\boldsymbol{\mu} = [\mu_1, ..., \mu_K]^T$ and $\mathbf{h}' = [\mathbf{h}'^T_1, ..., \mathbf{h}'^T_K]^T$ can be obtained through (7) by means of ML reasoning. However, this leads to a complex optimization problem over the $2K$ dimensional space spanned by $(\boldsymbol{\mu}, \boldsymbol{h})$.

III. Compressed Sensing Approach

Equation (7) can be rewritten to incorporate the STO, μ_k, into a new signal model encompassing the CIR as well

$$\mathbf{y} = \sum_{k=1}^{K}\mathbf{\Gamma}_k\mathbf{A}_k\boldsymbol{\xi}_k + \mathbf{v}, \tag{10}$$

where $\mathbf{A}_k$ and $\boldsymbol{\xi}_k$ are defined as

$$[\mathbf{A}_k]_{p,q} = [\mathbf{x}_k]_{|p-q|_N}, \quad 1 \leq p \leq N, 1 \leq q \leq N_g \tag{11}$$

$$\boldsymbol{\xi}_k = [\mathbf{0}^T_{\mu_k\times 1} \ \ \mathbf{h}'^T_k \ \ \mathbf{0}^T_{(N_g-\mu_k-L_h)\times 1}]^T. \tag{12}$$

In order to guarantee that the received vector (10) is not affected by IBI, the number of the CP symbols must satisfy

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$N_g \geq L_h + \mu_{\max}$. When the DFT is performed on the received samples, (10) becomes

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{C}_k \mathbf{X}_k \mathbf{F} \boldsymbol{\xi}_k + \mathbf{z}, \tag{13}$$

where $\mathbf{F}$ denotes the first N_g columns of the N point DFT matrix, $\mathbf{X}_k = \text{diag}\{X_k[0], ..., X_k[N-1]\}$, $\mathbf{z} = [V[0], ..., V[N-1]]$ and $\mathbf{C}_k$ is

$$\mathbf{C}_k = \begin{bmatrix} C_k[0] & C_k[N-1] & \dots & C_k[1] \\ C_k[1] & C_k[0] & \dots & C_k[2] \\ \vdots & \ddots & \ddots & \vdots \\ C_k[N-1] & C_k[N-2] & \dots & C_k[0] \end{bmatrix}. \tag{14}$$

For an OFDMA system with K active users equation (13) can be rewritten into folowing equivalent form

$$\mathbf{r} = \widetilde{\mathbf{Q}} \boldsymbol{\xi} + \mathbf{z}, \tag{15}$$

where $\widetilde{\mathbf{Q}}$ and $\boldsymbol{\xi}$ are given as

$$\widetilde{\mathbf{Q}} = [\mathbf{C}_1 \mathbf{X}_1 \mathbf{F}, \mathbf{C}_2 \mathbf{X}_2 \mathbf{F}, ..., \mathbf{C}_K \mathbf{X}_K \mathbf{F}] \tag{16}$$

$$\boldsymbol{\xi} = [\boldsymbol{\xi}_1^T, \boldsymbol{\xi}_2^T, ..., \boldsymbol{\xi}_K^T]^T. \tag{17}$$

By stacking K vectors of $\boldsymbol{\xi}_k$, the new unknown signal model, $\boldsymbol{\xi}$, consists of the STOs and the CIR of each user in the OFDMA system. As it can be observed from (12), the number of zero entries added due to the STO and CP symbols sparsify the vector $\boldsymbol{\xi}$ by a ratio of $\rho = (\sum_k L_k)/(KN_g)$. Since the CP length must always compensate for both STO and channel delay spread by design, $N_g \geq L_h + \mu_{\max}$, the sparsity ratio ρ can vary in the range of $\rho \in (0, 1)$ with larger ρ corresponding to more complex or less sparse signals. Lower sparsity ratios or more sparse signals can be obtained by increasing the CP length, N_g.

CS recovery methods search for a sparse representation of the unknown signal which fulfils the condition that the representation basis, $\widetilde{\mathbf{Q}}$, yields the observations

$$\begin{aligned} &\text{minimize} \quad ||\boldsymbol{\xi}||_0 \\ &\text{subject to} \quad \mathbf{r} = \widetilde{\mathbf{Q}} \boldsymbol{\xi} \end{aligned} \tag{18}$$

by minimizing the ℓ_0-pseudo-norm which is defined as the cardinality of the support set of the unknown signal. This optimization problem (18) is a nonconvex NP-hard problem since it involves an intractable combinatorial search. Sparse signal recovery methods can be grouped into two categories: greedy methods that are based on the matching pursuit algorithm [14], [15] and convex relaxation methods [9], [16]–[18] that are based on minimizing ℓ_1 norm of the unknown signal.

Sparsity promoting methods are functions of the data $(\mathbf{r}, \widetilde{\mathbf{Q}})$. Because of the additive noise term, $\mathbf{z}$, we know that our observations, $\mathbf{r}$, are perturbed. In addition to this the sensing matrix can also be considered as perturbed in the presence of CFO due to $\mathbf{C}_k$ matrices seen in (16). Since we do not have any prior knowledge about the CFO of each user in the OFDMA system, the sparsity promoting methods do not have access to the true CS matrix $\widetilde{\mathbf{Q}}$ and so they can only be functions of the data $(\mathbf{r}, \mathbf{Q})$ where $\mathbf{Q}$ can be written as

$$\mathbf{Q} = [\mathbf{X}_1 \mathbf{F}, \mathbf{X}_2 \mathbf{F}, ..., \mathbf{X}_K \mathbf{F}]. \tag{19}$$

The perturbation on the matrix $\mathbf{Q}$ can be interpreted as multiplicative noise due to the extra noise term which can be seen by substituting $\widetilde{\mathbf{Q}} = \mathbf{Q} + \mathbf{E}$ in (15) giving $\mathbf{E}\boldsymbol{\xi}$. The performances of the CS recovery methods under this general type of perturbations are studied in [19], [20].

The best case scenario of the perturbation analysis is the oracle case where both the CFOs, $\mathbf{C}_k$ matrices, and the support set of the unknown signal, $S = \{i \mid [\boldsymbol{\xi}]_i \neq 0\}$, are already known. Since $\widetilde{\mathbf{Q}}$ can be constructed now, the problem turns into the classical least squares problem where the solution is given as

$$\hat{\boldsymbol{\xi}}_S^{\#} = (\widetilde{\mathbf{Q}}_S^H \widetilde{\mathbf{Q}}_S)^{-1} \widetilde{\mathbf{Q}}_S^H \mathbf{r}. \tag{20}$$

Using the support S, it is straightforward to extend the solution, $\hat{\boldsymbol{\xi}}_S^{\#}$, to $\hat{\boldsymbol{\xi}}^{\#}$ by adding zeroes for the entries that are not on the support, $S^C = \{i \mid i \notin S\}$. The error for the oracle estimator [18] can be found as

$$E\{\|\hat{\boldsymbol{\xi}}^{\#} - \boldsymbol{\xi}\|_2^2\} = \sigma_v^2 \ \text{Tr}\{(\widetilde{\mathbf{Q}}_S^H \widetilde{\mathbf{Q}}_S)^{-1}\}. \tag{21}$$

The oracle bound (21), which also coincides with the Cramer Rao bound for the CIR estimates, can be used as a benchmark to compare the performances of the CS recovery methods since it represents the best case where the support of the unknown signal and the perturbation to the sensing matrix is given.

IV. Numerical Results

For simulations, an OFDMA system with $N = 128$ subcarriers is considered. The CIR of each user is generated with $L_k = \{8, 7, 6, 5\}$ paths. Independently and identically distributed complex Gaussian random variables with zero mean are used to generate the CIR coefficients. The power delay profile of the CIR is exponential, $E\{|h_k(l)|^2\} = \lambda_k e^{-l}$. In order to guarantee that the signal power of each user is unit, the constants of the power delay profile are computed as $\lambda_k = 1/(1 + e^{-1} + ... + e^{-L_k})$. There are $K = 4$ active users in the OFDMA uplink with 32 subcarriers reserved for each user. The values of the normalized CFO and STO for each user are set as $\epsilon_k = \{0.001, -0.0005, 0.003, -0.0002\}$, and $\mu_k = \{3, 2, 1, 1\}$ respectively. CP length is chosen as $N_g = 20$ to accommodate for both the CIR and the STO. For this CP value, the sparsity ratio of the signal in our model (15) becomes $\rho = 0.3375$. OMP algorithm is implemented as the sparse recovery method for the CS framework. The SNR is varied between 10dB and 40dB, and 10000 Monte Carlo iterations are run for each SNR value. Transmitted symbols are randomly chosen from quarternary phase shift keying (QPSK) constellation for each iteration and the subcarriers are randomly assigned for each user in accordance with the GCAS scheme. Figure 1 shows the mean squared error (MSE) of the STO estimates for each user in the OFDMA system versus varying SNR. The proposed method is able to produce estimates for STOs which improve as the SNR increases. The

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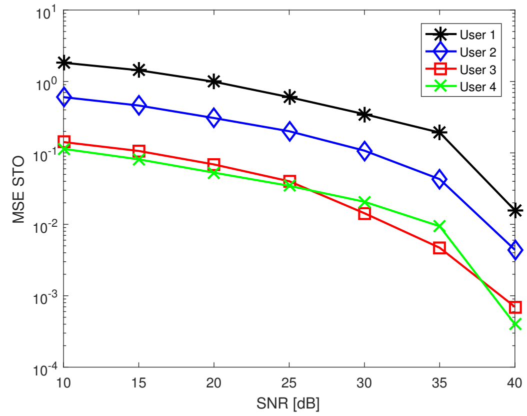

Fig. 1. MSE of STO estimates of each user versus SNR.

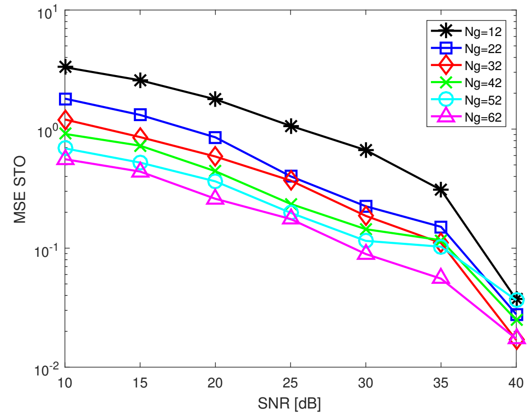

Fig. 3. MSE of STO estimates of user 1 with varying N_g.

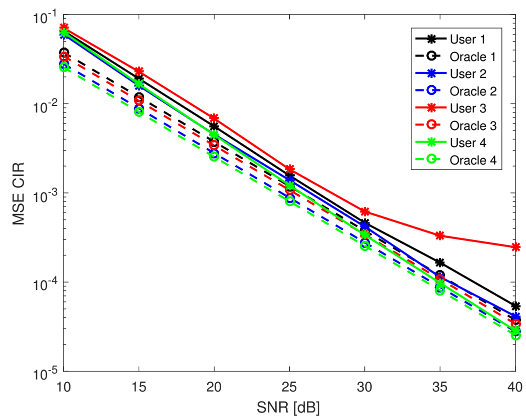

Fig. 2. MSE of CIR estimates of each user versus SNR.

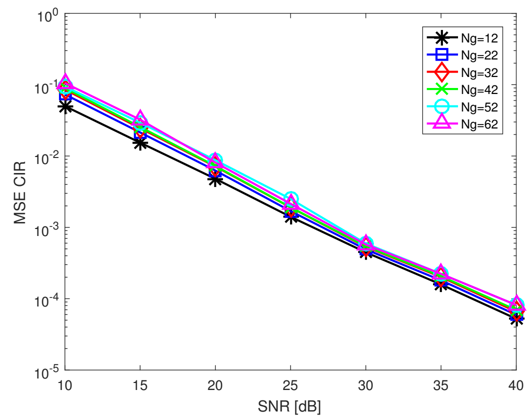

Fig. 4. MSE of CIR estimates of user 1 with varying N_g.

performance of the proposed estimator does not hit an error floor for any user across the given SNR range. The MSE of the estimates for users with greater STO such as user 1 and 2, is greater than the MSE of the other users with lower STOs. Figure 2 shows the MSE of the CIR estimates for each user in the OFDMA system. In order to compare the performance of the proposed CS method, the bound of the oracle estimator (20) is also given for each user. As it is observed from Figure 2, the proposed method provides estimates close to the oracle estimator (20) for users 1, 2 and 4 without using any prior knowledge about the CFO, STO or CIR of any of them. The estimate for user 3 reaches an error floor when the SNR becomes greater than 30dB. Since the CFO of user 3, $\epsilon_3 = 0.003$, is significantly greater than the CFO of other users, the multiplicative noise ends up being greater compared to other users which results in poor CIR estimation performance for user 3. It is also seen that the proposed method is able to produce robust CIR estimates for users 1, 2, and 4 although the CFO of user 3 is impairing the measurements of other users as well.

In the second simulation the channels generated for each user are kept same but the CP length, N_g, is varied between 12 and 62 (Figure 3). By changing the CP length, we also change the sparsity ratio of the unknown signal (15) from 0.5417 to 0.1048. As the CP length increases, the unknown signal becomes more sparse. The MSE of the STO estimates for user 1 with various CP lengths are shown in Figure 3. It is observed that increasing the CP length yields better estimates for the STO and thus decreases the MSE of the STO estimates. A drawback of the proposed method can be observed from Figure 4. Increasing the CP length also increases the the MSE of the CIR estimates.

V. Conclusion

The problem of estimating the STO and CIR of all active users with CFO in the uplink of an OFDMA system is investigated in this paper. We introduced a CS based framework to jointly estimate the STO and the CIR of each user. The proposed framework incorparates both the STO and CIR into the signal model and uses the CP symbols needed for the prevention of the IBI in order to exploit sparsity in this new signal model. By choosing the CP length, N_g, appropriately, this approach can sparsify the unknown signal in the proposed model by the ratio, ρ. Unlike other CS based CIR estimation methods, our method does not require the sparsity of the channel coefficients or channels with long delay spread. CS framework requires one block of training symbols to generate the sensing dictionary. Numerical results regarding the performance of the proposed method using the OMP algorithm are presented in the presence of CFO. Numerical results show that the proposed CS framework provides robust estimates under the presence of the CFO.

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