CFO and Channel Estimation for MISO-OFDM Systems

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***Abstract*—This study deals with the joint channel and carrier frequency offset (CFO) estimation in a Multiple Input Single Output (MISO) communications system. This problem arises in OFDM (Orthogonal Frequency Division Multiplexing) based multi-relay transmission protocols such that the geo-routing one proposed by A. Bader et al in 2012. Indeed, the outstanding performance of this multi-hop relaying scheme relies heavily on the channel and CFO estimation quality at the PHY layer. In this work, two approaches are considered: The first is based on estimating the overall channel (including the CFO) as a time-varying one using an adaptive scheme under the assumption of small or moderate CFOs while the second one performs separately, the channel and CFO parameters estimation based on the considered data model. The two solutions are analyzed and compared in terms of performance, cost and convergence rate.**

I. Introduction

Recently, an efficient beaconless geo-routing based multi-hop relaying protocol, namely OMR (OFDM-based Multi-hop Relaying) protocol, has been proposed in [1], [2]. As for other existing geo-routing protocols, in OMR the nodes can locally make their forwarding decisions using very limited knowledge of the overall network topology. Relaying decisions in OMR are taken in a distributed fashion at any given hop based on location information, in order to alleviate the overhead which rapidly grows with node density. In addition, to deal with the fact that the proposed paradigm leads to the creation of multiple copies of the same packet with different propagation delays, OMR relies on the orthogonal frequency division multiplexing (OFDM) which allows correct packet detection at a receiving node thanks to the use of the cyclic prefix (see [1] for more details).

In [2] and [1], it has been shown that the OMR overcomes existing contention based geo-routing relaying protocols in terms of end-to-end performance (throughput and time-space footprint). However, the performance analysis in [1], [3] relies on the assumption of perfect frequency synchronization between the nodes.

In standard OFDM systems, it is well known that frequency desynchronization leads to a carrier frequency offset (CFO) at the receiver node which deteriorates significantly the decoding performance. Fortunately, this problem is well mastered and many solutions exist to track and correct this CFO effect [4], [5].

The existing solutions from the literature are not adequate for our case, as we have several simultaneous transmitters (i.e. we have a particular MISO system where all relays transmit the same data packet through different channels) each with its own CFO and channel. The aim of this study is to provide solutions to this severe problem in order to preserve the end-to-end high performance of the OMR protocol.

II. MISO-OFDM communication system model

Consider an OFDM system with K subcarriers and using a cyclic prefix of length L larger than the channel impulse response size N. Assume the received signal is affected by a carrier frequency offset[1] (due generally to desynchronization between the transmitter and receiver's local oscillators). Then, for one single transmitter, after sampling and removing the guard interval, the received discrete baseband signal at time n_s (associated with the n_s-th OFDM symbol) is given by [5]:

$$\mathbf{y}(n_s) = \mathbf{\Gamma}(n_s)\frac{\mathbf{F}^H}{\sqrt{K}}\mathbf{H}\mathbf{x}(n_s) + \mathbf{v}(n_s) \tag{1}$$

where $\mathbf{y}(n_s) = [y_0(n_s),\ \cdots,\ y_{K-1}(n_s)]^T$, and $\mathbf{x}(n_s) = [x_0(n_s),\ \cdots,\ x_{K-1}(n_s)]^T$ ($x_k(n_s)$ being the transmitted symbol at time n_s and subcarrier k). The noise $\mathbf{v}(n_s)$ at time n_s, is assumed to be additive white Circular Complex Gaussian (CCG) satisfying $E\left[\mathbf{v}(k)\mathbf{v}(i)^H\right] = \sigma_{\mathbf{v}}^2\mathbf{I}_K\delta_{ki}$; $(.)^H$ being the Hermitian operator; $\sigma_{\mathbf{v}}^2$ the noise variance; $\mathbf{I}_K$ the identity matrix of size $K \times K$ and δ_{ki} the Dirac operator.

The channel frequency response matrix $\mathbf{H}$ of size $K \times K$, where channels are assumed constant over the packet transmission period is defined as:

$$\mathbf{H} = diag\left\{\frac{\mathbf{W}}{\sqrt{K}}\bar{\mathbf{h}}\right\} = diag\left\{H_0,\ \cdots,\ H_{K-1}\right\}, \tag{2}$$

H_k is the channel frequency response at the k-th subcarrier. $\bar{\mathbf{h}} = [h(0), \cdots, h(N-1)]^T$, $\mathbf{F}$ is the $(K \times K)$ Discrete Fourier Transform matrix; $\mathbf{W}$ the N first columns of $\mathbf{F}$; and

[1] In this study, the effect of time desynchronization is neglected.

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$\mathbf{\Gamma}(n_s)$ the normalized CFO matrix of size $K \times K$ at the n_s-th OFDM symbol given by:

$$\Gamma(n_s) = e^{j2\pi\phi n_s} diag\left\{1, \cdots, e^{j2\pi\phi(K-1)/K}\right\}. \quad (3)$$

$\phi = \Delta f \times T_s$ is the normalized CFO where Δf is the CFO and T_s is the symbol period.

Now, considering a MISO system where N_t nodes transmit simultaneously the same data to a single node, the received signal in (1) becomes:

$$\mathbf{y}(n_s) = \sum_{i=1}^{N_t} \mathbf{\Gamma}_i(n_s) \frac{\mathbf{F}^H}{\sqrt{K}} \mathbf{H}_i \mathbf{x}(n_s) + \mathbf{v}(n_s) \quad (4)$$

one can write equation (4) as:

$$\mathbf{y}(n_s) = \sum_{i=1}^{N_t} \mathbf{\Gamma}_i(n_s) \frac{\mathbf{F}^H}{\sqrt{K}} \mathbf{X}(n_s) \mathbf{h}_i + \mathbf{v}(n_s), \quad (5)$$

where

$$\begin{array}{ll} \mathbf{X}(n_s) = diag\left\{x_0(n_s), \cdots, x_{K-1}(n_s)\right\} \\ \mathbf{h}_i \quad\;\; = \left[H_{i,0}, \cdots, H_{i,K-1}\right]^T \\ \mathbf{\Gamma}_i(n_s) = e^{j2\pi\phi_i n_s} diag\left\{1, \cdots, e^{j2\pi\phi_i(K-1)/K}\right\}. \end{array} \quad (6)$$

$H_{i,k}$ refers to the frequency response of the i-th channel at the k-th frequency. Equation (5) can be re-written as :

$$\mathbf{y}(n_s) = \bar{\mathbf{H}}(n_s)\mathbf{x}(n_s) + \mathbf{v}(n_s), \quad (7)$$

where:

$$\bar{\mathbf{H}}(n_s) = \sum_{i=1}^{N_t} \mathbf{\Gamma}_i(n_s) \frac{\mathbf{F}^H}{\sqrt{K}} \mathbf{H}_i \quad (8)$$

III. Non-Parametric Channel Estimation

Since the transmitted data is common to all nodes, we consider in this approach the N_t channels with their CFOs as one global time varying channel given in (8). Let us assume a slow channel variation (i.e. small CFOs), in such a way the global channel is considered approximately constant over few OFDM symbols. In this case, and after doing the FFT, equation (5) can be approximated by :

$$\mathbf{y}(n_s) = \mathbf{X}(n_s)\mathbf{h} + \mathbf{v}, \quad (9)$$

$\mathbf{h}$ is the equivalent global time-varying channel vector corresponding to (8).

The channel estimation is performed using N_p pilot OFDM symbols[2],

Under Gaussian noise assumption, the (LS) Least Squares (LS coincide with the optimal Maximum Likelihood (ML) estimator in that case) estimation of $\mathbf{h}$ is given by:

$$\hat{\mathbf{h}} = \left(\mathbf{X}_p{}^H \mathbf{X}_p\right)^{-1} \mathbf{X}_p{}^H \mathbf{y}_\mathbf{p}. \quad (10)$$

Where $\mathbf{y}_p = \left[\mathbf{y}(1)^T \cdots \mathbf{y}(N_p)^T\right]^T$ and $\mathbf{X}_p = \left[\mathbf{X}(1)^T \cdots \mathbf{X}(N_p)^T\right]^T$.

[2]We assume the channel approximately invariant over the pilot sequence duration.

This algorithm can be implemented efficiently in the following way:

1) It is initialized by sending N_p successive pilot symbols.
2) Use the estimated channel for the equalization and detection of the current data symbol.
3) Then, pilots are replaced in (10) by the decided symbols using a sliding window of size N_p and following a decision directed approach, i.e. one replaces $\mathbf{X}(n_s)$ by $\hat{\mathbf{X}}(n_s)$ the decided symbol at time n_s.

The latter estimation method is valid only if the CFOs are small valued in which case the previous algorithm leads to good channel and symbol detection performance[3].

For the most general case where the CFO values are non controllable and not necessarily small, we propose next a more complex but more adequate method for the estimation of the global channel parameters.

IV. Parametric Channel Estimation

In the case of relatively large CFO values, the slow channel variation assumption is violated and the previous solution fails to provide an appropriate channel estimate. In that case, we need to resort to the direct estimation of the channel parameters (i.e. CFOs and channel impulse responses). Based on the data model in (5), one can use a Maximum Likelihood (ML) method for the estimation of the desired parameters. However, the ML cost function being highly non linear, we consider instead a reduced cost estimation method where we neglect the phase variation along one OFDM symbol, so that one can approximate:

$$\mathbf{\Gamma}_i(n_s) \approx e^{j2\pi\phi_i n_s} \mathbf{I}_K \quad (11)$$

Equation (11) leads to the approximate noise free model

$$\mathbf{y}(n_s) \approx \frac{\mathbf{F}^H}{\sqrt{K}} \mathbf{X}(n_s) \tilde{\mathbf{h}}(n_s), \quad (12)$$

where $\tilde{\mathbf{h}}(n_s) = \sum_{i=1}^{N_t} \mathbf{h}_i e^{j2\pi\phi_i n_s}$ refers to the equivalent time varying channel.

Now, by definition, the channel vector $\mathbf{h}_i$ represents the frequency response coefficients of the i-th channel, i.e. $\mathbf{h}_i = \mathbf{W}\bar{\mathbf{h}}_i \big/ \sqrt{K}$. One can rewrite $\tilde{\mathbf{h}}(n_s)$ in matrix form as:

$$\begin{aligned} \tilde{\mathbf{h}}(n_s) &= \tfrac{\mathbf{W}}{\sqrt{K}} \left[\bar{\mathbf{h}}_1, \cdots, \bar{\mathbf{h}}_{N_t}\right] \mathbf{e}(n_s) \\ &= \tfrac{\mathbf{W}}{\sqrt{K}} \bar{\mathbf{h}}(n_s), \end{aligned} \quad (13)$$

where $\mathbf{e}(n_s) = \left[e^{j2\pi\phi_1 n_s}, \cdots, e^{j2\pi\phi_{N_t} n_s}\right]^T$ and $\bar{\mathbf{h}}(n_s) = \left[\bar{\mathbf{h}}_1, \cdots, \bar{\mathbf{h}}_{N_t}\right] \mathbf{e}(n_s)$.

The estimate of the channel impulse response $\bar{\mathbf{h}}(n_s)$ can be easily obtained in the LS sense (using pilot symbols) as follows:

$$\mathbf{z}(n_s) = \frac{\mathbf{W}^H}{\sqrt{K}} \mathbf{X}(n_s)^{-1} \frac{\mathbf{F}}{\sqrt{K}} \mathbf{y}(n_s) \approx \bar{\mathbf{h}}(n_s) \quad (14)$$

[3]This suggests that one should consider a rough frequency synchronization between all nodes by exchanging for example a known and comon tone signal that can be used to mitigate the frequency offsets.

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By using N_p successive OFDM pilots, one can hence estimate:

$$\begin{aligned}\mathbf{Z} &= [\mathbf{z}(1), \cdots, \mathbf{z}(N_p)] \\ &\approx [\bar{\mathbf{h}}_1, \cdots, \bar{\mathbf{h}}_{N_t}] \begin{bmatrix} e^{j2\pi\phi_1} & \cdots & e^{j2\pi N_p\phi_1} \\ \vdots & \ddots & \vdots \\ e^{j2\pi\phi_{N_t}} & \cdots & e^{j2\pi N_p\phi_{N_t}} \end{bmatrix} \\ &= \breve{\mathbf{H}}\mathbf{E}^H \end{aligned} \quad (15)$$

From the rows of matrix $\mathbf{Z}$, one can obtain an estimate of the channels CFO while the column vectors provide an estimate of the channel impulse responses. Since, in general the CFO values are relatively small and hence closely separated and the sample size (i.e. N_p) is small too, one needs to use high resolution techniques for the frequency estimation. One can use ESPRIT[4] method to estimate the frequencies. To this end, by performing a regular SVD decomposition on the composite matrix $\mathbf{Z}$ one can write

$$\mathbf{Z} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \quad (16)$$

where, $\mathbf{V} : N_p \times N_t$ is a matrix of principal right singular vectors[5]. Since $\mathbf{E}$ and $\mathbf{V}$ span the same subspace (i.e. the row space of $\mathbf{Z}$), one can write $\mathbf{V} = \mathbf{E}\mathbf{Q}$, where $\mathbf{Q} : N_p \times N_p$ is a non singular unknown matrix.

Let $\mathbf{V}_1 = \mathbf{V}$ (without the last row) and $\mathbf{V}_2 = \mathbf{V}$ (without the first row), then

$$\mathbf{V}_1 = \mathbf{E}_1\mathbf{Q}, \qquad \mathbf{V}_2 = \mathbf{E}_2\mathbf{Q} \quad (17)$$

where, $\mathbf{E}_1 = \mathbf{E}$ without the last row and $\mathbf{E}_2 = \mathbf{E}$ without the first row. Hence, one can express $\mathbf{V}_2$ in terms of $\mathbf{E}_1$ as follows

$$\mathbf{E}_2 = \mathbf{E}_1\mathbf{\Phi}, \quad \mathbf{\Phi} = diag\left\{e^{-j2\pi\phi_1}, \cdots, e^{-j2\pi\phi_{N_t}}\right\} \quad (18)$$

Considering equations (17) and (18), we write $\mathbf{V}_2$ as:

$$\mathbf{V}_2 = \mathbf{E}_1\mathbf{\Phi}\mathbf{Q} \quad (19)$$

by evaluating $\mathbf{\Psi}$ as

$$\mathbf{\Psi} = \mathbf{V}_1{}^{\#}\mathbf{V}_2 = \mathbf{Q}^{-1}\mathbf{\Phi}\mathbf{Q} \quad (20)$$

where $(.)^{\#}$ refers to the pseudo-inverse operator. $\mathbf{\Phi}$ is estimated as the matrix of eigenvalues of $\mathbf{\Psi}$ and the CFOs are obtained from the phase arguments of the eigenvalues. Once $\mathbf{\Phi}$ is obtained, one can estimate $\breve{\mathbf{H}}$ as

$$\breve{\mathbf{H}} \approx \mathbf{Z}\big(\mathbf{E}^H\big)^{\#} \quad (21)$$

Remarks:

1) ESPRIT is an expensive method and can be replaced by Fourier search if the CFOs, are not too close as compared to the resolution limit of the DFT, i.e. $|(\phi_i - \phi_j)| \geq \frac{2}{N_p}$.
2) The channel and CFO estimates in (20) and (21) can be used to initialize a numerical method for ML optimization (e.g. for example with Levenberg-Marquardt method [7]) in order to improve the estimation performance, especially when the approximation in (11) is roughly satisfied.

[4]ESPRIT stands for Estimation of Subspace Parameters via Rotational Invariance Technique [6].

[5]We assume here that $N_p > N_t$ and that the CFOs are distinct, $\phi_i \neq \phi_j$ if $i \neq j$.

V. Simulations results

This section analyzes the channel estimation performance for the considered MISO-OFDM wireless system. The training sequence used in this paper is the Zadoff-Chu sequence considered in the LTE standard [8]. Fig. 1 represents the block-type pilot arrangement adopted in this work. Each field (or pilot) is represented by one OFDM symbol ($K = 64$ samples) where a CP ($L = 16$ samples) is added at its front. Simulation parameters are summarized in Table I.

The Signal to Noise Ratio (SNR) associated with pilots at the reception is defined as $SNR_p = \frac{\|\mathbf{X}_p\mathbf{h}\|^2}{KN_p\sigma^2_{\mathbf{v}}}$. The SNR, denoted SNR_d (in dB), associated with data is given by: $SNR_d = SNR_p - (Px_p - Px_d)$ where Px_p (respectively Px_d) is the power of pilots (respectively data) in dB.

Fig. 2 compares the Normalized Mean Square Error (NMSE) of the estimated data (related to he considered channel estimation methods followed by linear zero-forcing equalization) versus SNR_p at relatively low CFO. The NMSE curves show that the parametric method and the non-parametric one have similar performance in this context (for comparison, the plot in blue represents the CFO free context, while the magenta plot is for the channel estimate obtained by 'ignoring' the CFO effect).

One can observe also that the gap with CFO free context increases with the SNR which motivates for considering the ML or other advanced estimation approaches in future works to improve the estimation performance. Fig. 3 presents comparative results but for the symbol error rate with BPSK modulated signal.

In Figs. 4 and 5, we consider a similar experiment but for high CFO values. In that case the non-parametric approach is not adequate and does not allow correct detection of the data symbols. As in the previous figure, we still observe a large performance gap between the cases with and without CFO suggesting the use of more elaborated methods to compensate this performance loss.

In Figs 6, 7 and 8, we evaluate the Normalized Root Mean Squares Error (NRMSE) of the channel estimate versus the SNR or the pilot sequence size N_p. It is observed that for large SNR or large number of pilot symbols, the parametric approach performance improves significantly. Also, its performance for high CFO values is slightly better than for low CFOs due to the improved frequency resolution. On the other hand, the estimation quality of the non-parametric solution becomes worse for larger training sequences since the assumption that the channel remains invariant over all the pilot duration is ill satisfied when N_p increases.

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Parameters	Specifications
Channel model	Cost 207
Number of transmit antennas	$N_t = 3$
Number of receive antennas	$N_r = 1$
Channel length	$N = 4$
Number of pilot OFDM symbols	$N_p = 4$
Number of data OFDM symbols	$N_d = 5$
Pilot signal power	$P_{x_p} = 23$ dBm
Data signal power	$P_{x_d} = 20$ dBm
Number of sub-carriers	$K = 64$

TABLE I: Simulation parameters.

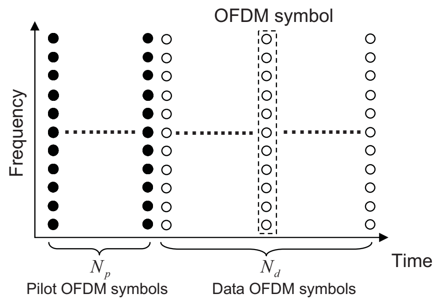

Fig. 1: Block-type pilot arrangement.

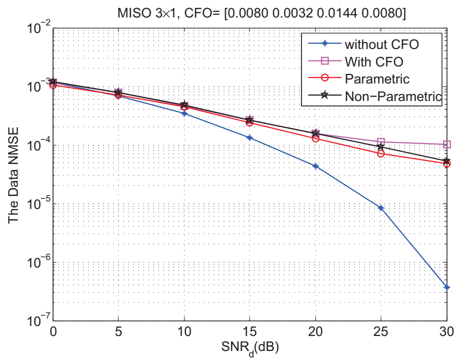

Fig. 2: NMSE of the data versus SNR_d (with and without CFO) at low CFO

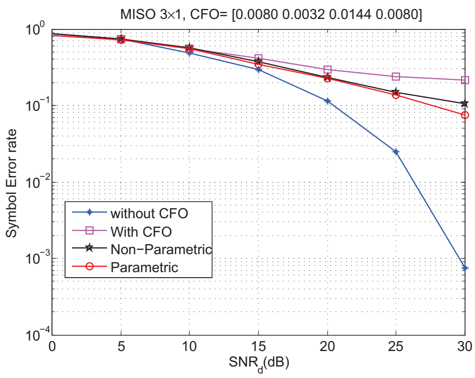

Fig. 3: Symbol error rate versus SNR_d (with and without CFO) at low CFO

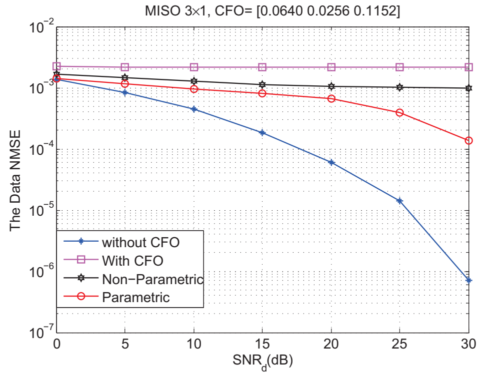

Fig. 4: NMSE of the data versus SNR_d (with and without CFO) at high CFO

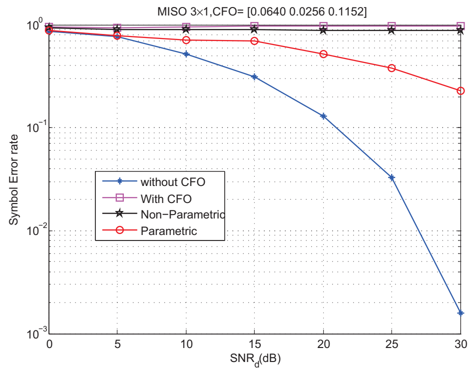

Fig. 5: Symbol error rate versus SNR_d (with and without CFO) at high CFO

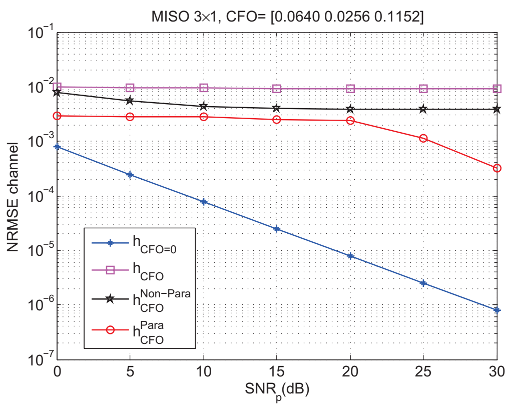

Fig. 6: NRMSE of the channel estimation versus SNR (with and without CFO).

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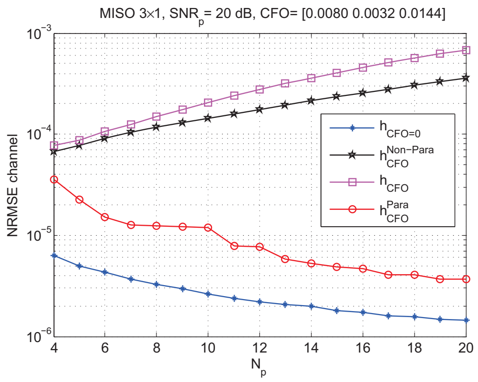

Fig. 7: NRMSE of the channel estimate versus N_p at low CFO.

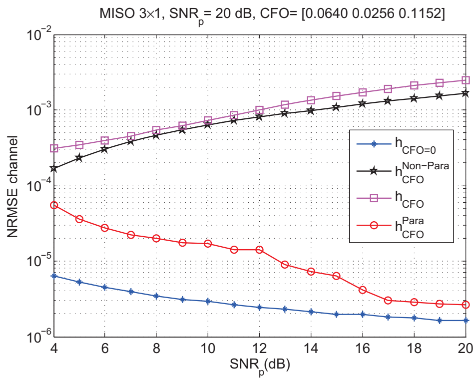

Fig. 8: NRMSE of the channel estimate versus N_p at high CFO.

VI. Conclusion

Based on the above theoretical study as well as on the experimental set-up of Dr Mohamed Tlich (not presented here)[9], we can draw the following remarks:

In this study we proposed a first solution for the channel+CFO estimation that is relatively cheap but can be used only if a rough frequency synchronization between all nodes is available to guarantee the small values of the CFOs and consequently the slow channel variation needed in this approach.

A second solution is provided based on parametric estimation. It is more expensive in terms of computational resources and pilots (i.e. requires longer pilots) but can work without any frequency synchronization.

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