Optimal Information Ordering for Sequential Detection with Cognitive Biases

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***Abstract*—The manner and order in which data is presented to a human observer can lead the human to make dramatically different decisions. This raises a question on how to best present data to an observer to achieve the best decision-making performance and minimum adverse effects. In this paper, we present a general framework to model cognitive biases that interfere in the human decision making process. We examine the problem of ordering observations in binary sequential detection. Our treatment considers the limited cognitive effort exerted by a decision-maker and the effect of the observations along with their distributions on the stopping time and accuracy of the sequential test. The complexity of the ordering algorithm is linear in the size of the observation set. Both the average time to make a decision and the probability of decision error are minimized.**

Index — Information Display, Decision-making, Sequential Test, Cognitive biases, Cognitive Effort

I. Introduction

Humans process information sequentially and the order in which they examine information can influence their decision. Different orderings affect both the accuracy of the decision and the time it takes to make a decision [1]. Information pieces are collected one after another, and once a sufficient level of confidence is achieved a decision is made. It is thus important to select the correct ordering of information to be presented to a human to promote "optimal" outcomes. The fact that the design of data display modifies the behavior and reasoning of a person is revealed in several domains. Institutions of the civil society conduct campaigns in which they properly choose their slogan, messaging tools and activities to initiate an action towards the public welfare. A doctor selectively presents the test results to the patient to influence the psychological state of the patient and the willingness to follow a certain treatment. Ads are often designed to emphasize exclusive features of a product over others for the private benefit of the business.

This work derives the proper ordering of the observations in binary sequential detection problems so that a decision on the true hypothesis is made quickly on average with a minimum probability of error by mitigating the effects of cognitive biases. A byproduct for ordering the observations is the identification of what minimum subset of a large dataset should be selectively displayed to an observer in order to initiate a decision but still minimize its error probability. While sequential detection is well-explored in the literature [2] [3], it is evident in the above examples that the decision-maker is a human rather than a machine. The two are different in several aspects. For instance, a machine considers all the sample distributions when setting the thresholds of the sequential test, but this cannot be expected for a human of limited processing capabilities. In addition, while the machine designs its decision thresholds by evaluating the sampled distributions, a human is highly affected by the observations themselves even under rational processing. Therefore the same N observations presented to a machine in different arbitrary orders do not alter its decision, which may not be the case for a human observer. The work is thus meant to be a first step towards accounting for cognitive biases at the time of designing the information display. The distinction made on purpose between the sampler of the data and the decision-maker is to push it forwards towards man-and-machine symbiosis.

The problem of ordering the observations in sequential detection is treated in [4]. The best ordering is defined to be the one that minimizes the average sample number (ASN). In that reference, the optimal ordering is determined a priori based on the known distributions of the data under the two hypotheses. In contrast, here we examine the real time ordering problem of an available observation set. A similar work to [4] is found in [5] and [6] except that a cognitive bias model is associated with the sequential test. In [7], the authors study the ordering of test items and its effect on the ability of humans to classify subsequent items presented to them. The purpose behind the work is to advise new learning models for humans in classification problems. In [8] and [9], the sequential acquisition of different information is assumed to be used by a rational decision-maker to update utility functions according to which the latter makes selections among various products. The framework is then extended to analyze how economic information signals propagate through the market.

This paper makes two main contributions. First, we propose a new formulation for the sequential test, which may provide a better model for human decision-making as its thresholds capture the time-varying and observation-dependent user's risk profile. Second, we show how to optimally order the N observations for sequential testing with minimal error and ASN by inspecting at most N permutations of the $N! = N \times (N-1) \times \cdots \times 1$ possible permutations of the observations. This is a major breakthrough as previous research indicated that the optimal solution of a real time observation selection problem for the traditional sequential test has combinatorial complexity [10]. The paper is organized as follows. In Section II a formal statement of the problem is made. In Section III we present the human sequential test model. The ordering of the observations is studied in Section IV. A human reaction to sampled information is simulated against a sequential probability ratio test (SPRT) in Section V. Section VI concludes the paper.

II. Problem Statement

Available is a set of information that is assumed to convey one of two possible hypotheses H_1 and H_0. The information should be optimally displayed to the human observer to minimize an objective function that captures the average

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time that the observer will take to make a decision and the cost of making an incorrect decision. We recognize that different observers have different mental capabilities and prior experiences. Therefore, an estimate of the observer's reaction to the information display or way of processing the different items along with the parameters of any fitted bias model are assumed to be obtained through prior testing and interviewing. The problem becomes as follows:

Given a vector of samples $Y_1, \ldots, Y_N$ of respective distributions $f_1(Y_1|H_k), \ldots, f_N(Y_N|H_k)$ under hypothesis $H_k, k \in \{0,1\}$. The designer of the information display presents samples sequentially and in a predetermined order to the subject who is only aware of the sample distributions. The selected order is designed to minimize the average time the observer takes to make a decision matching his or her risk profile roughly known by the designer. *The observer is assumed to be unaware of the fact that information is being reordered and therefore uses the original density functions of the observations.*

III. Human Sequential Test

In this section we define a novel sequential test based on which the selection and ordering of the samples are derived. The test is novel in the sense that it utilizes both the past samples and their distributions to design thresholds for future samples. Still, it does not run into the computation of joint distributions that will overload a human and are typically avoided in a natural thinking process . The test also directly reflects the impact of observation ordering on its thresholds and thus the stopping time. Since humans adjust their decision thresholds periodically as more information is collected, the test stands as a candidate model for human decision-making.

In a Bayesian setting, a log-likelihood term l_i of Y_i is defined as

$$l_i = \ln\left(\frac{f_i(Y_i|H_1)}{f_i(Y_i|H_0)}\right) \tag{1}$$

Given the independence of the observations, the cumulative log-likelihood metric up to time i is given by

$$L_i = \sum_{k=1}^{i} l_k \tag{2}$$

For independent identically distributed (iid) observations Y_i, SPRT is the optimal sequential test. Its higher and lower thresholds are maintained constant and given respectively by $B = \ln\left(\frac{1-P_M}{P_F}\right)$ and $A = \left(\frac{P_M}{1-P_F}\right)$. P_F is the design probability of a false alarm and P_M is the design probability of a miss. Generalized SPRT (GSPRT) is optimal for non-iid observations and has time-varying thresholds A_i and B_i.

There are two limitations for using a Bayesian test to model human-decision making. First the update of the sufficient statistic in (2) does not model how humans generally update their beliefs. While we define the human sequential test and the observation-ordering algorithm according to (2), we show how they can be extended to more general belief models in Section IV-C. The second limitation, to be treated in this section, is that Bayesian tests like SPRT and GSPRT derive their thresholds from the joint distribution of L_i over time i and do not consider the particular values of the observations Y_i in their computation. On one hand, an observer with constraints on time and thinking and lack of knowledge of the upcoming samples aborts such computations. On the other hand, these computations do not accurately reflect the human thinking process. For instance, in [11] an interviewer aims at estimating the willingness-to-pay (WTP) of a person by inquiring a response to a sequence of bid values. It is shown that the order in which the bid values are presented to the interviewee actually modifies the decision-threshold according to which the interviewee responds with 'yes' or 'no' for the presented values. Since this is the case even for the same sequence of bid values, decision-thresholds have to be observation-dependent.

Let p_{i-1} denote the prior probability of H_1 at the start of step $i-1$. Upon intaking observation Y_{i-1}, prior p_i can be updated recursively:

$$p_i = \frac{p_{i-1} \times f_{i-1}(Y_{i-1}|H_1)}{(1-p_{i-1}) \times f_{i-1}(Y_{i-1}|H_0) + p_{i-1} \times f_{i-1}(Y_{i-1}|H_1)} \tag{3}$$

Note the dependence of p_i only on observations up to time step $i-1$. We re-define the instantaneous probabilities of a false alarm and a miss $\hat{P}_{F_i}$ and $\hat{P}_{M_i}$ to include the updated priors. Let $F'_i(L_i|H_j, L_{i-1})$ be the cumulative distribution function (cdf) of L_i under hypothesis $H_j, j \in \{0,1\}$, conditioned on L_{i-1}. We are allowed to condition on L_{i-1} since observations $Y_1, \ldots, Y_{i-1}$ are known at step i. The instantaneous probabilities of false alarm and miss $\hat{P}_{F_i}$ and $\hat{P}_{M_i}$ are given by

$$\hat{P}_{F_i} = (1-p_i) \times (1 - F'_i(B_i|H_0, L_{i-1})) \tag{4}$$

$$\hat{P}_{M_i} = p_i \times F'_i(A_i|H_1, L_{i-1}) \tag{5}$$

$\hat{P}_{F_i}$ and $\hat{P}_{M_i}$ represent the observer's risk profile. By maintaining $\hat{P}_{F_i}$ and $\hat{P}_{M_i}$ constant over i, past observations will alter the prior p_i in (4) and (5) and consequently thresholds B_i and A_i. Therefore the test thresholds become directly correlated with the observer's actual decision thresholds since they capture both the observer's risk profile and the impact of the past observations on making a decision. Note that different observation sequences up to stage i yield different thresholds and may produce different outcomes.

Let $F_i(l_i|H_j)$ be the cdf of l_i under hypothesis $H_j, j \in \{0,1\}$. We have

$$\begin{aligned} F'_i(X|H_j, L_{i-1}) &= \text{Prob}\left(L_i \le X|H_j, L_{i-1}\right) \\ &= \text{Prob}\left(L_{i-1} + l_i \le X|H_j, L_{i-1}\right) \\ &= \text{Prob}\left(l_i \le X - L_{i-1}|H_j, L_{i-1}\right) \\ &= F_i\left(X - L_{i-1}|H_j\right) \end{aligned} \tag{6}$$

where the second equality in (6) follows from (2). Define

$$B'_i = B_i - L_{i-1} \tag{7}$$

$$A'_i = A_i - L_{i-1} \tag{8}$$

Then using (4), (5) and (6) we have

$$\hat{P}_{F_i} = (1-p_i) \times (1 - F_i(B'_i|H_0)) \tag{9}$$

$$\hat{P}_{M_i} = p_i \times F_i(A'_i|H_1) \tag{10}$$

Consequently, at every step i we have

$$B'_i = {F_i}_c^{-1}\left(\frac{\hat{P}_{F_i}}{1-p_i}\middle|H_0\right) \tag{11}$$

$$A'_i = F_i^{-1}\left(\frac{\hat{P}_{M_i}}{p_i}\middle| H_1\right) \tag{12}$$

where ${F_i}_c^{-1}(l_i|H_j)$ is the complementary cdf of l_i under $H_j, j \in \{0,1\}$. Given (11) and (12) the sequential test is defined as follows: at step i, if $l_i > B'_i$, H_1 is declared true. If $l_i < A'_i$, H_0 is declared true. Otherwise, a new sample is collected and the test repeats. The sequential test proceeds in real-time, which is a further requirement to model human decision-making.

IV. Observation Ordering

Inspecting (11) and (12), thresholds B'_i and A'_i are dependent on p_i and thus on past observations $Y_1, \ldots, Y_{i-1}$. This implies that different samples displayed to the observer will generate different future thresholds and consequently different stopping times. However, since $\hat{P}_{F_i}$ and $\hat{P}_{M_i}$ in (9) and (10) are maintained constant, selections and orders for which the test statistically terminates earlier lead to better error performance. Therefore, it is required to select the subset of observations and the order of its elements so that the test terminates as soon as possible.

A. Non-recursive expression for p_i

To derive the observation ordering, we first obtain a non-recursive expression for p_i. Note that (3) can be rearranged into

$$p_i = \frac{\frac{f_{i-1}(Y_{i-1}|H_1)}{f_{i-1}(Y_{i-1}|H_0)}}{\frac{1}{p_{i-1}} - 1 + \frac{f_{i-1}(Y_{i-1}|H_1)}{f_{i-1}(Y_{i-1}|H_0)}} \tag{13}$$

Using (1) we have

$$p_i = \frac{\exp(l_{i-1})}{\frac{1}{p_{i-1}} - 1 + \exp(l_{i-1})} \tag{14}$$

A similar expression of p_{i-1} to that of p_i can be obtained from (14) as function of p_{i-2} and l_{i-2} and then substituted directly into (14) to give

$$p_i = \frac{\exp(l_{i-1} + l_{i-2})}{\frac{1}{p_{i-2}} - 1 + \exp(l_{i-1} + l_{i-2})} \tag{15}$$

Proceeding recursively on i and using (2) we thus have

$$p_i = \frac{\exp(L_{i-1})}{\frac{1}{p_0} - 1 + \exp(L_{i-1})} \tag{16}$$

where p_0 is the prior probability of H_1 before any data is observed.

B. Selection and ordering of l_i

We choose to define the selection and ordering on $l_1, \ldots, l_N$, which can be mapped back to $Y_1, \ldots, Y_N$. We impose the constraint $B'_i > 0 > A'_i$ for all i. Let l_i be the last log-likelihood term upon which the test terminates. From (16) p_i is monotonically increasing in L_{i-1}. Since F_i in (11) and (12) is a cdf, B'_i and A'_i are monotonically decreasing in L_{i-1}. Therefore, there always exists a unique sum $L^{(i)}$ defined implicitly as the solution to

$$\begin{aligned} l_i &= B'_i\left[p_i\left[L^{(i)}\right]\right] \quad \text{if } l_i > 0 \\ l_i &= A'_i\left[p_i\left[L^{(i)}\right]\right] \quad \text{if } l_i < 0 \end{aligned} \tag{17}$$

where we emphasize that B'_i and A'_i are functions of p_i which is in turn a function of L_{i-1}. Using (11) and (12) we have

$$p^{(i)} = p_i\left[L^{(i)}\right] = \begin{cases} 1 - \frac{\hat{P}_{F_i}}{F_{ic}(l_i|H_0)}, & \text{if } l_i > 0 \\ \frac{\hat{P}_{M_i}}{F_i(l_i|H_1)}, & \text{if } l_i < 0 \end{cases} \tag{18}$$

and using (16), $L^{(i)}$ is then given by

$$L^{(i)} = \ln\left(\frac{\left(\frac{1}{p_0} - 1\right) \times p^{(i)}}{1 - p^{(i)}}\right) \tag{19}$$

When l_i is large enough in absolute value, l_i and $L^{(i)}$ are opposite in sign. Introducing l_i as the first log-likelihood term is enough to terminate the sequential test. Else, $L^{(i)} + \epsilon$ should accumulate in L_{i-1} before l_i is introduced as the last term, where ϵ is arbitrarily small and has the same sign as l_i. This accumulation should be done in minimum time steps. Since l_i could be any term of $l_1, \ldots, l_N$, the algorithm becomes as follows: split set $l_1, \ldots, l_N$ into the set of positive log-likelihoods $\mathcal{P} = \{l_{p_1}, \ldots, l_{p_{N'}}\}$ and the set of negative log-likelihoods $\mathcal{N} = \{l_{n_1}, \ldots, l_{n_{N-N'}}\}$, where N' is the number of positive log-likelihoods in the original set. Compute the N threshold sums $L^{(1)}, \ldots L^{(N)}$ using (18) and (19). If any of those thresholds is opposite in sign to the corresponding log-likelihood term l_i, select that term and ask the observer to make a decision in the first time step. Else, for each l_i consider set $\mathcal{P}\backslash l_i$ if $l_i > 0$ and set $\mathcal{N}\backslash l_i$ if $l_i < 0$. If the first case is true, select from $\mathcal{P}\backslash l_i$ the largest positives whose sum will exceed $L^{(i)}$. If the second case is true, select from $\mathcal{N}\backslash l_i$ the largest negatives whose sum will drop below $L^{(i)}$. Append l_i to the selection at the end. Repeat the procedure for all N possible l_i terms. Of the N obtained sequences, the one with the least number of samples is the global solution. If different outcomes are yielded by two solutions of the same length, the sequence in favor of the hypothesis suggested by the whole set of log-likelihood terms is selected. If no sequence terminates the test, the designer either chooses whole set $\mathcal{P}$ or whole set $\mathcal{N}$ but without intermixing. This way both choices produce extreme values of p_i. Thus, in the worst case, N sequences are examined before the global solution is identified, which is too fast compared to combinatorial or exponential procedures in N that do the selection. The solution is then mapped back to the observations, which are presented sequentially to the observer.

C. Non-Bayesian Belief Update

We now show how the human sequential test model and the observation-ordering algorithm can be extended to capture different forms of belief-updates. In [1] the authors assume that belief updating is an anchor-and-adjustment process, where the anchor is the current belief S_{i-1} at time index $i-1$, and the adjustment is brought by the i^{th} piece of evidence x_i as

$$S_i = S_{i-1} + w_i(s(x_i) - R) \tag{20}$$

This is to say that a personal evaluation $s(x_i)$ of x_i is compared to a reference point R, and the gap weighted by w_i changes the degree of belief in a hypothesis to S_i. Typically $0 \le w_i \le 1$ and may be a function of S_{i-1}. Now let $R = 0$, and $w_i = 1\ \forall i$. Notice that (20) becomes of the same form as (2), where S_{i-1}, $s(x_i)$ and x_i are the counterparts of L_{i-1},

l_i and Y_i respectively. Since L_i is a sufficient statistic, the human sequential test can be extended to model non-Bayesian belief updates. As an illustration, we replace L_{i-1} in the expressions of p_i, B_i' and A_i' with L_{i-1}' given by

$$L_{i-1}' = \sum_{k=1}^{i-1} w_k l_k \tag{21}$$

where $0 \leq w_k \leq 1$. From [1], a monotonically non-increasing sequence w_k in k models the primacy effect, where additional pieces of evidence for the same hypothesis become less weighted over time. Assuming independence between w_k and l_k, the same observation-ordering algorithm may be applied.

V. Numerical Analysis

To inspect the impact of ordering the observations on the accuracy of the decision-making, it is required to have a reference sequential test for comparison. Since in the literature only SPRT is optimally designed, the observations are chosen to be Gaussian iid. Then $Y_1, \ldots, Y_N$ follow $\mathcal{N}_{\mathcal{R}}(\mu, \sigma^2)$ under H_1 and $\mathcal{N}_{\mathcal{R}}(0, \sigma^2)$ under H_0, where the first argument in $\mathcal{N}_{\mathcal{R}}(\cdot, \cdot)$ is the mean of the distribution and the second argument is its variance. Thresholds B_i' and A_i' are given by

$$B_i' = Q^{-1}\left(\frac{\hat{P}_{F_i}}{1 - p_i}\right) \times \frac{\mu}{\sigma} - \frac{\mu^2}{2\sigma^2} \tag{22}$$

$$A_i' = Q^{-1}\left(1 - \frac{\hat{P}_{M_i}}{p_i}\right) \times \frac{\mu}{\sigma} + \frac{\mu^2}{2\sigma^2} \tag{23}$$

where $Q^{-1}(x)$ is the inverse Q-function. Let $N = 20$ and $\frac{\mu}{\sigma} = 1$. We simulate the sequential test in MATLAB under true hypothesis H_1. Though H_1 is the true hypothesis, we let $p_0 = 0.5$. We vary the specifications of the false alarm and miss probabilities and inspect the resultant variation of the obtained error probability of a miss versus the ASN. At every setting, all sequential tests are run $M = 1\text{e}5$ times to average the results. The variation of the test miss rate versus the ASN is

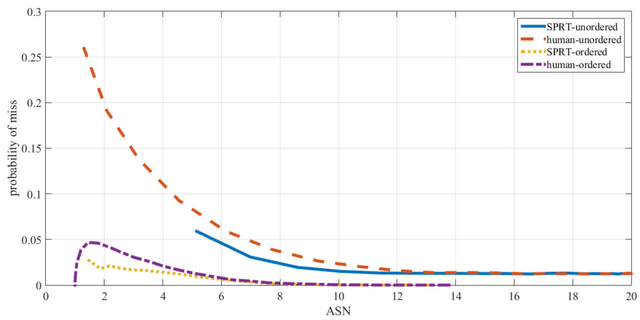

Fig. 1. Miss probability versus ASN for SPRT and the human sequential test model for unordered and ordered observations.

shown in figure 1. For the case where no order is applied to the log-likelihood terms for either test, lower error probabilities are attained for higher sample numbers and thus the curves are monotonically decreasing. Since the SPRT considers all observation distributions starting from time $n = 1$ to ∞ in the design of its thresholds, it outperforms the human sequential test model unless the observer is ready to take enough samples. While similar performance is attained for high ASN, the SPRT maintains lower error probability for decisions made after a few number of samples. This is because the thresholds of the SPRT are fixed, while the thresholds set by a human subject are observation dependent. Since no order is imposed on the observations, decisions made early tend to follow the first few encountered samples and thus hold a higher error probability. Note that too early stopping times are recorded only for the human sequential test model with high error. The curves are generated by varying the specifications $\hat{P}_{M_i}$ and $\hat{P}_{F_i}$ for the human sequential test and P_M and P_F for SPRT from about 0 up to 0.1.

Since the human thresholds are dependent on the encountered observations, ordering the latter should reduce the randomness of the stopping time of the sequential test that is source of high error probability. For the human sequential test model, we apply the order defined in section IV-B. This order is intended to both minimize the test error probability and stopping time. For fair comparison, an observation ordering should also be defined for SPRT. Since the SPRT thresholds are constant and using (2), the earliest stopping time of the SPRT is obtained by introducing the log-likelihood terms in favor of a single hypothesis, and in non-increasing order of absolute value. Two such orders are possible corresponding to H_1 and H_0. For both the human sequential test and the SPRT, the shortest of the two orders is selected. If two orders have the same length, the designer computes the overall cumulative log-likelihood of all the observations and selects the order in favor of the suggested hypothesis.

Given the two observation orderings as described for the human sequential test and the SPRT, we examine again the variation of the test miss rate versus the ASN upon varying the specifications $\hat{P}_{M_i}$, $\hat{P}_{F_i}$, P_M and P_F from about 0 up to 0.1. The curves are appended to figure 1. We notice that for both tests the curves pushed towards lower error probability and lower ASN. Moreover, zero error probability is now attained for an ASN below $N = 20$. This improvement is brought by the ordering of the observations. The threshold ASN above which the human sequential test and the SPRT match in performance is now reduced. For too early stopping times, the human sequential test counter-intuitively maintains low error probabilities. This represents the case where the observer does not target low error probabilities $\hat{P}_{M_i}$ and $\hat{P}_{F_i}$ and thus is ready to believe any hypothesis conveyed by the observations quite fast. In such a case, the observer is effectively following the decision made by the ordering algorithm which has access to the entire observation set and is therefore making a relatively reliable decision. In addition, the designer of the information display is benevolent and aims at reducing the decision errors that might be committed by the observer. Thus the designer presents a minimal information selection that favors the same hypothesis optimally suggested by the whole information set. The resultant performance drastically varies from the case where the samples are not ordered. It also differs from that of SPRT. Though SPRT is the optimal sequential test for iid observations, we note that the observations are no more iid when they are ordered, for which case SPRT is no more the optimal test. Still the maximum error probability for SPRT is less than that for a human sequential test. For large values of ASN, the curve captures the performance of less gullible human observers. As such observers see more data, their performance asymptomatically approaches that of the SPRT despite the fact that they suffer from cognitive biases, establishing the effectiveness of the proposed method.

Each pair value (miss rate, ASN) in figure 1 corresponds to

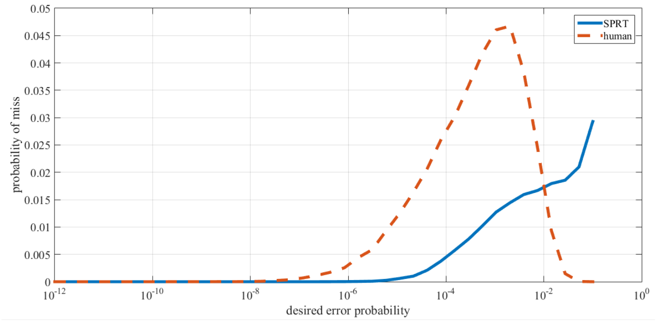

Fig. 2. Overall miss probability versus input/desired error probability for SPRT and the human sequential test model for ordered observations.

an error pair $(\hat{P}_{M_i}, \hat{P}_{F_i})$ targeted by the observer or (P_M, P_F) specified for SPRT. Choosing $\hat{P}_{M_i} = \hat{P}_{F_i}$ and $P_M = P_F$ we plot the resultant test miss rate versus $\hat{P}_{M_i}$ and P_M in figure 2 for the case of the ordered observations. As expected for SPRT, higher overall miss probability is obtained for higher P_M. For the human sequential test, this is not the case since the impact of the presented samples varies for different choices of $\hat{P}_{M_i}$. For instance, for high enough value of $\hat{P}_{M_i}$ the ASN is unity. However, the designer selects the observation by considering all the observation set. Probabilistically, the error risk is that of the designer's selection rather than the observer's decision. The design of the information display compensates for the high risk incurred in the observer's fast decision. Note that the accuracy of the values presented in figure 2 is computationally limited by the number M of the simulations.

In figure 3 we show the variation of the test miss rate

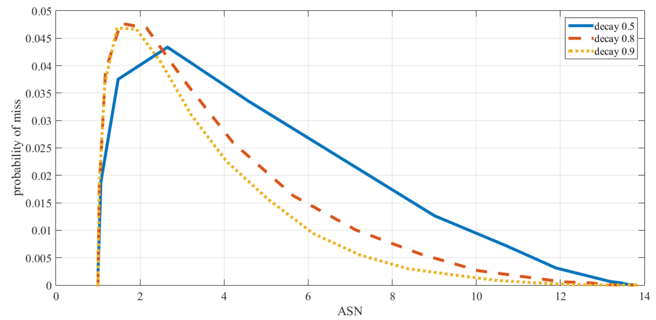

Fig. 3. Miss probability versus ASN for non-Bayesian belief update with different weightings of the log-likelihood terms.

versus the ASN for the human sequential test with ordered observations for the case of the non-Bayesian belief update model (21). We use $w_k = \lambda^{k-1}$, $k \geq 1$, and we show three curves corresponding to three values of the weight decay factor λ: 0.5, 0.8 and 0.9. As expected, a higher error rate is recorded per value of ASN for the three curves corresponding to the Bayesian case shown in figure 1. This is because the log-likelihood terms are weighted less importantly due to the primacy effect. Inspecting the curves of figure 1, as weight w_k approaches the Bayesian weight (unity), the corresponding miss rate-versus-ASN curve more closely approximates that for Bayesian belief-updating.

VI. Conclusion

In this paper the ordering of the observations is derived in order to minimize the number of samples required by an observer to decide on one of two hypotheses with a minimum probability of error. The phenomenon is modeled with a sequential test whose thresholds reflect the impact of the observations on a human besides their distributions in choosing when to make a decision. The threshold computation does not burden the observer with a high cognitive load and is carried out in real time. Compared to a machine sequential detector, a human subject falls short in terms of abiding by the specifications. However, by ordering the observations together with low error targeted by the observer at every stage of the sequential test, similar performance as that of the machine sequential detector can be obtained. An extension to the belief-updating model is presented and simulated. Exploiting the experience exclusively held by a human should boost man-machine cooperation for ultimate system performance.

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