Two and Three Inputs Widely Linear FRESH Receivers for Cancellation of a Quasi-Rectilinear Interference with Frequency Offset

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***Abstract*—Widely linear (WL) receivers can fulfill single antenna interference cancellation (SAIC) of one rectilinear (R) or quasi-rectilinear (QR) co-channel interference (CCI). The SAIC technology for QR signals has been shown to be less powerful than SAIC for R signals. To overcome this limitation, a SAIC/MAIC enhancement using three-inputs WL frequency-shift (FRESH) receiver has been introduced for QR signals. However, this receiver loses its efficiency for an interference having a residual frequency offset (FO) above a fraction of the baud rate. This may appear for airborne communications and it is the case for the inter-carrier interference of filter-bank based multicarrier waveforms using OQAM constellations which are candidate for 5G mobile networks. This paper extends the standard two-inputs SAIC/MAIC receiver and the three-inputs WL FRESH receiver for QR signals with FO. Analytical results and simulations are presented to study the impact of this FO on the performance of these receivers.**

I. Introduction

These two last decades, since the pioneer works on the subject [1–4], WL filtering has raised up a great interest for second-order (SO) non-circular signals [5] in many areas. Nevertheless, the subject which has received the greatest interest is CCI mitigation in radio communication networks using R or QR modulations. R modulations correspond to mono-dimensional modulations such as ASK or BPSK modulations. QR modulations are complex modulations corresponding, after a simple derotation operation, to a complex filtering of a R modulation. Examples of QR modulations are MSK, GMSK or OQAM modulations. One of the most important properties of WL filtering is its ability to fulfill SAIC of one R or QR multi-user CCI, allowing the separation of two users from only one receive antenna [6–8]. The effectiveness of this technology jointly with its low complexity explain why it is operational in most of GSM handsets, allowing significant networks capacity gains for the GSM system [8]. Extension of the SAIC concept to a multi-antenna reception is called MAIC. However, it has been shown recently in [9] that the SAIC/MAIC technology for QR signals may be less powerful than the one for R signals. This result is directly related to the different cyclostationarity and non-circularity properties of R and QR signals. To overcome this limitation, a SAIC/MAIC enhancement based on the concept of three inputs WL FRESH receiver has been introduced in [10] for QR signals. However, it has been shown in [11] in the GSM context that SAIC/MAIC receivers, optimal in a minimum mean square error sense, lose their efficiency if, after the signal of interest (SOI) synchronization, the CCI has a residual FO above a small fraction of the baud rate. This may be the case for airborne communications, due to a potential high differential Doppler shift between the SOI and the CCI. This is also the case in the context of FBMC-OQAM waveforms, which are considered as promising candidates for 5G mobile networks in particular [12]. Indeed, the ICI of FBMC-OQAM waveforms, which is present at reception for highly frequency selective channels or for multiple-input multiple-output systems, have FO corresponding to multiple of 50% of the (real) baud rate. Note that the scarce WL filtering based solutions available for this problem [13], [14] or for CCI mitigation [15] are two-inputs WL receivers which do not exploit explicitly the FO information.

In this context, the purpose of this paper is twofold. The first one is to extend, for arbitrary propagation channels, both the standard two-inputs SAIC/MAIC receiver and the concept of three inputs WL FRESH receiver introduced in [10] for QR signals with non zero differential FO. The second one is to analyse, both analytically and by simulations, the impact of this differential FO on the performance of these two SAIC/MAIC receivers. Note that the scarce papers dealing with WL FRESH filtering for demodulation of QR signals [16–18] do not consider the proposed WL FRESH receivers and do not present any analysis of potential differential FO on the performance. To our knowledge, the impact of differential FO on the performance of WL receivers has been analysed in [19] for R signals but has never been analysed for QR signals before this paper.

II. Models and SO Statistics

A. Observation model and SO statistics

We consider an array of N narrow-band antennas receiving the contribution of a QR SOI, one QR multi-user CCI and a background noise. The vector of complex amplitudes of the signals at the output of these antennas after frequency synchronization can then be written as

$$\mathbf{x}(t) = \sum_k j^k b_k \mathbf{g}(t-kT) + \sum_k j^k e_k \left[v(t-kT)e^{j2\pi\Delta_f t}\right] * \mathbf{h}_I(t) + \mathbf{u}(t)$$

$$\mathbf{x}(t) = \sum_k j^k b_k \mathbf{g}(t-kT) + \sum_k j^k e_k e^{j2\pi\Delta_f kT}\mathbf{g}_{I_o}(t-kT) + \mathbf{u}(t)$$
$$\mathbf{x}(t) \triangleq \sum_k j^k b_k \mathbf{g}(t-kT) + \mathbf{n}(t). \quad (1)$$

Here, b_k and e_k are real-valued zero-mean i.i.d. random variables, directly related to the SOI and CCI symbols respectively [7], [20], T is the symbol period for MSK and GMSK signals and half the symbol period for OQAM signals, $\mathbf{g}(t) = v(t) * \mathbf{h}(t)$ is the impulse response of the SOI global channel, $*$ is the convolution operator, $v(t)$ and $\mathbf{h}(t)$ are the impulse responses of the SOI pulse shaping filter and propagation channel respectively, Δ_f is the residual FO of the CCI, which is assumed to be known (as for the ICI of FBMC-OQAM waveforms) or perfectly estimated a priori, $\mathbf{g}_{I_o}(t) = v_o(t) * \mathbf{h}_I(t)$ where $v_o(t) = v(t)e^{j2\pi\Delta_f t}$ and $\mathbf{h}_I(t)$ is the impulse response of the propagation channel of the CCI, $\mathbf{u}(t)$ is the background noise vector, assumed zero-mean, circular, stationary, temporally and spatially white and $\mathbf{n}(t)$ is the total noise vector composed of the CCI and background noise. Note that model (1) is exact for MSK and OQAM signals whereas it is only an approximated model for GMSK signals.

The SO statistics of $\mathbf{n}(t)$ are characterized by the two correlation matrices $\mathbf{R}_n(t,\tau)$ and $\mathbf{C}_n(t,\tau)$, defined by

$$\mathbf{R}_n(t,\tau) \triangleq \mathrm{E}\left[\mathbf{n}(t+\tau/2)\mathbf{n}^H(t-\tau/2)\right] \quad (2)$$
$$\mathbf{C}_n(t,\tau) \triangleq \mathrm{E}\left[\mathbf{n}(t+\tau/2)\mathbf{n}^T(t-\tau/2)\right] \quad (3)$$

where $(.)^T$ and $(.)^H$ mean transpose and conjugate transpose respectively. Using (1), it is easy to verify that $\mathbf{R}_n(t,\tau)$ is a periodic function of t with period equal to T. In a same way, it is easy to show that $\mathbf{C}_n(t,\tau) = \mathbf{C}'_n(t,\tau)e^{j4\pi\Delta_f t}$ where $\mathbf{C}'_n(t,\tau)$ is a periodic function of t with period equal to $2T$. Matrices $\mathbf{R}_n(t,\tau)$ and $\mathbf{C}_n(t,\tau)$ have then Fourier series expansions given by

$$\mathbf{R}_n(t,\tau) = \sum_{\alpha_i} \mathbf{R}_n^{\alpha_i}(\tau)e^{j2\pi\alpha_i t} \quad (4)$$
$$\mathbf{C}_n(t,\tau) = \sum_{\beta_i} \mathbf{C}_n^{\beta_i}(\tau)e^{j2\pi\beta_i t}. \quad (5)$$

Here, α_i and β_i are the so-called non-conjugate and conjugate SO cyclic frequencies of $\mathbf{n}(t)$ such that $\alpha_i = i/T$ and $\beta_i = (2i+1)/2T + 2\Delta_f$ ($i \in \mathbb{Z}$) [21], $\mathbf{R}_n^{\alpha_i}(\tau)$ and $\mathbf{C}_n^{\beta_i}(\tau)$ are the first and second cyclic correlation matrices of $\mathbf{n}(t)$ for the cyclic frequencies α_i and β_i and the delay τ, defined by

$$\mathbf{R}_n^{\alpha_i}(\tau) \triangleq \left\langle \mathbf{R}_n(t,\tau)e^{-j2\pi\alpha_i t}\right\rangle \quad (6)$$
$$\mathbf{C}_n^{\beta_i}(\tau) \triangleq \left\langle \mathbf{C}_n(t,\tau)e^{-j2\pi\beta_i t}\right\rangle \quad (7)$$

where $\langle\cdot\rangle$ is the temporal mean operation in t over an infinite observation duration.

B. Current two and three inputs FRESH model

Conventional linear processing of $\mathbf{x}(t)$ [9], [10] only exploits the information contained in the zero non-conjugate ($\alpha = 0$) SO cyclic frequency of $\mathbf{x}(t)$.

Standard WL filtering of QR signals requires a derotation preprocessing to make QR signals look like R signals. Using (1), the derotated observation vector can be written as

$$\mathbf{x}_d(t) \triangleq j^{-\frac{t}{T}}\mathbf{x}(t) = \sum_k b_k \mathbf{g}_d(t-kT) + \mathbf{n}_d(t) \quad (8)$$

where $\mathbf{g}_d(t) \triangleq j^{-t/T}\mathbf{g}(t)$ and $\mathbf{n}_d(t) \triangleq j^{-t/T}\mathbf{n}(t)$. Standard WL filtering of $\mathbf{x}_d(t)$ only exploits the information contained in the zero non-conjugate and conjugate $(\alpha_d, \beta_d) = (0,0)$ SO cyclic frequencies of $\mathbf{x}_d(t)$, or equivalently in the SO cyclic frequencies, $(\alpha,\beta) = (0, 1/2T)$, of $\mathbf{x}(t)$. This is done through the exploitation of the temporal mean of the first correlation matrix of the extended derotated model $\widetilde{\mathbf{x}}_d(t) \triangleq [\mathbf{x}_d^T(t), \mathbf{x}_d^H(t)]^T$, or equivalently of $\mathbf{x}_{F_2}(t) \triangleq [\mathbf{x}^T(t), e^{j2\pi t/2T}\mathbf{x}^H(t)]^T = j^{t/T}\widetilde{\mathbf{x}}_d(t)$, also called two-inputs FRESH model in [10]. However, for $\Delta_f = 0$, the two most energetic conjugate SO cyclic frequencies of a QR CCI are $\beta_d = 0$ and $\beta_d = -1/T$, if the CCI is derotated, and $\beta = \pm 1/2T$, without any derotation [21], which proves the sub-optimality of model $\widetilde{\mathbf{x}}_d(t)$, or $\mathbf{x}_{F_2}(t)$, which only exploits one of these two cyclic frequencies.

To exploit the information contained in at least $\alpha_d = 0$, $\beta_d = 0$ and $\beta_d = -1/T$, or equivalently in at least $\alpha = 0$ and $\beta = \pm 1/2T$, a three-inputs FRESH model, $\mathbf{x}_{dF_3}(t) \triangleq [\widetilde{\mathbf{x}}_d^T(t), e^{-j4\pi t/2T}\mathbf{x}_d^H(t)]^T$, or equivalently $\mathbf{x}_{F_3}(t) \triangleq [\mathbf{x}^T(t), e^{j2\pi t/2T}\mathbf{x}^H(t), e^{-j2\pi t/2T}\mathbf{x}^H(t)]^T = j^{t/T}\mathbf{x}_{dF_3}(t)$, has been introduced in [10] for QR signals. It has been shown in [10] that the temporal mean of the first correlation matrices of $\mathbf{x}_{dF_3}(t)$ and $\mathbf{x}_{F_3}(t)$ exploit the information contained in $\alpha = 0$, $\alpha = \pm 1/T$ and $\beta = \pm 1/2T$.

However, for $\Delta_f \neq 0$, the two most energetic conjugate SO cyclic frequencies of a QR CCI become $\beta_d = 2\Delta_f$ and $\beta_d = 2\Delta_f - 1/T$ for a derotated CCI, and $\beta = 2\Delta_f \pm 1/2T$ without any derotation. This gives $\beta_d \neq 0$ if $\Delta_f \neq 0$ and $\Delta_f \neq 1/2T$ and $\beta_d \neq -1/T$ if $\Delta_f \neq 0$ and $\Delta_f \neq -1/2T$. In other words, for $\Delta_f \neq 0$ and $\Delta_f \neq \pm 1/2T$, both the two and three-inputs FRESH models and associated receivers give poor performance and extended two and three inputs models are required.

C. Extended two and three inputs FRESH model

The extended two-inputs FRESH model must exploit the information contained in $\beta_d = 2\Delta_f$ (or $\beta = 2\Delta_f + 1/2T$) instead of $\beta_d = 0$ (or $\beta = 1/2T$). Similarly, the extended three-inputs FRESH model has to exploit the information contained in $\beta_d = 2\Delta_f$ and $\beta_d = 2\Delta_f - 1/T$ (or $\beta = 2\Delta_f \pm 1/2T$) instead of $\beta_d = 0$ and $\beta_d = -1/T$ (or $\beta = \pm 1/2T$). We then propose to exploit and to analyse the performance, both analytically and by simulations, of the extended two and three-inputs FRESH models respectively defined by

$$\begin{aligned}\mathbf{x}_{EF_2}(t) &\triangleq \left[\mathbf{x}^T(t), e^{j2\pi(2\Delta_f + \frac{1}{2T})t}\mathbf{x}^H(t)\right]^T \\ &= j^{\frac{t}{T}}\left[\mathbf{x}_d^T(t), e^{j4\pi\Delta_f t}\mathbf{x}_d^H(t)\right]^T \triangleq j^{\frac{t}{T}}\mathbf{x}_{EdF_2}(t) \\ &= \sum_k j^k b_k \mathbf{g}_{EF_2,k}(t-kT) + \mathbf{n}_{EF_2}(t) \quad (9)\end{aligned}$$

$$\begin{aligned}\mathbf{x}_{EF_3}(t) &\triangleq \left[\mathbf{x}^T(t), e^{j2\pi(2\Delta_f+\frac{1}{2T})t}\mathbf{x}^H(t), e^{j2\pi(2\Delta_f-\frac{1}{2T})t}\mathbf{x}^H(t)\right]^T \\ &= j^{\frac{t}{T}}\left[\mathbf{x}_d^T(t), e^{j4\pi\Delta_f t}\mathbf{x}_d^H(t), e^{j4\pi(\Delta_f-1/2T)t}\mathbf{x}_d^H(t)\right]^T \\ &\triangleq j^{\frac{t}{T}}\mathbf{x}_{EdF_3}(t) = \sum_k j^k b_k \mathbf{g}_{EF_3,k}(t-kT) + \mathbf{n}_{EF_3}(t) \quad (10)\end{aligned}$$

Here, $\mathbf{n}_{EF_2}(t) \triangleq [\mathbf{n}^T(t), e^{j2\pi(2\Delta_f+1/2T)t}\mathbf{n}^H(t)]^T$, $\mathbf{n}_{EF_3}(t) \triangleq [\mathbf{n}^T(t), e^{j2\pi(2\Delta_f+1/2T)t}\mathbf{n}^H(t), e^{j2\pi(2\Delta_f-1/2T)t}\mathbf{n}^H(t)]^T$, $\mathbf{g}_{EF_2,k}(t) \triangleq [\mathbf{g}^T(t), e^{j4\pi\Delta_f kT}e^{j2\pi(2\Delta_f+1/2T)t}\mathbf{g}^H(t)]^T$ and $\mathbf{g}_{EF_3,k}(t) \triangleq [\mathbf{g}^T(t), e^{j4\pi\Delta_f kT}e^{j2\pi(2\Delta_f+1/2T)t}\mathbf{g}^H(t), e^{j4\pi\Delta_f kT}e^{j2\pi(2\Delta_f-1/2T)t}\mathbf{g}^H(t)]^T$.

III. Generic Pseudo-MLSE Receiver

A. Pseudo-MLSE approach

To extend, in an efficient original way and for an arbitrary propagation channel, the two and three-inputs FRESH SAIC/MAIC receivers for QR signals with differential FO, and to analyse the impact of this FO on the performance, we use the continuous time pseudo-maximum likelihood sequence estimation (MLSE) approach, introduced recently in [10], and we apply it to the models (9) and (10) respectively. This approach consists in computing the continuous time MLSE receiver from (9) or (10), assuming that the associated two or three-inputs FRESH total noise, $\mathbf{n}_{EF_2}(t)$ or $\mathbf{n}_{EF_3}(t)$, is Gaussian, circular and stationary.

B. Generic extended (E) pseudo-MLSE receiver

We denote by $\mathbf{x}_{EF_M}(t)$ and $\mathbf{n}_{EF_M}(t)$ the generic extended M ($M = 1, 2, 3$) inputs FRESH observation and associated FRESH total noise vectors respectively. We assume that $\mathbf{x}_{EF_1}(t)$ and $\mathbf{n}_{EF_1}(t)$ correspond to $\mathbf{x}(t)$ and $\mathbf{n}(t)$ respectively. Assuming a stationary, circular and Gaussian generic extended FRESH total noise $\mathbf{n}_{EF_M}(t)$, it is shown in [22], [23] that the sequence $\mathbf{b} \triangleq (b_1, ..., b_K)$ which maximizes its likelihood from $\mathbf{x}_{EF_M}(t)$ is the one which minimizes the following criterion[1]

$$\Lambda(\mathbf{b}) = \sum_{k=1}^{K}\sum_{k'=1}^{K} b_k b_{k'} r_{k,k'} - 2\sum_{k=1}^{K} b_k z_{EF_M}(k) \quad (11)$$

where $z_{EF_M}(k) \triangleq \Re[j^{-k}y_{EF_M}(k)]$ with

$$y_{EF_M}(k) = \int \mathbf{g}^H_{EF_M,k}(f)\left[\mathbf{R}^0_{n,EF_M}(f)\right]^{-1}\mathbf{x}_{EF_M}(f)e^{j2\pi fkT}df \quad (12)$$

$$r_{k,k'} = j^{k'-k}\int \mathbf{g}^H_{EF_M,k}(f)\left[\mathbf{R}^0_{n,EF_M}(f)\right]^{-1}\mathbf{g}_{EF_M,k'}(f)e^{j2\pi f(k-k')T}df. \quad (13)$$

Here, $\mathbf{R}^0_{n,EF_M}(f)$ is the Fourier transform of (6), where α_i and $\mathbf{n}(t)$ are replaced by 0 and $\mathbf{n}_{EF_M}(t)$ respectively, whereas $\mathbf{g}_{EF_M,k}(f)$ corresponds to $\mathbf{g}(f)$, for $M = 1$.

[1] All Fourier transforms of vectors $\mathbf{x}$ and matrices $\mathbf{X}$ use the same notation where t or τ is simply replaced by f.

C. Interpretation of the E-pseudo-MLSE receiver

We deduce from (12) that $y_{EF_M}(k)$ is the sampled version, at time $t = kT$, of the output of the filter whose frequency response is

$$\mathbf{w}^H_{EF_M,k}(f) \triangleq \left(\left[\mathbf{R}^0_{n,EF_M}(f)\right]^{-1}\mathbf{g}_{EF_M,k}(f)\right)^H \quad (14)$$

and whose input is $\mathbf{x}_{EF_M}(t)$. The structure of the extended M inputs pseudo-MLSE receiver is then depicted at Fig. 1. It is composed of the TI M inputs filter (14), followed by a sampling at the symbol rate, a derotation operation, a real part capture and a decision box implementing the Viterbi algorithm.

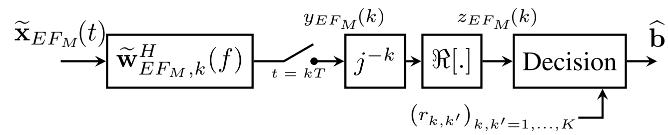

Fig. 1. Extended M inputs pseudo-MLSE receiver

D. SINR at the output of the E-pseudo-MLSE receiver

For real-valued symbols b_k, the symbol error rate (SER) at the output of the extended M inputs pseudo-MLSE receiver is directly linked to the signal to interference plus noise ratio (SINR) on the current symbol before decision, i.e. at the output $z_{EF_M}(n)$ [24, Sec 10.1.4], while the inter-symbol interference is processed by the decision box. For this reason, we compute this output SINR and we will analyse its variations in section IV. It is easy to verify from (1), (9), (10), (12) and (13) that $z_{EF_M}(n)$ can be written as

$$z_{EF_M}(n) = b_n r_{n,n} + \sum_{k\neq n} b_k \Re[r_{n,k}] + z_{n,EF_M}(n) \quad (15)$$

where $z_{n,EF_M}(n) \triangleq \Re[j^{-n}y_{n,EF_M}(n)]$ and $y_{n,EF_M}(n)$ is defined by (12) for $k = n$ with $\mathbf{n}_{EF_M}(f)$ instead of $\mathbf{x}_{EF_M}(f)$. The SINR on the current symbol is then defined by

$$\mathrm{SINR}_{M,n} \triangleq \frac{\pi_b r^2_{n,n}}{\mathrm{E}\left[\Re\left[j^{-n}y_{n,EF_M}(n)\right]^2\right]}. \quad (16)$$

where $\pi_b \triangleq \mathrm{E}[b_n^2]$. To analyse this $\mathrm{SINR}_{M,n}$ for particular scenarios, it is necessary to compute $r_{n,n}$ from (13) and $\mathrm{E}[\Re[j^{-n}y_{n,EF_M}(n)]^2]$ from the SO cyclic statistics of $y_{n,EF_M}(n)$. The latter statistics can be computed from (14) and the SO cyclic statistics of $\mathbf{n}_{EF_M}(t)$, themselves function of (6) and (7). The detailed way to derive the $\mathrm{SINR}_{M,n}$ from the cyclic statistics of $\mathbf{n}_{EF_M}(t)$ is tedious and not reported in this paper due to a lack of space. It will be presented elsewhere. For this reason, we just present and analyse in section IV, for particular scenarios, the final analytical expressions of $\mathrm{SINR}_{M,n}$.

IV. SINR Analysis For One CCI

A. Assumptions

To show the effectiveness of our pseudo-MLSE receiver, acting on (9) and (10) with respect to the conventional receiver and the better performance of (10) with respect to (9), we assume that the total noise is composed of one multi-user CCI and a background noise. We assume a raised cosine pulse shaping filter $v(t)$ with a roll-off γ. The SOI and CCI have the same bandwidth $B = (1+\gamma)/T$, and spectrally overlap if $0 \le |\Delta_f|T \le 1+\gamma$, as illustrated in Fig. 2, what we assume in the following. Due to space limitations, we limit the analysis to deterministic propagation channels with no delay spread such that

$$\mathbf{h}(t) = \mu\delta(t)\mathbf{h} \quad \text{and} \quad \mathbf{h}_I(t) = \mu_I\delta(t-\tau_I)\mathbf{h}_I. \tag{17}$$

Here, μ and μ_I control the amplitude of the SOI and CCI, $\delta(t)$ is the Dirac pulse, τ_I is the delay of the CCI with respect to the SOI whereas $\mathbf{h}$ and $\mathbf{h}_I$, such that $\mathbf{h}^H\mathbf{h} = \mathbf{h}_I^H\mathbf{h}_I = N$, are the channel vectors of the SOI and CCI.

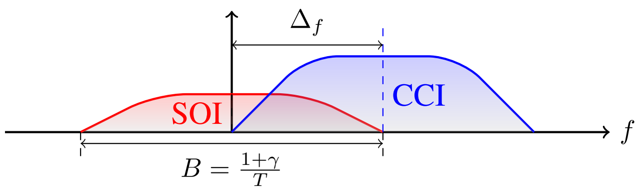

Fig. 2. Spectral representation of the SOI and CCI

B. SINR computations and analysis

Under the previous assumptions, analytical interpretable expressions of the SINRs (16) are only possible for a zero roll-off. In this case, we denote by $\pi_s \triangleq \mu^2\pi_b$, $\pi_I \triangleq \mu_I^2\pi_e$ and η_2 the power of the SOI, the CCI (for $\Delta_f = 0$) and the background noise per antenna at the output of the pulse shaping matched filter respectively, $\pi_e \triangleq \mathrm{E}[e_n^2]$, $\varepsilon_s \triangleq \pi_s\mathbf{h}^H\mathbf{h}/\eta_2$ and $\varepsilon_I \triangleq \pi_I\mathbf{h}_I^H\mathbf{h}_I/\eta_2$. Moreover, assuming $N = 1$ and a strong CCI ($\varepsilon_I \gg 1$) for models (9), (10) and also for model (1) if $\Delta_f \neq 0$, we obtain after tedious computations not reported here

$$\mathrm{SINR}_1 \approx 2\varepsilon_s|\Delta_f|T; \qquad \Delta_f \neq 0 \tag{18}$$

$$\mathrm{SINR}_1 = \frac{2\varepsilon_s}{1+\varepsilon_I\left[1+\cos(2\phi_{Is})\cos\left(\frac{\pi\tau_I}{T}\right)\right]}; \quad \Delta_f = 0 \tag{19}$$

$$\mathrm{SINR}_2 \approx 2\varepsilon_s|\Delta_f|T; \qquad 0.5 \le |\Delta_f|T \le 1 \tag{20}$$

$$\mathrm{SINR}_2 \approx \frac{\varepsilon_s[1+2|\Delta_f|T]}{2}; \qquad 0.25 \le |\Delta_f|T \le 0.5 \tag{21}$$

$$\mathrm{SINR}_{2,n} \approx 2\varepsilon_s\left[1-0.5\left\{1+|\Delta_f|T+(1-4|\Delta_f|T)\cos^2(\Psi_{sI,n})\right\}\right]$$
$$0 \le |\Delta_f|T \le 0.25; \quad (\Delta_f, \Psi_{sI,n}) \neq (0, k\pi) \tag{22}$$

$$\mathrm{SINR}_2 \approx \frac{9\varepsilon_s}{2\varepsilon_I\left[3+2\cos(4\phi_{Is})\right]}; (\Delta_f, \Psi_{sI,n}) = (0, k\pi) \tag{23}$$

$$\mathrm{SINR}_{3,n} \approx 2\varepsilon_s\left[1-\frac{c_1+c_2|\Delta_f|T+c_3(|\Delta_f|T)^2}{1+c_4|\Delta_f|T}\right]. \tag{24}$$

Here $\phi_{Is} \triangleq \mathrm{Arg}(\mathbf{h}_I^H\mathbf{h})$ and $\Psi_{sI,n} \triangleq [-\phi_{Is} + 2\pi\Delta_f(nT - \tau_I) - \pi\tau_I/2T]$. The c_i quantities, $1 \le i \le 4$, are complex coefficients, functions of ϕ_{Is}, Δ_f, T, n and τ_I, whose expressions are different for $|\Delta_f|T \in [0, 0.25], [0.25, 0.5], [0.5, 0.75]$ and $[0.75, 1]$ respectively. As these expressions are complicate, they are not given in the paper by lack of space.

A receiver performs SAIC as $\varepsilon_I \to \infty$ at time nT if the associated $\mathrm{SINR}_{M,n}$ does not converge toward zero. We deduce from (18) that the conventional receiver performs SAIC as long as there is a spectral discrimination between the sources ($\Delta_f \neq 0$). In this case, it is not sensitive to the phase of the signals and the output SINR does not depend on n. The SINR is maximum and equal to $2\varepsilon_s$, the one obtained without CCI, if the sources do not overlap ($|\Delta_f|T = 1$). Otherwise, the output SINR strongly decreases as the overlap between the sources strongly increases. For a complete overlap ($\Delta_f = 0$), (19) shows that SAIC at the output of the conventional receiver is generally no longer possible, except when $(\tau_I/T, \phi_{Is}) = (2k_1, (2k_2-1)\pi/2)$ or $(2k_1+1, k_2\pi)$, where k_1 and k_2 are integers.

Moreover, we deduce from (20) and (21) that for a spectral overlap which is less than 75% and for a strong CCI, the extended two-inputs WL FRESH receiver still performs SAIC thanks to a spectral discrimination between the sources only. Nevertheless, while its performance correspond to those of the conventional receiver for a spectral overlap which is less than 50%, it has better performance than the conventional receiver for a spectral overlap comprised between 50% and 75%. For a spectral overlap which is greater than 75% and for a strong CCI, (22) shows that the extended two-inputs WL FRESH receiver discriminates the sources spectrally and by phases and the output SINR depends on the differential phase of the sources and then on n. It completely cancels the CCI as long as there is at least one of the two discriminations (spectrum or phase) between the sources $(\Delta_f, \Psi_{sI,n}) \neq (0, k\pi)$. However (22) shows that the relative weight of the phase discrimination with respect to the spectral one increases with the spectral overlap. In other words, the phase discrimination takes over from the spectral one when the latter becomes too weak, which generates better performance than the conventional receiver for SAIC. Note that such an analysis from analytical SINR results has never been reported elsewhere.

The complicate expression (24) of $\mathrm{SINR}_{3,n}$ that we have obtained allows us to show that for a strong CCI, the extended three-inputs WL FRESH receiver performs SAIC thanks to a spectral discrimination only when the spectral overlap is less than 25%, with $\mathrm{SINR}_3 \ge \mathrm{SINR}_2$. For a spectral overlap greater than 25%, the extended three-inputs WL FRESH receiver performs SAIC thanks to both a spectral and a phase discrimination between the sources with an increasing weight of the latter as the overlap increases. In all cases, we have shown that $\mathrm{SINR}_3 \ge \mathrm{SINR}_2$, hence the great interest of the extended three inputs receivers with respect to the two-inputs one.

To give a statistical perspective of these results for arbitrary values of γ, we now assume that ϕ_{Is}, $\pi\tau_I/2T$ and n are independent random variables uniformly distributed on $[0, 2\pi]$, $[0, 2\pi]$ and $[0, \lfloor 1/|\Delta_f|T \rfloor]$ for $\Delta_f \neq 0$ respectively. Under these assumptions, choosing $\varepsilon_s = 10$ dB and $\varepsilon_I = 20$ dB, Figures 3 and 4 show, for $\gamma = 0$ and 0.5 respectively, $M = 1, 2, 3$, and $|\Delta_f|T = 0$, 0.25 and 0.5, $\Pr[(\mathrm{SINR}_{M,n}/2\varepsilon_s)\text{ dB} \ge x\text{ dB}] \triangleq p_M(x)$ as a function of x (dB), where $\Pr[.]$ means probability. Note, whatever γ and $|\Delta_f|T$, both good performance of the extended M-

inputs WL FRESH receivers for $M = 2$ and 3 and increasing performance with M of these receivers, proving the interest of (10) with respect to (9). Note also, for a given value of M, increasing performance with $|\Delta_f|T$ of the M-inputs WL FRESH receiver. For example, for $\gamma = 0.5$ and $x = -3$ dB, we note that $p_3(x) = 50\%$, 62%, 85% for $|\Delta_f|T = 0, 0.25$ and 0.5 respectively, showing very good performance of the extended 3-inputs WL FRESH receiver whatever $|\Delta_f|T$.

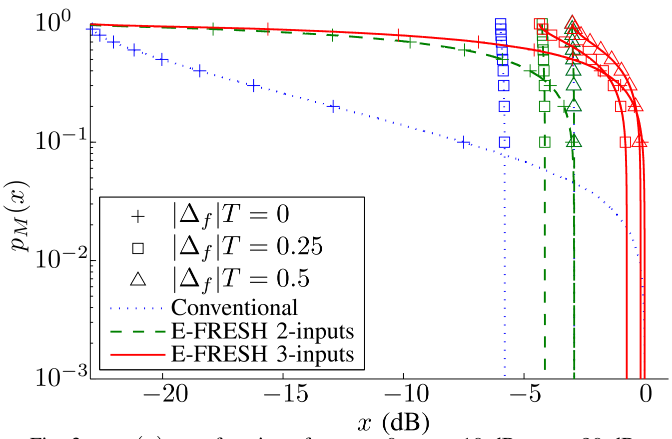

Fig. 3. $p_M(x)$ as a function of x, $\gamma = 0$, $\varepsilon_s = 10$ dB, $\varepsilon_I = 20$ dB.

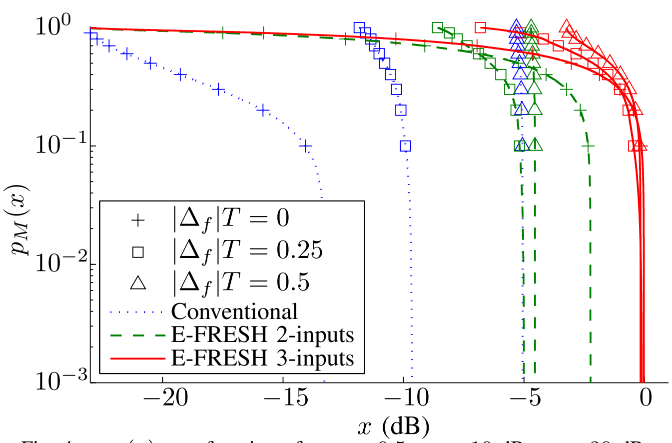

Fig. 4. $p_M(x)$ as a function of x, $\gamma = 0.5$, $\varepsilon_s = 10$ dB, $\varepsilon_I = 20$ dB.

V. Conclusion

The two and three-inputs SAIC/MAIC WL FRESH receivers have been extended, for arbitrary propagation channels and from a MLSE-based approach, for QR signals having differential FO. Performance of the proposed receivers have been analysed for deterministic channels with no delay spread, both analytically and by simulations, enlightening the impact of the FO parameter on the performance. Roles of spectral and phase discrimination between the sources have been explained for one receive antenna, from original analytical expressions of the output SINR. Finally, it has been shown that contrary to the conventional receiver, the proposed WL FRESH receivers have good performance whatever the value of the FO, increasing with the number M of inputs. These receivers may open new perspectives, for ICI mitigation of FBMC-OQAM waveforms in particular.

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