# Distributed PLKF Using Delayed Information Sharing for 3D **AOA** Target Tracking

Sheng Xu School of Information Technology and Mathematical Sciences, University of South Australia, Australia Email: sheng.xu@mymail.unisa.edu.au

Kutluyıl Doğançay School of Engineering, University of South Australia, Australia Email: kutluyil.dogancay@unisa.edu.au

Hatem Hmam Cyber and Electronic Warfare Division, DST Group, Australia Email: hatem.hmam@dsto.defence.gov.au

Abstract—This paper investigates the problem of distributed angle-ofarrival (AOA) target tracking in 3D space using unmanned aerial vehicles (UAVs). Because of communication constraints arising from distance and bandwidth constraints in a distributed UAV system, some UAVs may not be able to share their information with all other UAVs. This will lead to reduced tracking performance. In order to improve the estimation performance, a 3D distributed pseudolinear Kalman filter (DPLKF) using delayed information through intermediate UAVs is proposed. To track a moving target, a new estimation method using 1-step delayed information is developed which has low computational complexity. The communication topology with delayed information sharing is analyzed. In order to reduce communication traffic, a direct neighbors selection strategy is proposed. The effectiveness of the proposed estimation strategy is demonstrated with simulation examples.

#### I. INTRODUCTION

Unmanned aerial vehicles (UAVs) have been widely used for angle-of-arrival (AOA) target localization [1, 2]. In 3D AOA target localization, the target state is estimated by azimuth and elevation angle measurements collected by multiple UAVs [2, 3]. To solve the AOA target localization problem, many different estimation methods have been proposed. A pseudolinear estimator (PLE) which is a linear least squares estimator with a closed-form solution, was introduced to locate a target in [4]. The authors of [5] presented a maximum likelihood estimator (MLE) for AOA target localization. The extended Kalman filter (EKF) was applied to the nonlinear AOA target localization problem in [6]. In [7], the authors proposed a 3D PLE employing a bias compensation method developed in [8]. Another 3D AOA PLE was developed in [9] including an xy-PLE and a z-PLE. In [7], the bias compensation algorithm has been simplified which is similar to a 2D bias compensation. Recently, a closed-form solution for 3D AOA localization capable of handling sensor position errors was proposed [10]. However, being a batch estimate, the computational complexity of these estimators increase significantly as more measurements are acquired. In [11], a recursive 3D pseudolinear Kalman filter (PLKF) was developed with a better stability than an EKF method.

Using multiple UAVs has advantages in terms of estimation accuracy and system reliability, and becomes a good choice for AOA target localization. The key advantages of the distributed estimation strategy compared with the centralized estimation are: (1) low-energy communication cost, (2) parallel processing, and (3) independent UAVs which are robust to link failure problem [12-14]. In [12] and [15], two different kinds of diffusion Kalman filters for distributed sensor network were proposed with different diffusion update strategies. [16] introduced a distributed EKF method to locate a single target. To reduce computational complexity and communication traffic [17] proposed a method to reduce unnecessary cooperation in distributed sensor network. In the partial diffusion method in [18] and [19], each node transmits a part of the entries

of its estimate vector to its neighbors at each iteration. The tradeoff between communication cost and estimation performance was analyzed in [18]. However, in a distributed UAV target localization system, as the information acquired by each UAV is limited, the estimation accuracy will decrease. In this paper we aim to use all the available information including the delayed information transmitted by intermediate UAVs to improve estimation performance. This leads to a novel 3D distributed PLKF (DPLKF) using delayed information to track a moving target.

In this paper, we focus on distributed estimation using delayed information from intermediate UAVs to track a moving target in 3D space. The process of delayed information transmission is presented and the estimation strategies using 1-step delayed information are developed. To reduce network communication, a direct neighbor selection method is developed whereby each UAV communicates with a subset of their neighbors. The rest of the paper is organized as follows. Section II presents the problem formulation. The detailed delayed information transmission process is introduced in Section III. Section IV presents an estimation algorithm using delayed information and the path optimization introduced in [11]. Communication topology of the distributed UAV tracking system is analyzed in Section V. The direct neighbors selection strategy is introduced in VI. Simulation results are presented in Section VII. The paper concludes in Section VIII.

## II. PROBLEM FORMULATION

Consider multiple distributed UAVs equipped with AOA sensors tracking a single target in 3D space. Every mobile UAV can get their own estimate from different noisy azimuth and elevation measurements taken at discrete-time instants k = 1, 2, 3, ... The target tracking geometry and the ideal 3D angle measurements are depicted in Fig. 1. The ideal angle measurements of the ith UAV are

$$\theta_{i,k} = \tan^{-1} \frac{p_{yk} - r_{i,yk}}{p_{xk} - r_{i,xk}}, \quad -\pi < \theta_{i,k} \le \pi$$
 (1a)

$$\theta_{i,k} = \tan^{-1} \frac{p_{yk} - r_{i,yk}}{p_{xk} - r_{i,xk}}, \quad -\pi < \theta_{i,k} \le \pi$$

$$\phi_{i,k} = \tan^{-1} \frac{p_{zk} - r_{i,zk}}{\|[p_{xk}, p_{yk}] - [r_{i,xk}, r_{i,yk}]\|}, \quad -\frac{\pi}{2} < \phi_{i,k} \le \frac{\pi}{2}$$
(1b)

where  $\boldsymbol{p}_k = [p_{xk}, p_{yk}, p_{zk}], \, \boldsymbol{r}_{i,k} = [r_{i,xk}, r_{i,yk}, r_{i,zk}]$  are locations of the target and the *i*th UAV at time k, respectively.  $||\cdot||$  denotes the Euclidean norm and tan<sup>-1</sup> is the 4-quadrant arctangent. The velocities of the target and the *i*th UAV are represented by  $[\dot{p}_{xk}, \dot{p}_{yk}, \dot{p}_{zk}]$ and  $[\dot{r}_{i,xk},\dot{r}_{i,yk},\dot{r}_{i,zk}]$ . The noisy azimuth and elevation measurements can be written as

$$\tilde{\theta}_{i,k} = \theta_{i,k} + n_{i,k}, \quad -\pi < \tilde{\theta}_{i,k} \le \pi$$
 (2a)

$$\tilde{\phi}_{i,k} = \phi_{i,k} + m_{i,k}, \quad -\frac{\pi}{2} < \tilde{\phi}_{i,k} \le \frac{\pi}{2}$$
 (2b)

where  $n_{i,k}$  and  $m_{i,k}$  are independent additive zero-mean Gaussian white noise with variance  $\sigma_{\theta,i}^2$  and  $\sigma_{\phi,i}^2$ , respectively. The position and velocity of the target are unknown. Assume the UAVs' positions and velocities are known with negligible error. From the measurement models, a nonlinear relationship exists between angle measurements and target state. Our objective is to estimate the target state using the noisy angle measurements from different UAVs.

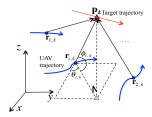


Fig. 1. Multiple UAV AOA tracking geometry and 3D angle measurements.

#### III. DELAYED INFORMATION TRANSMISSION

The UAVs share their information such as measurements, locations, local estimates and related weight factors with neighboring UAVs (neighbors) using on-board communication equipment. The UAVs that can communicate with each other directly are called direct neighbors. Assume each UAV has sufficiently large memory to save the data collected at different sampling time instants. Furthermore, every UAV can relay the outdated information from other neighbors acquired at previous time instants. As a result, two indirect neighbors can share outdated information through an intermediate UAV.

First, we present two assumptions to make the delayed information transmission problem analyzable.

#### **Assumptions:**

1. The communication process happens after all UAVs get their measurements at time k and all the UAVs communicate at same time. 2. The intermediate UAVs cannot transmit the information they receive at time k until the next time instant k+1.

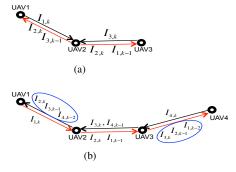


Fig. 2. Information transmission schematic at time k: (a) one intermediate UAV, (b) multiple intermediate UAVs.

In the distributed UAV target tracking system, neighboring UAVs can not only share measurements and locations of time k but also the information including the diffusion updated estimate with a weight coefficient of time k-1. We call the saved information at last sampling time k-1 as 1-step delayed information. Similarly, the outdated information of time k-M is called M-step delayed information. Each UAV can be an intermediate node to pass the outdated information. As shown in Fig. 2(a), UAV 2 can communicate with both UAV 1

and UAV 3, but UAV 1 and UAV 3 cannot communicate with each other directly. UAV 2 becomes an intermediate UAV which can relay the delayed information from UAV 1 or 3 to UAV 3 or 1 at time k. In Fig. 2(a),  $I_{1,k}$  means the UAV 1's information at time k and the arrow denotes the information transmission direction.

Note that when two UAVs are connected by multiple intermediate UAVs the delayed information needs multiple communication times to transmit one by one through all the intermediate UAVs. Thus, in the delayed information sharing network, we have a constraint that restricts the available delayed information of the ith UAV at time k. Constraint: If two UAVs are connected through M intermediate UAVs, at time k they can only get the k-M-step delayed information from each other.

Fig.2(b) depicts the constraint with an example of M=2. In order to eliminate the repeated delayed information, a rule is proposed.

**Rule**: If the delayed information has already been used before or received from another intermediate UAV, it will be ignored. Thus, data incest problem should be avoided.

To realize the rule, two detailed logical judgements are developed:

- 1. If two UAVs can communicate directly at last sampling time (such as UAV 1 and UAV 2 in Fig.2(b)), their delayed information will be ignored.
- 2. If the delayed information from an indirect neighbor l (such as UAV 1 and UAV 3 in Fig.2(b)) is first time transmitted by an intermediate UAV, this intermediate UAV will be recorded. The delayed information of UAV l transmitted by other intermediate UAVs will be ignored.

## IV. MOVING TARGET TRACKING ALGORITHM WITH 1-STEP DELAYED INFORMATION

The main estimator used in this paper is a DPLKF, drawing on [11]. The target state vector in xy-plane is defined as  $\boldsymbol{a}_k = [p_{xk}, \dot{p}_{xk}, p_{yk}, \dot{p}_{yk}]^T$  where  $^T$  denotes matrix transpose. The target dynamic model is

$$\boldsymbol{a}_{k+1} = \boldsymbol{U}_k \boldsymbol{a}_k + \boldsymbol{u}_k \tag{3}$$

where

$$oldsymbol{U}_k = \left[ egin{array}{cc} oldsymbol{A}_k & oldsymbol{0}_{2 imes 2} \ oldsymbol{0}_{2 imes 2} & oldsymbol{A}_k \end{array} 
ight], oldsymbol{A}_k = \left[ egin{array}{cc} 1 & T \ 0 & 1 \end{array} 
ight],$$

 $\boldsymbol{u}_k$  is the process noise including model uncertainty and T denotes the constant time interval between discrete-time instants. The process noise exists if the target moves with acceleration or irregular changing velocities. In a classical distributed estimator, l denotes the neighboring UAVs (including the ith UAV itself) that can communicate with the ith UAV. It also includes the neighborhood of neighboring UAVs whose delayed information is transmitted to the ith UAV.

The algorithm becomes very complex if multiple-step delayed information is used which is similar to tracking with out-of-sequence measurements [20]. Therefore, in this paper we only use the 1-step delayed information in the moving target tracking algorithm. Thus, the UAVs connected through more than one intermediate UAVs cannot share delayed information. The proposed algorithm comprises two steps: estimate update with delayed information and estimation with real-time information (the information sampled at time k). The diagram of the distributed target tracking system is shown in Fig. 3. The xy-DPLKF has two steps. First, estimate update with delayed information  $\tilde{\theta}_{l,k-1}$ ,  $r_{l,xyk-1}$ ,  $W_{l,k-1|k-1}$  and  $a_{l,k-1|k-1}$ ,

$$\mathbf{h}_{l,k-1} = [-\sin\tilde{\theta}_{l,k-1}, 0, \cos\tilde{\theta}_{l,k-1}, 0]$$
 (4a)

$$\xi_{l,k-1} = \hat{d}_{l,xyk-1}^2 \frac{\left(1 - e^{-2\sigma_{\theta,l}^2}\right)}{2} \tag{4b}$$

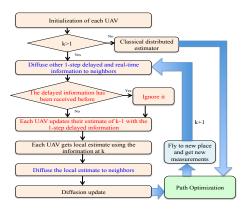


Fig. 3. Flow diagram of the distributed moving target tracking system.

$$\hat{d}_{l,xyk-1} = ||[p_{l,xk-1|k-1}, p_{l,yk-1|k-1}]^T - [r_{l,xk-1}, r_{l,yk-1}]^T||$$
(4c)

$$y_{i,k-1} \leftarrow h_{l,k-1} \left( r_{l,xyk-1} - a_{i,k-1|k-1} \right)$$
 (4d)

$$\mathbf{k}_{i,k-1} \leftarrow \mathbf{W}_{i,k-1|k-1} \mathbf{h}_{l,k-1}^T (\mathbf{h}_{l,k-1} \mathbf{W}_{i,k-1|k-1} \mathbf{h}_{l,k-1}^T + \xi_{l,k-1})^{-1}$$
(4e)

$$a_{i,k-1|k-1} \leftarrow a_{i,k-1|k-1} + k_{i,k-1}y_{l,k-1}$$
 (4f)

$$W_{i,k-1|k-1} \leftarrow (I - k_{i,k-1}h_{l,k-1})W_{i,k-1|k-1}$$
 (4g)

where ← means a parallel and sequential process and ^denotes an estimated value. N denotes the number of both direct and indirect neighboring UAVs.  $W_{i,k-1|k-1}$  is the updated estimate covariance matrix with 1-step delayed information. Second, with the updated  $a_{i,k-1|k-1}$  and  $W_{i,k-1|k-1}$ , the estimation process using the realtime information is

$$\mathbf{a}_{i,k|k-1} = U_{k-1}\mathbf{a}_{i,k-1|k-1} \tag{5a}$$

$$\mathbf{W}_{i,k|k-1} = \mathbf{U}_{k-1} \mathbf{W}_{i,k-1|k-1} \mathbf{U}_{k-1}^T + \mathbf{M}_{k-1}$$
 (5b)

$$\psi_{i,k} \leftarrow a_{i,k|k-1} \tag{5c}$$

$$\Psi_{i,k} \leftarrow W_{i,k|k-1} \tag{5d}$$

$$\Psi_{i,k} \leftarrow W_{i,k|k-1}$$
 (30)

$$\boldsymbol{h}_{l,k} = \left[ -\sin\tilde{\theta}_{l,k}, 0, \cos\tilde{\theta}_{l,k}, 0 \right] \tag{5e}$$

$$\xi_{l,k} = \frac{1}{2} ||[p_{l,xk|k-1}, p_{l,yk|k-1}]^T - [r_{l,xk}, r_{l,yk}]^T||^2 \left(1 - e^{-2\sigma_{\theta,l}^2}\right)$$
(5f)

$$y_{i,k} \leftarrow \boldsymbol{h}_{l,k} \left( \boldsymbol{r}_{l,xyk} - \boldsymbol{\psi}_{i,k} \right) \tag{5g}$$

$$\mathbf{k}_{l,k} \leftarrow \mathbf{\Psi}_{l,k} \mathbf{h}_{l,k}^T (\mathbf{h}_{l,k} \mathbf{\Psi}_{l,k} \mathbf{h}_{l,k}^T + \xi_{l,k})^{-1}$$
 (5h)

$$\psi_{i,k} \leftarrow \psi_{i,k} + \mathbf{k}_{i,k} y_{i,k} \tag{5i}$$

$$\Psi_{i,k} \leftarrow (I - k_{i,k} h_{l,k}) \Psi_{i,k} \tag{5j}$$

$$\boldsymbol{a}_{i,k|k} \leftarrow \sum_{l=1}^{N} \lambda_{l,k} \boldsymbol{\psi}_{l,k}, \quad \lambda_{l,k} = \frac{\frac{1}{\operatorname{tr}(\boldsymbol{W}_{l,k|k})}}{\sum_{k=1}^{N} \frac{1}{\operatorname{tr}(\boldsymbol{W}_{m,k|k})}}$$
(5k)

$$W_{i,k|k} \leftarrow \Psi_{i,k}$$
 (51)

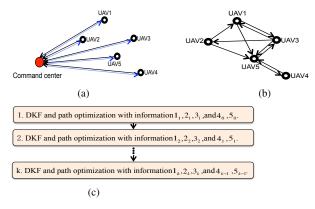
where  $M_{k-1}$  is the process noise covariance matrix [2] and  $M_{k-1} = \begin{bmatrix} q^x \boldsymbol{B}_{k-1} & \boldsymbol{0}_{2 \times 2} \\ \boldsymbol{0}_{2 \times 2} & q^y \boldsymbol{B}_{k-1} \end{bmatrix}$  and  $\boldsymbol{B}_{k-1} = \begin{bmatrix} \frac{T^4}{4} & \frac{T^3}{2} \\ \frac{T^3}{2} & T^2 \end{bmatrix}$ , where  $\lambda_{l,k}$  is the diffusion weights of each neighbor and the sum of  $\lambda_{l,k}$  equals

Similarly, an improved z-DPLKF using both the delayed and realtime information is constructed in a similar way. A path optimization strategy [11] is used for each UAV.

#### V. COMMUNICATION TOPOLOGY

Communication topology examples of a centralized and a distributed strategies are shown in Fig. 4. The directions of the information delivery are indicated by the arrows. In Fig. 4(a), the centralized strategy entails a command center with a fixed communication topology. In the distributed strategy, there is no command center and different UAVs have different direct neighbors that can dynamically change.

We provide an example of a distributed UAV target tracking network in Fig. 4(b). UAV 1 can only get the information from 1, 2 and 3 directly. Similarly, UAV 2 can only get 2 and 3's information, 3 can get 1, 3, 4, and 5's, 5 can get all the UAV's direct information. In addition, in our proposed strategy, the time-delayed information is used to improve the estimation. For example, at a sampling time, UAV 1 can get the real-time information of UAV 1, 2, 3, 1-step delayed information of 5 transmitted by 3. The algorithm structure is shown in Fig.4(c) where  $1_k$  mean the information of UAV 1 at sampling time k. Thus, the UAV distributed network becomes close to a centralized one and the estimation performance of each UAV will be improved with the delayed information. The communication complexity of our proposed strategy is larger than both the centralized and a normal distributed strategy. However, from the aspects of communication energy cost and estimation performance, the proposed distributed strategy has significant advantages.



Schematic diagram of UAVs communication topology: (a) centralized strategy, (b) distributed strategy, (c) algorithm diagram of the improved distributed strategy.

A communication topology comparison between a normal distributed method and the novel method using 1-step delayed information is depicted in Fig. 5. From Fig. 5, our proposed method extend the information zone of UAV i. As UAV i can acquire more information, its estimation performance will be improved.

### VI. DIRECT NEIGHBORS SELECTION

In a distributed UAV tracking system, UAV i may have multiple direct neighbors and these neighbors also have many direct neighbors. Thus, UAV i may be required to carry a big information traffic in each communication process. In order to reduce the communication traffic, we propose a selection method that decides on the best information from direct neighbors. The schematic diagram of this neighbors selection process is shown in Fig. 6.

UAV i at most can receive  $\gamma$  direct neighbors' information (including UAV i itself). The cost function to judge which neighbor is the best at time k is defined as

$$C_{l^*,k} = \frac{1}{\text{tr}(\mathbf{W}_{l^*,k|k})} \tag{6}$$

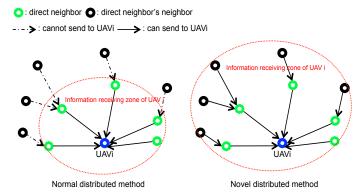


Fig. 5. Comparison between a normal distributed method and the novel method.

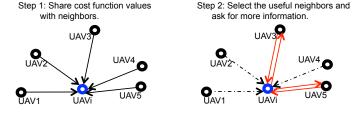


Fig. 6. The process of useful direct neighbors selection.

where  $l^*$  is the direct neighbor (except UAV i itself here). Then, compare all the neighbors cost functions and pick  $\gamma$  UAVs with  $\gamma$  smallest  $C_{l^*,k}$  as the final seclected neighbors. The selection algorithm can be extended also to consider the selection of neighbours of direct neighbours. As we assume the measurement noises of different UAVs are all the same and thus, the measurement noise will not impact the direct neighbors selection. The summary of the selection process is given in Table I.

#### TABLE I SUMMARY OF THE STEEPEST SLOPE OF COST FUNCTION ESTIMATION

- Step 1: UAVs share their cost function values  $C_{l^*,k}$  with all neighbors;
- Step 2: Make a sequence of  $C_{l^*,k}$  and select  $\gamma$  smallest ones as their useful neighbors;
- Step 3: Ask for other information from the useful neighbors and run the estimation algorithm;

#### VII. SIMULATION STUDIES

The initial positions of 5 UAVs are [200, 1200, 300]m, [-300, 500, 100]m[500, -300, 0]m[1000, 0, 0]m[2000, 0, 0]m.Α moving target is originally located at [1000, 1000, 1000]m. The initial target velocity  $[5, 5\sin\frac{\pi k}{30}, 1]m/s$ , with  $[1^2, 1^2, 1^2]m^2/s^4$ . In 5,  $q^x$ acceleration error variances  $= q^y =$  $10^{-7}$ . We  $q^z$ assume the target is moving with a constant velocity. The time interval is T = 1s with 30 measurement points. All the UAVs have the same initial state matrix  $m{X}_{0|0}$  $[\mathbf{a}_{0|0}, \mathbf{b}_{0|0}]^T = [1400, 9, 800, 13, 1100, 5]^T$  and covariance matrix  $P_{0|0} = \text{diag}[W_{0|0}, S_{0|0}] = \text{diag}[10^4, 10^4, 10^4, 10^4, 10^4, 10^4].$   $b_{0|0}$ and  $S_{0|0}$  are the initial state and covariance matrices of the z-DPLKF. All the UAVs have the same moving velocity 70m/s, as the UAVs are fixed wing, the elevation speed has a constraint that [-35, 35] m/s. Sensor measurement noises are  $\sigma_{\theta} = \sigma_{\phi} = 1^{\circ}$  and  $\gamma = 3$  that each UAV at most can receive two other neighbors' information. There is a no fly zone with 200m radius around the target. The flying collision problem is ignored. A fixed communication topology which is same as that in Fig.7(b) is applied. In the fixed communication

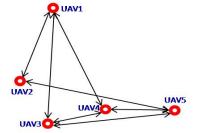


Fig. 7. The fixed communication topology.

topology, each UAV at most can communicates with 2 other direct neighbors. Besides, the 1-step delayed information from an indirect neighbor will not be impacted by the best neighbor selection strategy. The tracking process is repeated with 300 Monte Carlo runs. The mean-squared-error (MSE) is acquired from trace of the covariance matrices. The root-mean-squared-error (RMSE) is calculated from all the estimates and target true state. Besides, the trajectories are the mean positions of the UAVs and target with the data from 300 Monte Carlo runs.

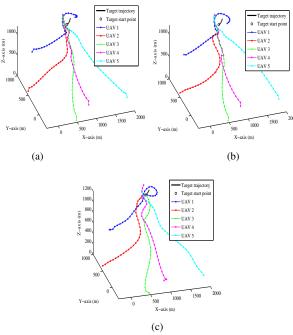


Fig. 8. Trajectories of UAVs tracking a moving target using different methods with process noise: (a) normal DPLKF, (b) improved DPLKF, (c) centralized PLKF.

A centralized PLKF [11] and a normal DPLKF are applied for comparison. Fig. 8 shows the UAVs trajectories using different strategies. It is obvious that the UAVs' trajectories are different using different target tracking methods. The estimation performance of different strategies are shown in Fig. 8. The mean MSE and RMSE shown in Fig. 9 are averaged over the 5 UAVs' data. Form Fig. 9, both the MSE and RMSE of the improved DPLKF are better than the normal DPLKF which are close to a centralized method. At beginning, the performance of the proposed method is not very good because neighbor selection strategy limited the

available information. As time instant increases, the performance of the new method becomes better than the normal distributed method. Furthermore, neither the MSE nor the RMSE can converge to zero because of the process noise.

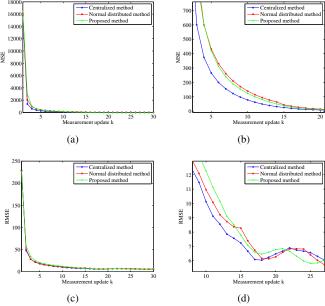


Fig. 9. Performance comparison of different methods for a moving target tracking with process noise: (a) Mean MSE, (b) amplified view of (a), (c) Mean RMSE, (d) amplified view of (c).

#### VIII. CONCLUSION

In a distributed UAV target tracking system, some UAVs cannot share information because of communication constraints. In order to use more information from different UAVs, a delayed information transmission strategy is proposed to improve estimation performance. A 3D distributed pseudolinear Kalman filter using 1-step delayed information was developed. The communication topology of the improved distributed PLKF method is analyzed. A direct neighbors selection strategy is proposed to reduce communication traffic. Simulation examples verified that the estimation performance of using delayed information is better than distributed estimation only using direct neighbors' information. The future work will consider using multiple UAVs to track multiple targets with dynamic communication topology taking into account multi-step delayed information.

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