COGNITIVE RADIO NETWORKS BASED ON OPPORTUNISTIC BEAMFORMING WITH QUANTIZED FEEDBACK

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ABSTRACT

In this paper, we consider an opportunistic beamforming scheduling scheme of secondary users (SUs) which can share the spectrum with a primary user (PU) in an underlay cognitive radio network. In the scheduling process, the cognitive base station (CBS) having multi-antennas, generates orthogonal beams which insure the minimum interference to the PU. Then, each SU feeds back its maximum signal to interference and noise ratio (SINR) and the corresponding beam index to the CBS. The CBS selects the users having the largest SINRs for transmission. The aim of our work is to study the effect of SINR feedback quantization on the throughput of the secondary system. To do this, we derive an accurate statistical characterization of ordered beams SINR and then we derive the closed-form expression of the system throughput with SINR feedback quantization based on Lloyd-Max algorithm.

Index Terms— Cognitive radio, opportunistic beamforming, feedback quantization, Lloyd-Max quantizer

1. INTRODUCTION

Cognitive radio (CR) is a novel approach for improving the utilization of the radio electromagnetic spectrum [1]. It allows unlicensed (secondary) users to share the spectrum with licensed (primary) users without significantly impacting their communication [2]. In [3], we proposed an opportunistic beamforming scheduling scheme of secondary users (SUs) which can share the spectrum with a primary user (PU) in an underlay cognitive radio network. We assumed that the cognitive base station (CBS) does not have full channel state information (CSI) from SUs while it has an imperfect CSI from the PU and we proposed a two-steps scheduling algorithm based on opportunistic beamforming (OB) [4]. In the first step, orthogonal beams are generated by the CBS to minimize the interference to the PU. In the second step, each SU calculates the signal to interference plus noise ratios (SINRs) for each beam and feeds back its maximum SINR and the corresponding beam index to the CBS. The CBS selects for transmission the users with the highest SINRs and assigns to each of these users the beam corresponding to the highest SINR. In practice, the SINRs are quantized before being fed back to the base station in order to make more efficient use of the limited resources (bandwidth and power). The SINR quantization for OB non-cognitive system has been recently studied in the literature [5–7]. In these studies, the impact of the feedback quantization on the throughput of OB system is analyzed.

The context of this work is different since it deals with cognitive radio network. Indeed, we consider the cognitive users scheduling scheme of [3] and we propose to quantize the SINR feedback using the minimum mean squared error (MMSE) optimal quantizer i.e. the Lloyd-Max quantizer [8]. We propose to identify the optimal set of quantization thresholds and to study analytically the impact of quantization on the throughput of the secondary system. To do this, we study analytically the statistics of the ordered beam SINRs for a particular user. The difficulty here comes from the fact that these SINRs are correlated random variables.

2. SYSTEM MODEL

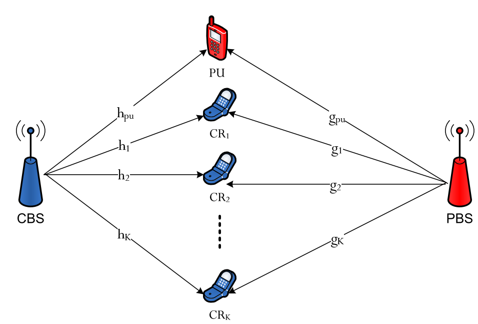

Fig. 1. System model

We consider the system model illustrated in Figure 1 and described in [3], where a cognitive radio network coexists with a primary network. The primary network consists of a primary base station (PBS) with a single transmitting antenna and one PU with a single receiving antenna. The cognitive network comprises K SUs, with a single receiving antenna each, and a CBS with M transmitting antennas. Throughout this paper, we assume that $M \ll K$ and the frequency division duplex (FDD) mode is used for both primary and secondary links. We consider the downlink of the cognitive radio network in which the CBS transmits independent signals to N_s scheduled secondary users, $1 \leq N_s \leq M-1$ (the scheduling will be explained in the following). We denote by $\mathbb{S}$ the set of the N_s selected cognitive users. Since the same carrier frequency is used within the primary and the secondary networks, the received signals at the SUs are corrupted by the signal transmitted by the PBS. Let $\mathbf{h}_k = [h_{k,1}, h_{k,2}, \cdots, h_{k,M}]$, where $h_{k,t}$ is the channel tap gain between the t-th transmit antenna of the CBS and the k-th secondary user, for $1 \leq t \leq M$ and $1 \leq k \leq K$. Let $\mathbf{g} = [g_1, g_2, \cdots, g_K]$, where g_k, for $1 \leq k \leq K$, denotes the channel tap gain between the transmit antenna at the PBS and the k-th cognitive user receive antenna. The entries of channel vectors $\mathbf{h}_k$ and $\mathbf{g}$ are independent and identically distributed (i.i.d.) complex Gaussian samples of a random variable with zero mean and unit variance. We assume that the channels are constant during the transmission of a burst of T symbols and vary independently from burst to burst. The received signal at the k-th cognitive user, for $1 \leq k \leq K$, can be written as:

$$y_k = \sqrt{P_s}\mathbf{h}_k \sum_{i \in \mathbb{S}} \mathbf{w}_i x_i + \sqrt{P_{pu}} g_k x_{pu} + n_k, \qquad (1)$$

where P_s and P_{pu} denote the transmitted power for each selected cognitive user and for the primary user, respectively. In this work, fixed power allocation for all selected users is adopted. The quantities x_{pu} and x_i denote the transmitted data from the PBS to the PU and from the CBS to the i-th SU, respectively, n_k denotes the noise at the k-th cognitive user which is a zero-mean Gaussian random variable with variance σ_k^2. We assume that the variances σ_k^2 (for $1 \leq k \leq K$) are equal to σ^2. The weighting vector $\mathbf{w}_i$ (of size $M \times 1$) denotes the beamforming weight vector for the i-th selected secondary user.

3. OPPORTUNISTIC BEAMFORMING SCHEDULING WITH SINR QUANTIZATION

In our work, we consider the two steps SUs scheduling method proposed in [3]. We assume that the CBS has an imperfect estimate of the interference channel $\mathbf{h}_{pu}$ (channel between the CBS and the PU) and has only partial channels knowledge about the secondary links (channels between the CBS and the SUs). In order to reduce the interference to the PU, the CBS generates, in the first step, orthogonal beams to the interference channel estimate $\hat{\mathbf{h}}_{pu}$ using the Gram-Schmidt algorithm. In the second step, the CBS selects a set $\mathbb{S}$ of N_s secondary users by applying the opportunistic beamforming approach proposed in [4]. Thus, the cognitive base station transmits the generated beams to all SUs. Then, by using (1), each SU k calculates the following N_s SINRs by assuming that x_j, $1 \leq j \leq N_s$, is the desired signal and the others x_i, $i \neq j$, $1 \leq i \leq N_s$, are interfering signals as:

$$SINR_{k,j} = \frac{|\mathbf{h}_k \mathbf{w}_j|^2 P_s}{\sum_{i=1,\, i\neq j}^{N_s} |\mathbf{h}_k \mathbf{w}_i|^2 P_s + |g_k|^2 P_{pu} + \sigma_k^2}. \qquad (2)$$

In practice, each SU feeds back to the CBS a quantized version of its maximum SINR.Thus, the value range of SINRs is divided into $Q = 2^b$ intervals, with boundaries values given as:

$$b_0 < b_1 < \ldots < b_Q. \qquad (3)$$

If the largest SINR value of user k, denoted by γ_k^1, is in the q-th interval, where $1 \leq q \leq Q$, i.e. $b_{q-1} < \gamma_k^1 \leq b_q$, then the k-th user will feedback the index q of that interval together with the index of its best beam. The CBS allocates the beams to selected users based on the feedback information. Specifically, a beam will be assigned to the user who has the largest SINR interval index, among all the users requesting that beam. Notice that if many users feed back the same quantization interval index for the same beam, one of these users is selected at random. In addition, it may happen that no user requests one or several beams. In this case, the CBS will assign that beam to a randomly chosen SU.

We propose to use in this paper the Lloyd-Max quantizer [8]. Then, we study the impact of feedback quantization on the secondary system sum rate. In order to identify the optimal set of quantization thresholds and to compute the sum rate, we derive the statistics of the ordered beam SINR for a given user.

4. LLOYD-MAX QUANTIZATION

In this section, we consider the Lloyd-Max quantization [8]. Let Γ_q, for $1 \leq q \leq Q$, be the reconstruction levels of the Q-level quantizer $\Omega(.)$ defined as:

$$\Omega\left(\gamma_k^1\right) = \Gamma_q \text{ if } b_{q-1} < \gamma_k^1 \leq b_q \qquad (4)$$

The quantizer is designed to minimize the average distortion D_Q given by:

$$D_Q = \sum_{q=1}^{Q} \int_{b_{q-1}}^{b_q} \left(\gamma - \Gamma_q\right)^2 f_{\gamma^1}(\gamma)\, d\gamma \qquad (5)$$

where $E[.]$ denotes the statistical expectation and $f_{\gamma^1}(\gamma)$ is the pdf of the largest SINR per user which is independent of k as will be seen in section 6. The expression of $f_{\gamma^1}(\gamma)$ will be given in (18).The necessary conditions to minimize D_Q are:

$$\begin{cases} \frac{\partial D_Q}{\partial b_q} = 0 \\ \frac{\partial D_Q}{\partial \Gamma_q} = 0 \end{cases} \qquad (6)$$

Solving (6) using the Lloyd-Max algorithm, we obtain the necessary conditions for minimization as:

$$\Gamma_q = \frac{\int_{b_{q-1}}^{b_q} \gamma f_{\gamma^1}(\gamma)\, d\gamma}{\int_{b_{q-1}}^{b_q} f_{\gamma^1}(\gamma)\, d\gamma} \qquad (7)$$

$$b_q = \frac{\Gamma_q + \Gamma_{q+1}}{2} \quad (8)$$

Mathematically, the decision and the reconstruction levels are solutions of the above set of nonlinear equations. In general, closed form solutions to equations (7) and (8) do not exist and they can be solved by numerical techniques in an iterative way by first assuming an initial set of values for the decision levels b_q. For simplicity, one can start with decision levels corresponding to uniform quantization, where decision levels are equally spaced. Based on the initial set of decision levels, the reconstruction levels can be computed using equation (7). These reconstruction levels are used in equation (8) to obtain the updated values of b_q. Solutions of equations (7) and (8) are iteratively repeated until convergence is achieved. In the next section, we propose to study the impact of SINR quantization on the secondary system sum rate.

5. SUM RATE OF THE SECONDARY SYSTEM

In this section, we study the throughput of the secondary system. The loss in throughput due to the quantization is equal to:

$$R_{loss} = R_A - R_Q \quad (9)$$

where R_A (respectively R_Q) is the throughput of the secondary system with analog (respectively quantized) feedback.

The throughput of the secondary system with analog feedback is expressed as [3]:

$$R_A = N_s \int_0^{+\infty} \frac{1}{1+\gamma}\left(1 - (F_S(\gamma))^K\right) d\gamma, \quad (10)$$

where $F_S(x)$ denotes the cdf of $SINR_{k,j}$ in (2) and is given in [3]:

$$F_S(x) = 1 - \left(\sum_{j=1}^{N_s-1} \frac{a_j \exp(-x/\rho)}{(x+1)^j} + \frac{a_{N_s} \exp(-x/\rho)}{(\alpha x + 1)} \right), \quad (11)$$

where a_j, for $1 \leq j \leq N_s$, are constants.

The exact sum rate expression for the secondary system with quantized feedback can be calculated as [[9], Eq. 7.71]:

$$R_Q = N_s \left(\sum_{k=1}^{K} \binom{K}{k} \left(\frac{1}{N_s}\right)^k \left(\frac{N_s-1}{N_s}\right)^{K-k} \times \left(\sum_{q=1}^{Q} \frac{\left(F_{\gamma^1}(b_q)\right)^k - \left(F_{\gamma^1}(b_{q-1})\right)^k}{F_{\gamma^1}(b_q) - F_{\gamma^1}(b_{q-1})} \int_{b_{q-1}}^{b_q} \log_2(1+\gamma) f_{\gamma^1}(\gamma)\, d\gamma \right)\right) + \left(\frac{N_s-1}{N_s}\right)^{K-1} \sum_{u=2}^{N_s} \int_0^{\infty} \log_2(1+\gamma) f_{\gamma^u}(\gamma)\, d\gamma \quad (12)$$

where $f_{\gamma^u}(\gamma)$ is the pdf of the u-th largest SINR per user and will be given in (15).

In the next section, we derive the statistics of the ordered beam SINR for a given user needed in (7), (8) and (12).

6. STATISTICS OF THE ORDERED BEAM SINRS FOR A PARTICULAR USER

Because the SINR values for a particular user are not independent, the order statistics used in [3] cannot be applied. Indeed, the beam SINRs for the same user k, i.e. $SINR_{k,j}$ in (2), for $1 \leq j \leq N_s$, are correlated random variables as they involve the same channel vector $\mathbf{h}_k$. The ordered SINRs for a given user k are denoted by $\gamma_k^{N_s} \leq \cdots \leq \gamma_k^u \leq \cdots \leq \gamma_k^1$. Since for a user k, the channels $\mathbf{h}_k$ and the noises n_k have statistics which are independent of k, we omit the index k in the following. We show that the cdf of γ^u, for $2 \leq u \leq N_s$, is given by:

$$F_{\gamma^u}(\gamma) = \int_0^{\infty}\int_0^{\infty}\int_0^{\frac{(N_s-u)y_1}{(u-1)}}\int_0^{\gamma\left(y_1+y_2+z+\frac{1}{\rho}\right)} f_{X^u,Y^-,Y^+,Z}(x,y_1,y_2,z)\, dx dy_1 dy_2 dz. \quad (13)$$

where $\rho = \frac{P_s}{\sigma^2}$ and

$$f_{X^u,Y^-,Y^+,Z}(x,y_1,y_2,z) = \frac{N_s!\,(y_1-(u-1)x)^{u-2}}{\alpha\,(N_s-u)!\,(u-1)!} \frac{\exp\left(-\left(x+y_1+y_2+\frac{z}{\alpha}\right)\right) U(y_1-(u-1)x)}{(N_s-u-1)!\,(u-2)!} \sum_{i=0}^{N_s-u}\binom{N_s-u}{i}(-1)^i (y_2-ix)^{N_s-u-1} U(y_2-ix)$$

$$x>0,\ y_1>(u-1)x,\ y_2<(N_s-u)x,\ z>0 \quad (14)$$

where $U(.)$ denotes the unit step function and $\alpha = \frac{P_{pu}}{P_s}$.

Proof: See the Appendix.

By taking derivative of (13) with respect to γ, the pdf of γ^u is given by

$$f_{\gamma^u}(\gamma) = \int_0^{\infty}\int_0^{\infty}\int_0^{\frac{(N_s-u)y_1}{(u-1)}} \left(y_1+y_2+z+\frac{1}{\rho}\right) f_{X^u,Y^-,Y^+,Z}\left(\gamma\left(y_1+y_2+z+\frac{1}{\rho}\right), y_1, y_2, z\right) dy_1 dy_2 dz. \quad (15)$$

We also show that the cdf of the largest SINR for a particular user, denoted by γ^1, is given by

$$F_{\gamma^1}(\gamma) = \int_0^{\infty}\int_0^{\infty}\int_0^{\gamma\left(y+z+\frac{1}{\rho}\right)} f_{X^1,Y,Z}(x,y,z)\, dx dy dz. \quad (16)$$

where

$$f_{X^1,Y,Z}(x,y,z) = \frac{N_s}{\alpha\,(N_s-2)!}\sum_{i=0}^{N_s-1}\binom{N_s-1}{i}(-1)^i (y-ix)^{N_s-2}\exp\left(-\left(x+y+\frac{z}{\alpha}\right)\right) U(y-ix) \quad (17)$$

We omit here the proof of (17) due to the lack of space.

After taking derivative with respect to γ, by applying the binomial theorem and using equation (2.323) in [10], we can obtain the closed-form expression for the pdf of the largest SINR per user, $f_{\gamma^1}(\gamma)$, given by:

$$f_{\gamma^1}(\gamma) = \frac{N_s}{\alpha(N_s-2)!} \sum_{i=0}^{N_s-1} \binom{N_s-1}{i} (-1)^i \sum_{j=0}^{N_s-2} \binom{N_s-2}{j}$$
$$\times \frac{(-i\gamma)^j}{(\gamma+\frac{1}{\alpha})^{j+1}} \exp\left(-\frac{\gamma}{\rho}\left(1+\frac{i(\gamma+1)}{1-i\gamma}\right)\right) \sum_{d=0}^{N_s-2-j}\binom{N_s-2-j}{d}$$
$$\times (1-i\gamma)^d \left(-\frac{i\gamma}{\rho}\right)^{N_s-2-j-d} \left(\Sigma^1_{i,j,d}(\gamma)+\Sigma^2_{i,j,d}(\gamma)\right) U(1-i\gamma) \tag{18}$$

where $\Sigma^1_{i,j,d}(\gamma) = \sum_{l=0}^{d} \left(\frac{d!\left(\frac{(j+1)!}{(\gamma+\frac{1}{\alpha})}+\frac{j!}{\rho}\right)\left(\frac{i\gamma}{\rho(1-i\gamma)}\right)^{d-l}}{(d-l)!(x+1)^{l+1}} \right)$ and

$\Sigma^2_{i,j,d}(\gamma) = \sum_{l=0}^{d+1}\left(\frac{(d+1)!j!\left(\frac{i\gamma}{\rho(1-i\gamma)}\right)^{d+1-l}}{(d+1-l)!(\gamma+1)^{l+1}}\right).$

7. SIMULATION RESULTS

In this section, we present simulation results of our proposed scheduling algorithm based on quantized SINR feedback. We compare the performance of the secondary system sum rate for the Lloyd-Max quantization and the uniform quantization.

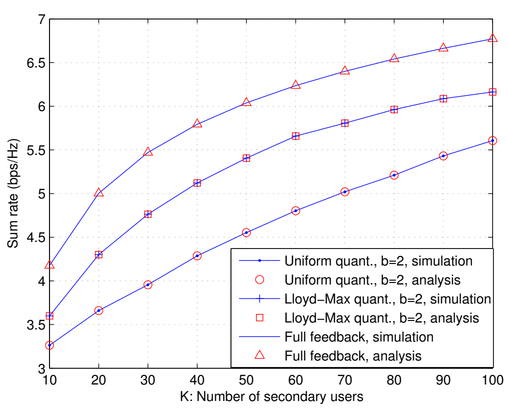

Fig. 2. Sum rate versus the number K of secondary users for different quantization schemes for $b = 2$, $M = 4$, $N_s = 3$ and $\rho = 5\,dB$

Figure 2 shows the sum rate versus the number K of secondary users for $b = 2$, $M = 4$, $N_s = 3$ and $\rho = 5\,dB$ for the Lloyd-Max quantization and the uniform quantization. It also shows the sum rate for the ideal scheme [3] with analog best beam SINR feedback (full feedback). In figure 2, the curves obtained by using the numerical results in (10) and (12) (red curves) are compared to the simulation results (blue solid curves) for the ideal scheme and the two quantization schemes. The figure shows that the analytical curves are inline with the curves obtained by simulations. We notice that the sum rate increases with the total number of cognitive users since the multi-user diversity increases [4]. Moreover, it can be seen from the figure that the use of different quantization schemes leads to a loss in terms of throughput. However, the Lloyd-Max quantizer clearly outperforms the uniform quantizer as expected.

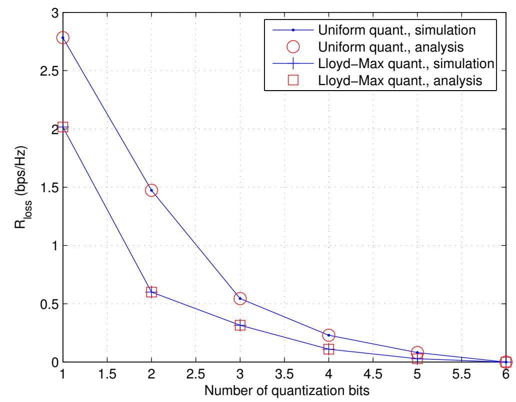

Fig. 3. Sum rate loss versus the number b of quantization bits for different quantization schemes for $K = 50$, $M = 4$, $N_s = 3$ and $\rho = 5\,dB$

Figure 3 shows the sum rate loss versus the number b of quantization bits for the Lloyd-Max quantizer and the uniform quantizer for $K = 50$, $M = 4$, $N_s = 3$ and $\rho = 5\,dB$. We notice that the curves obtained by using the numerical results in (9) (red curves) are inline with the simulation results (blue solid curves) for the two quantization schemes considered in this figure. We remark that the gap between the Lloyd-Max quantizer and the uniform quantizer in terms of sum rate decreases as b increases. Moreover, for both quantizers, the sum rates converge to the one obtained with full feedback as b increases.

8. CONCLUSIONS

In this paper, we considered a SUs scheduling scheme based on opportunistic beamforming. We analyzed the impact of SINR feedback quantization on the throughput of the secondary system. In particular, we derived the accurate statistical characterizations of ordered beams SINRs and we gave the exact analytical expression of the sum rate for the scheduling scheme based on quantized SINR feedback. We considered the Lloyd-Max quantizer and we compared its performance with that of the uniform quantizer.

9. APPENDIX

Let $X_{k,j} = |\mathbf{h}_k\mathbf{w}_j|^2$ for $1 \le k \le K$ and $1 \le j \le N_s$, ordered as $X_k^1 > X_k^2 > \ldots > X_k^{N_s}$, where X_k^u denotes the u-th largest value ($2 \le u \le N_s$) among the N_s values $X_{k,j}$ for $1 \le j \le N_s$. We notice that $SINR_{k,j}$ is the u-th largest beam SINR, for user k, if and only if $X_{k,j} = X_k^u$. Based on this observation, the u-th largest SINR of user k, denoted γ_k^u, can be written as:

$$\gamma_k^u = \frac{X_k^u}{Y_k^- + Y_k^+ + Z_k + \frac{1}{\rho}} \tag{19}$$

where $Y_k^- = \sum_{i=1}^{u-1} X_k^i$, $Y_k^+ = \sum_{i=u+1}^{N_s} X_k^i$, $Z_k = |g_k|^2 \alpha$, $\alpha = \frac{P_{pu}}{P_s}$ and $\rho = \frac{P_s}{\sigma^2}$. Since for a user k, the channels $\mathbf{h}_k$ and the noises n_k have statistics which are independent of k, we omit the index k in the following. The cdf of γ^u can be calculated in terms of the joint pdf of X^u , Y^-, Y^+ and Z, denoted by $f_{X^u,Y^-,Y^+,Z}(x, y_1, y_2, z)$, as

$$F_{\gamma^u}(\gamma) = \int_0^\infty \int_0^\infty \int_0^{\frac{(N_s-u)y_1}{(u-1)}} \int_0^{\gamma\left(y_1+y_2+z+\frac{1}{\rho}\right)} f_{X^u,Y^-,Y^+,Z}(x, y_1, y_2, z)\, dx dy_1 dy_2 dz. \tag{20}$$

After taking derivative with respect to γ, the pdf of γ^u is given by

$$f_{\gamma^u}(\gamma) = \int_0^\infty \int_0^\infty \int_0^{\frac{(N_s-u)y_1}{(u-1)}} \left(y_1 + y_2 + z + \frac{1}{\rho}\right) f_{X^u,Y^-,Y^+,Z}\left(\gamma\left(y_1 + y_2 + z + \frac{1}{\rho}\right), y_1, y_2, z\right) dy_1 dy_2 dz. \tag{21}$$

Applying Bayesian rules, the joint pdf $f_{X^u,Y^-,Y^+,Z}(x, y_1, y_2, z)$, can be obtained as follows [9]

$$f_{X^u,Y^-,Y^+,Z}(x, y_1, y_2, z) = f_{X^u}(x) \times f_{Y^-}(y_1) \times f_{Y^+}(y_2) \times f_Z(z) \tag{22}$$

where $x > 0$, $y_1 > (u-1)x$, $y_2 < (N_s-u)x$, $z > 0$. Since the random variables $X_{k,j}$, for a given k and $1 \le j \le N_s$, are i.i.d. random variables and have a gamma distribution $\Gamma(1,1)$, $f_{X^u}(x)$, $1 \le u \le N_s$, can be obtained as [[11], Eq.2.1.6]:

$$f_{X^u}(x) = \frac{N_s!}{(N_s-u)!\,(u-1)!} (1 - \exp(-x))^{N_s-u} \exp(-ux) \tag{23}$$

The pdfs $f_{Y^-}(y_1)$ and $f_{Y^+}(y_2)$ can be obtained, respectively, as [[12], Eqs. 26, 27]:

$$f_{Y^-}(y_1) = \frac{(y_1 - (u-1)x)^{u-2} \exp(-y_1 + (u-1)x)}{(u-2)!} U(y_1 - (u-1)x), \tag{24}$$

$$f_{Y^+}(y_2) = \frac{1}{(N_s-u-1)!} \sum_{i=0}^{N_s-u} \binom{N_s-u}{i} (-1)^i \times \frac{(y_2 - ix)^{N_s-u-1} \exp(-y_2)}{(1-\exp(-x))^{N_s-u}} U(y_2 - ix), \tag{25}$$

where $U(.)$ denotes the unit step function. The random variable Z follows the gamma distribution $\Gamma(1, \alpha)$ and the PDF $f_Z(z)$ is given by:

$$f_Z(z) = \frac{1}{\alpha} \exp\left(-\frac{z}{\alpha}\right). \tag{26}$$

Finally, after proper substitution, we can obtain the analytical expression given in (14).

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