MISO Estimation of Asynchronously Mixed BPSK Sources

Vincent Gouldieff[1], Kostas Berberidis[2], Jacques Palicot[1]
[1]CentraleSupelec/IETR, CentraleSupelec, Campus de Rennes, Avenue de la Boulaie - CS 47601
35576 Cesson-Sevigne Cedex, France
[2]University of Patras, Department of Computer Engineering and Informatics & CTI "Diophantus"
26500 Patras, Greece
Email: {vincent.gouldieff, jacques.palicot}@supelec.fr, berberid@ceid.upatras.gr

***Abstract*—In this paper, a method for discrete sources extraction from *underdetermined*, *finite-bandwidth* and *delayed* mixtures of BPSK sources with a single antenna receiver is proposed. Unlike most of the already existing algorithms which consider the unavoidable delay between the sources as undesirable, the proposed method takes advantage of such a delay. Indeed, it turns out that it is possible to recover the symbols even if there is neither gain nor phase diversity. The complexity of the proposed algorithm is quite low, which makes it efficient for real-time sources separation. The effectiveness of the method is illustrated via numerical simulations for different *scenarios*.**

***Keywords*—Blind source separation, BPSK, underdetermined mixture, MISO, time diversity.**

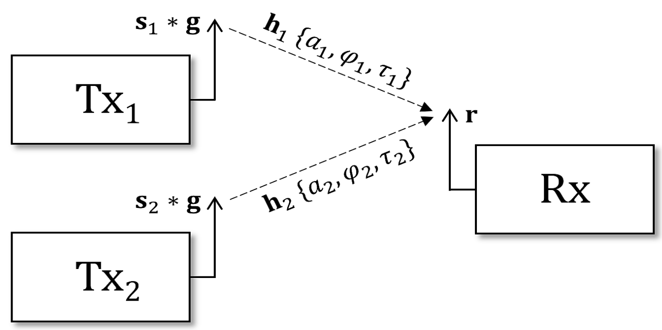

Fig. 1. Schematic representation of the separation problem

I. Introduction

Blind Source Separation (BSS) problems have been thoroughly studied in the literature over the past two decades. When there are less sensors than sources, the problem is known to be *underdetermined* and turns out to be quite challenging. To remove the indeterminacy, we need to exploit any *a priori* knowledge and diversity induced by the system. Usually, the blind separation of underdetermined mixtures of discrete sources is performed in two joint stages: the mixing parameters estimation stage and the discrete sources extraction stage.

Some of the most popular approaches for the parameters estimation step are actually greedy algorithms based on Expectation-Maximization [1] or Maximum-Likelihood [2] methods. Analytical solutions, based on polynomial criterion [3] [4] or on eigenvalue decomposition [5] have been proposed in order to reduce the involved complexity. Clustering methods [6] and constant modulus algorithms [7] have also been proposed as promising solutions to the involved estimation problem. The method in [8] aims at solving a similar problem by exploiting the high-frequency diversity induced by the shift between the carrier-frequencies.

The next step, *i.e.* that of extracting the symbol sequences, depends mainly on the way the sources are mixed. In the instantaneous situation, the extraction is commonly performed by applying a MAP (Maximum A Posteriori)-based method (*e.g.*, Least Square method in [8]) on each symbol-spaced sample. This method performs poorly if the sources have close phases and gains, or if the symbols are not well synchronized at the receiver. Such situations may appear in cellular phone or satellite transmissions, where delayed (single-path) or convolutive (multi-path) channels are often encountered. A similar scenario has been studied extensively in the so-called Multiuser Detection context for Code Division Multiple Access based systems [9].

In the proposed approach, there is no need to assume that the sources are mixed in a synchronous manner. In other words, the involved channels may introduce different delays, *i.e.* $\tau_1 \neq \tau_2$ in Fig. 1. The delay between the sources is no longer seen as a drawback and the induced *Inter-Users Interferences* (IUI), instead of being considered as noise, is viewed as an exploitable temporal diversity. Per-Survivor Processing [10] or Particle Filtering [11] based methods might be employed but, unfortunately, their good performance is offered at the expense of high complexity which may be prohibitive in real life scenarios.

Note that in the proposed method the focus is on the extraction of the symbol sequences from a noisy mixture of two asynchronously received BPSK signals transmitted over finite-bandwidth channels. The problem of blindly estimating the mixing parameters is not addressed here and these parameters are assumed to be perfectly known. It should also be stressed that the proposed method could be extended to more sources, to all PSK/QAM modulations and to the case of multi-path and time-varying channels.

The paper is organized as follows: In Section 2 the problem under study is formulated and basic definitions are provided, while in Section 3 the introduced IUI is analyzed. In Section 4 the new algorithm is derived and summarized while its performance is studied via extensive simulations in Section 5. Finally, Section 6 concludes the paper.

II. Problem Formulation

Let us assume that two users transmit *i.i.d.* BPSK symbol sets $\mathbf{s_1}$ and $\mathbf{s_2}$, such that,

$$\forall n \in \mathbb{N}, (s_1(n), s_2(n)) \in \{-1; +1\}^2$$

The symbol times are denoted t_{S1} and t_{S2}, and we consider for simplification that $t_{S1} = t_{S2} = t_S$.

At the transmitters sides we consider that the symbols are filtered by square-root raised cosine filters, denoted by $\mathbf{g}$ (with the same roll-off β) and the resulting signals are transmitted through respective base-band channels. The latter assumption is taken only for simplification and ease of notation. In case of passband transmission, the corresponding base-band signal can be obtained via a procedure which includes estimation of the carrier frequency f_C, multiplication of the received mixture by $e^{-j \cdot 2\pi \cdot f_C}$ and then proper filtering. In the situation where $f_{C1} \neq f_{C2}$, the method described in [8] could be directly applied. Finally, for $u \in \{1; 2\}$, the channel $\mathbf{h_u}$ between user *u* and the receiver is considered as invariant within a time slot, single-path and delayed, such that,

$$h_u(t) = a_u \cdot e^{j \cdot \phi_u} \cdot \delta(t - \tau_u)$$

Note that, in the following, if u stands for *1*, then $\breve{u}$ stands for *2*, and vice versa. Thus the received mixture can be written as,

$$r(t) = a_1 \cdot e^{j \cdot \phi_1} \cdot \sum_{k=-\infty}^{+\infty} s_1(k) \cdot g(t - k \cdot t_s - \tau_1) + a_2 \cdot e^{j \cdot \phi_2} \cdot \sum_{k=-\infty}^{+\infty} s_2(k) \cdot g(t - k \cdot t_s - \tau_2) + w(t) \quad (1)$$

where $w(t)$ is white Gaussian noise of variance σ_w^2.

III. Preprocessing Operations and Preliminaries

A. Matched Filtering

In a typical single-carrier communication system context, the received signal is filtered with a matched filter and then sampled every $n \cdot t_s$ (*symbol-spaced sampling*). Under standard assumptions, such a technique is efficient since it removes *Inter-Symbol Interference* (ISI) while reducing the effects of noise.

A similar type of filtering would also be employed in the case of IUI. Since the roll-off factors at transmitters 1 and 2 are identical, it would be meaningful to filter $r(t)$ with a matched filter (*i.e.* a square root raised cosine filter of roll-off β). Denoting $\mathbf{h} = \mathbf{g} * \mathbf{g}$, we may get,

$$y(t) = a_1 \cdot e^{j \cdot \phi_1} \cdot \sum_{k=-\infty}^{+\infty} s_1(k) \cdot h(t - k \cdot t_s - \tau_1) + a_2 \cdot e^{j \cdot \phi_2} \cdot \sum_{k=-\infty}^{+\infty} s_2(k) \cdot h(t - k \cdot t_s - \tau_2) + b(t) \quad (2)$$

B. Fractionally-spaced non-uniform oversampling

In the case of two *delayed* users and in order to exploit the inherent time diversity, we propose sampling the filtered signal every $n \cdot t_s + \tau_1$ and every $n \cdot t_s + \tau_2$, such that two discrete-time signals, not fully correlated, are obtained. Recall that $u \in \{1; 2\}$ and that, if $u = 1$, then $\breve{u} = 2$ and *vice versa*.

$$y_u(n) \triangleq y(n \cdot t_s + \tau_u) = a_u \cdot e^{j \cdot \phi_u} \cdot s_u(n) + a_{\breve{u}} \cdot e^{j \cdot \phi_{\breve{u}}} \cdot \sum_{k=-\infty}^{+\infty} s_{\breve{u}}(n+k) \cdot h(k \cdot t_s + \tau_{\breve{u}} - \tau_u) + b_u(n) \quad (3)$$

Denoting as $\xi_{\breve{u}}(n)$ the IUI component of $y_u(n)$ caused by symbols $\mathbf{s}_{\breve{u}}$, we can write (3) in the following way,

$$y_u(n) = a_u \cdot e^{j \cdot \phi_u} \cdot s_u(n) + a_{\breve{u}} \cdot e^{j \cdot \phi_{\breve{u}}} \cdot \xi_{\breve{u}}(n) + b_u(n) \quad (4)$$

where :

$$\xi_{\breve{u}}(n) = \sum_{k=-\infty}^{+\infty} s_{\breve{u}}(n+k) \cdot h(k \cdot t_s + \tau_{\breve{u}} - \tau_u) \quad (5)$$

Note that since the received signals are oversampled their noise components b_1 and b_2 are no longer independent. However, they both have the same variance σ_b^2.

The symbols $\mathbf{s}_1$ and $\mathbf{s}_2$ when $a_1 \neq a_2$ or $\phi_1 \not\equiv \phi_2[\pi]$ can be easily extracted via standard thresholding algorithms. Conversely, if $a_1 \simeq a_2$ and $\phi_1 \cong \phi_2[\pi]$, the methods based on symbol-spaced sampling (MAP algorithms performed at $n \cdot t_s + \tau$ with $\tau \in [\tau_1; \tau_2]$) will give a mean *Bit Error Ratio* (BER) of around $20 - 25\%$.

C. Introduced IUI

Now, let us approximate $\xi_u(n)$ by a finite sum in order to determine the finite set in which $q_u(n) = y_u(n) - b_u(n) = a_u \cdot e^{j \cdot \phi_u} \cdot s_u(n) + a_{\breve{u}} \cdot e^{j \cdot \phi_{\breve{u}}} \cdot \xi_{\breve{u}}(n)$ takes values.

Notice that the absolute value of the raised cosine filter function is bounded by a strictly decreasing function:

$$\forall \beta \in]0; 1], \exists \gamma > 1 \,\text{s.t.}\, \forall t \in \mathbb{R}_+^*, |h(t)| \leq \frac{1}{(t/t_s)^\gamma + 1}$$

Denote $\forall l \in \mathbb{N}_+^*, \xi_u = \xi_{u,l} + \xi_{u,\infty}$, where, $\forall n \in \mathbb{N}$,

$$\xi_{u,l}(n) = \sum_{k \in \mathcal{K}_{u,l}} s_u(n+k) \cdot h(k \cdot t_s + \tau_u - \tau_{\breve{u}}) \quad (6)$$

where $\mathcal{K}_{u,l} = \left\{ \left\lfloor -\frac{l-\breve{u}}{2} \right\rfloor, ..., \left\lfloor \frac{l-u+1}{2} \right\rfloor \right\}$ (l nearest symbols).

Using the previous inequality, we can show that,

$$\forall n \in \mathbb{N}, |\xi_{u,\infty}(n)| \leq 2 \cdot \sum_{k=\left\lfloor \frac{l}{2} \right\rfloor}^{+\infty} \frac{1}{k^\gamma + 1} \quad (7)$$

Thus: $\exists l$ *s.t.* $\forall n \in \mathbb{N}, |\xi_{u,\infty}(n)| \ll \sigma_b^2$. For such a l,

$$y_u(n) = a_u \cdot e^{j \cdot \phi_u} \cdot s_u(n) + a_{\breve{u}} \cdot e^{j \cdot \phi_{\breve{u}}} \cdot \xi_{\breve{u},l}(n) + b_{u,l}(n) \quad (8)$$

where $\sigma_{b_l}^2 \simeq \sigma_b^2$.

Let us now further study the windowed IUI term given by (6). It can be written as $\xi_{u,l}(n) = \mathbf{h}_{u,l,\tau} \cdot \mathbf{s}_{u,l}^T(n)$, where $\mathbf{s}_{u,l}(n) = (s_u(n+k))_{k\in\mathcal{K}_{u,l}}$ and $\mathbf{h}_{u,l,\tau} = (h(k \cdot t_s + \tau_u - \tau_{\breve{u}}))_{k\in\mathcal{K}_{u,l}}$. Since $\mathbf{s}_{u,l}(n)$ belongs to a 2^l alphabet, $\xi_{u,l}(n)$ can take at most to 2^l different values.

Based on the above we can conclude that the samples $s_{\breve{u}}(n+k)$ where $k \notin \mathcal{K}_{u,l}$ have a negligible contribution to the value of sample $y_u(n)$. Note that the value of l depends mainly on $|\tau_u - \tau_{\breve{u}}|$, β and σ_b^2.

Thus, in terms of BER, a well chosen *windowed* algorithm can be as efficient as a greedy *global* method (a global brute-force method for instance, whose complexity however is exponential with respect to the size of the samples vector).

IV. An Iterative Soft Local Algorithm

In this section, we propose a new algorithm to separate the sources. It performs a local Bayesian re-estimation of the emitted symbols $\mathbf{s}_u$ based on the previously computed estimates of the symbols $\mathbf{s}_{\breve{u}}$.

At each iteration, the mean reliability of the estimates u is improved, which in turn results in reliability improvement of the estimates $\breve{u}$. This mechanism leads to a mean decrease in BER if a smart local and alternate re-estimation of symbols $\mathbf{s}_u$ and $\mathbf{s}_{\breve{u}}$ is performed.

A. Derivation

Recall that $u \in \{1; 2\}$ denotes user 1 or user 2. If $u = 1$, then $\breve{u} = 2$, and *vice versa*. In this subsection, we develop the re-estimation of the symbols $\mathbf{s}_u$ given the previously computed estimates of the symbols $\mathbf{s}_{\breve{u}}$. The counterpart is straightforward.

In the following, we suppose that we have computed for all n in $\left\{\frac{l}{2}+1 : N-\frac{l}{2}\right\}$ the probability $p_{\breve{u},i}^{\pm}(n)$ that $s_{\breve{u}}(n)$ equals ± 1 at iteration index i.

First, the Bayesian re-estimation of $s_u(n)$ given $y_u(n)$ is,

$$p_{u,i+1}^{\pm}(n) = p[s_u(n) = \pm 1 | y_u(n)] \quad (9)$$

We denote by $\mathcal{S}_l$ the set containing all the 2^l l-sized $[\pm 1 \pm 1 \cdots \pm 1]$ vectors, and we recall that $\mathbf{s}_{\breve{u},l}(n) = (s_{\breve{u}}(n+k))_{k\in\mathcal{K}_{\breve{u},l}}$.

Based on results derived in Subsection 3.3, we may get,

$$p_{u,i+1}^{\pm}(n) = \sum_{\mathbf{s}\in\mathcal{S}_l} p[s_u(n) = \pm 1, \mathbf{s}_{\breve{u},l}(n) = \mathbf{s} | y_u(n)] \quad (10)$$

Using Bayes theorem and assuming that the symbols are $i.i.d.$, we obtain from above equation,

$$p_{u,i+1}^{\pm}(n) \propto \sum_{\mathbf{s}\in\mathcal{S}_l} p[y_u(n) | s_u(n) = \pm 1, \mathbf{s}_{\breve{u},l}(n) = \mathbf{s}] \cdot p[\mathbf{s}_{\breve{u},l}(n) = \mathbf{s}] \quad (11)$$

We denote, for $k \in \mathcal{K}_{u,l}$, $s(k)$ the k-th term of $\mathbf{s} \in \mathcal{S}_l$, and hence we define,

$$p_{\breve{u},i}(n+k) = \begin{cases} p_{\breve{u},i}^{+}(n+k) & \text{if } s(k) = +1 \\ p_{\breve{u},i}^{-}(n+k) & \text{if } s(k) = -1 \end{cases} \quad (12)$$

Since the symbols are $i.i.d.$, the right hand side probability in (11) can be computed for $\mathbf{s} \in \mathcal{S}_l$ according to,

$$p[\mathbf{s}_{\breve{u},l}(n) = \mathbf{s}] = \prod_{k\in\mathcal{K}_{\breve{u},l}} p_{\breve{u},i}(n+k) \quad (13)$$

The left hand side probability in (11) can be computed for $\mathbf{s} \in \mathcal{S}_l$ according to,

$$p[y_u(n) | s_u(n) = \pm 1, \mathbf{s}_{\breve{u},l}(n) = \mathbf{s}] = p[y_u(n) | q_{u,\pm 1,\mathbf{s}}] \quad (14)$$

where $q_{u,\pm 1,\mathbf{s}} = \pm a_u \cdot e^{j\cdot\phi_u} + a_{\breve{u}} \cdot e^{j\cdot\phi_{\breve{u}}} \cdot \mathbf{h}_{\breve{u},l,\tau} \cdot \mathbf{s}^T$ corresponds to the noiseless value of the mixture if $s_u = \pm 1$ and $\mathbf{s}_{\breve{u},l} = \mathbf{s}$ with $\mathbf{s} \in \mathcal{S}_l$.

Assuming that the introduced noise is Gaussian with variance σ_b^2, we obtain,

$$p[y_u(n) | q_{u,\pm 1,\mathbf{s}}] \propto e^{-\frac{(d_{u,\pm 1,\mathbf{s}}(n))^2}{2\sigma_b^2}} \quad (15)$$

where,

$$d_{u,\pm 1,\mathbf{s}}(n) = |\operatorname{Re}(y_u(n) - q_{u,\pm 1,\mathbf{s}}) \cdot \cos(\phi_u) + \operatorname{Im}(y_u(n) - q_{u,\pm 1,\mathbf{s}}) \cdot \sin(\phi_u)| \quad (16)$$

$d_{u,\pm 1,\mathbf{s}}(n)$ corresponds to the distance between the projections of $y_u(n)$ and $q_{u,\pm 1,\mathbf{s}}$ along vector $(cos(\phi_u); sin(\phi_u))$.

Such a projection allows better rejection of the interferer compared to the standard Euclidean distance.

We may finally get from (13) and (15),

$$p_{u,i+1}^{\pm}(n) \propto \sum_{\mathbf{s}\in\mathcal{S}_l} e^{-\frac{(d_{u,\pm,\mathbf{s}}(n))^2}{2\sigma_b^2}} \cdot \prod_{k\in\mathcal{K}_{\breve{u},l}} p_{\breve{u},i}(n+k) \quad (17)$$

Notice that the LLR (Log Likelihood Ratio) can be easily computed at iteration i using,

$$L(s_u(n) | y_u(n)) = \ln \frac{p_{u,i}^{+}(n)}{1 - p_{u,i}^{+}(n)} \quad (18)$$

This ratio can be used as the *soft* input of the outer channel decoder if appropriate. Otherwise, a hard decision is made as:

$$\widehat{s_u}(n) = \operatorname{sgn}[L(s_u(n) | y_u(n))] \quad (19)$$

B. Summary of the new algorithm

As already mentioned, the proposed algorithm performs an iterative and alternate *soft* Bayesian re-estimation of the emitted symbols probabilities, as depicted in Algorithm 1.

Notice that we only need to store $2 \cdot N$ probabilities for the proposed algorithm to work, which is significantly smaller than the memory required by trellis-based sequence estimation algorithms.

Moreover, the proposed algorithm is well suited for parallel and pipelined computing, which is of great interest when real-time operation is needed.

Algorithm 1 The proposed algorithm (Standard Version)

Initialization
Choose proper l and compute $q_{u,\pm 1,\mathbf{s}}$ for all $\mathbf{s} \in \mathcal{S}_l$
Initialize $\mathbf{p}^{\pm}_{u,1}$ to $[0.5 \cdots 0.5]$

Iterative part
for $i = 1 : maxiter$ **do**
  for $n = \frac{l}{2} + 1 : N - \frac{l}{2}$ **do**
    Estimate $p^{\pm}_{1,i+1}(n)$ according to (17)
    Estimate $p^{\pm}_{2,i+1}(n)$ according to (17)
  end for
  if $||\mathbf{p}^{\pm}_{mean,i+1} - \mathbf{p}^{\pm}_{mean,i}||_2 \leq th_{glo}$ **then**
    Stop algorithm
  end if
end for

C. *Improved versions of the algorithm*

Proper modifications on the algorithm summarized above may result in improved convergence speed, stability and achieved BER.

First, the estimates $\breve{u}$ at the current iteration step i may be directly used to obtain new estimates u. In this *forward* version, we do not wait iteration $i+1$ to exploit newly computed estimates. This modification results in a convergence speed improvement since the BER is now strictly decreasing.

Moreover, the symbols may be estimated after sorting the samples (3) by absolute value in descending order. In this *sorted* version, estimating first the well-conditioned symbols results in improved estimates of the ill-conditioned ones. This version results in a convergence speed improvement compared to both *standard* and *forward* versions.

V. Numerical Results

A. *Achievable BER*

In this section, the performance of the proposed algorithms is studied via some typical numerical experiments. All simulations have been performed over 2×10^5 symbols for different parameters.

Here, in case of two users, the noise power is defined such that $SNR_{dB} = 10 \cdot \log(\frac{P_1+P_2}{P_b})$ and the mean BER such that $BER = \frac{BER_1+BER_2}{2}$.

Also, let us define the following involved parameters: $a = \frac{a_u}{a_{\breve{u}}} \in [0.25; 1]$, $\phi = |\phi_u - \phi_{\breve{u}}| \in [0; \frac{\pi}{2}]$ and $\tau = |\tau_u - \tau_{\breve{u}}| \in [0; 0.5] \cdot t_s$.

We will compare the performance of the proposed method with the theoretical BER for a single QPSK signal, since such a signal is equivalent to two equipowered and synchronized BPSK signals, properly phase shifted by $\frac{\pi}{2}$. This situation corresponds to the optimal mixing conditions in terms of BER.

We will also compare the performance of the proposed algorithm with the performance of the instantaneous MAP algorithm, arbitrary synchronized between τ_1 and τ_2. The estimates are given by $(\widehat{s_1}(n), \widehat{s_2}(n))_{MAP} = \text{argmax}_{(s_1,s_2)}\, p((s_1,s_2)|y(n); a, \phi, \tau = 0)$.

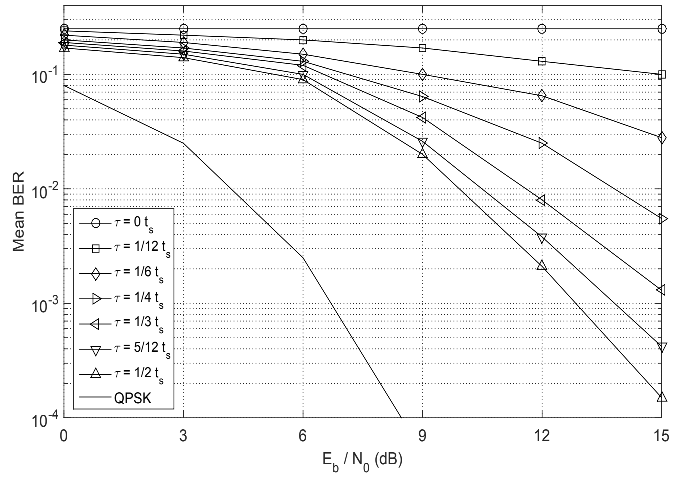

Fig. 2. Mean BER vs. SNR (dB) for τ as a parameter when $a_1 = a_2$ and $\phi_1 = \phi_2$. The Standard Version of the proposed algorithm is used.

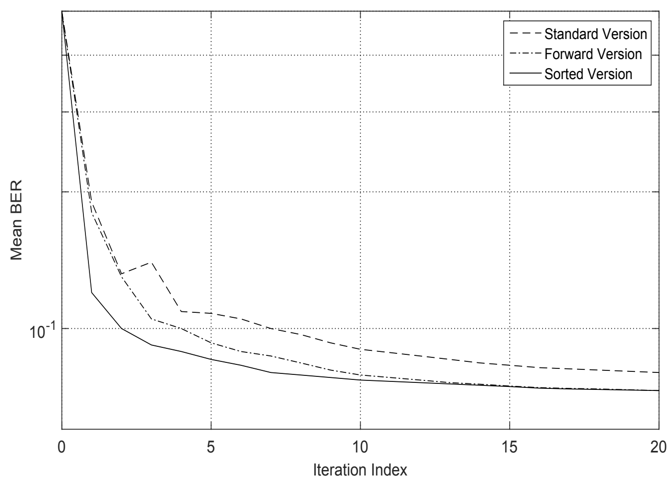

Fig. 3. Mean BER of recovered signals vs. iteration index for the different versions of the proposed algorithm.

Fig. 2 shows the performance of the proposed algorithm when $a = 1$ and $\phi \equiv 0[\pi]$, for different values of τ. As expected, the greater the value of τ, *i.e.* the greater the time diversity, the smallest the BER. Notice that the instantaneous MAP method would give a mean BER greater than 0.2 independently of the values of τ and SNR. For clarity, we did not represent the curves obtained with the instantaneous method.

Fig. 3 shows the convergence speed of the proposed algorithms, for $SNR = 6dB$, $\tau = 0.5t_s$, $a = 1$, $\phi \equiv 0[\pi]$. Notice that the *forward* and *sorted* versions are strictly converging. Observe also that only a few iterations are sufficient to achieve the related limit.

Finally, Fig. 4 shows the performance of the sorted algorithm when multiple degrees of diversity (here phase and delay) are available. The obtained results prove that the proposed algorithm may properly exploit the diversity induced by the delay even for small values of phase diversity.

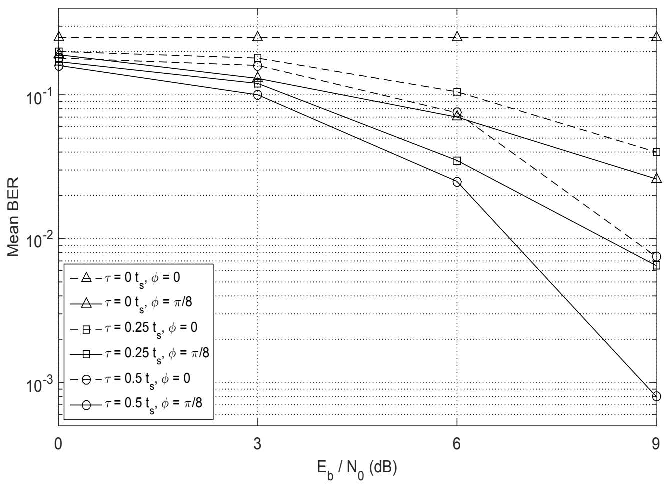

Fig. 4. Mean BER vs. SNR (dB) for τ and ϕ as parameters when a is set equal to 1.

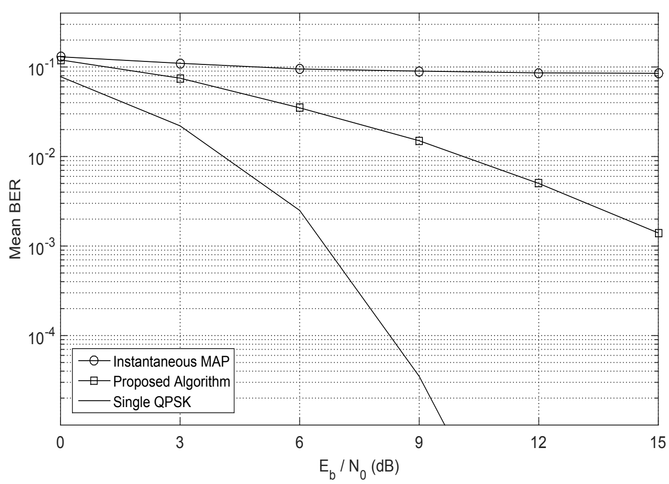

Fig. 5. Overall mean BER vs. SNR (dB) when all the mixing parameters are uniformly distributed over their range of values.

B. Overall performance

In order to study the overall algorithm performance, we consider that τ, ϕ and a are uniformly distributed over their corresponding range of values. Within the time frame of each separate experiment the values of these parameters are kept fixed and are still considered as known.

Such a situation corresponds well to a real-life multi-user detection scenario, in which the time is slotted but the users not perfectly synchronized. In this situation, the assumption that the parameters are known holds since pilot-aided algorithms may generally be used to efficiently estimate the mixing parameters.

Fig. 5 shows the mean performance of the proposed algorithm compared to the instantaneous MAP algorithm, when all the parameters (a, ϕ and τ) are uniformly and randomly chosen as described above.

VI. CONCLUSION

In this paper, a new efficient way to separate two BPSK sources by taking advantage of the time diversity induced by the channel and/or the imperfect synchronization of the sources at the receiver has been proposed. In real situation (*i.e.* with randomly delayed sources), the proposed algorithm turns out to be more efficient than already existing methods relying on symbol-spaced sampling combined with instantaneous MAP methods.

The issue related to real-time estimation of the mixing parameters still has to be investigated in order to completely solve the blind BPSK separation. Global optimization of parameters estimation/symbols extraction will also have to be carried out. Furthermore, high-frequency shifts and other exploitable diversity should be taken into account in order to better match real conditions and to further improve the BER.

REFERENCES

[1] A. Belouchrani and J.-F. Cardoso, "Maximum Likelihood Source Separation By the Expectation-Maximization Technique: Deterministic and Stochastic Implementation," in *Proc. NOLTA*, 1995.

[2] A. Kannan and V. U. Reddy, "Maximum Likelihood Estimation of Constellation Vectors for Blind Separation of Co-Channel BPSK Signals and its Performance Analysis," *IEEE Transactions on Signal Processing*, vol. 45, no. 7, pp. 1736–1741, 1997.

[3] O. Grellier and P. Comon, "Blind Separation of Discrete Sources," *IEEE Signal Processing Letters*, vol. 5, no. 8, pp. 212–214, 1998.

[4] ——, "Analytical Blind Discrete Source Separation," in *Eusipco*, 2000, pp. 7–10.

[5] A.-J. van der Veen, "Analytical Method for Blind Binary Signal Separation," *IEEE Transaction on Signal Processing*, vol. 45, no. 4, pp. 1078–1082, 1997.

[6] K. I. Diamantaras and E. Chassioti, "Blind Separation of N binary Sources from one Observation: a Deterministic Approach," *IEEE Transactions on Signal Processing*, pp. 93–98, 2000.

[7] A.-J. van der Veen and A. Paulraj, "An Analytical Constant Modulus Algorithm," *IEEE Transactions on Signal Processing*, vol. 44, no. 5, pp. 1–19, 1996.

[8] M. Pedzisz and A. Mansour, "Instantaneous MISO Separation of BPSK Sources," in *ICA*, 2006, pp. 862–867.

[9] S. Verdu, "Multiuser Detection," *Cambridge University Press*, 1998.

[10] T. Shilong, Z. Hui, and G. Na, "Single-Channel Blind Separation of Two QPSK Signals using Per-Survivor Processing," in *IEEE Asia-Pacific Conference on Circuits and Systems, Proceedings, APCCAS*, vol. 610041, 2008, pp. 473–476.

[11] T. Shilong, C. Shaohe, Z. Hui, and W. Jian, "Particle Filtering based Single-Channel Blind Separation of Co-Frequency Signals," in *International Symposium on Intelligent Signal Processing and Communication Systems*, 2007, pp. 582–585.