## BIT LOADING IN MIMO-PLC SYSTEMS WITH THE PRESENCE OF INTERFERENCE

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#### **ABSTRACT**

In broadband indoor power line communication (PLC) systems, multiple input multiple output (MIMO) techniques have been introduced to address the increasing demand for high data rates under the constraint of limited allocated bandwidth. Whereas the self inter-antenna interference can be dealt with on each subcarrier, both inter-carrier and inter-symbol interference can occur yielding sub-optimal bit loading if not considered. In this paper, we extend to the MIMO case the lowcomplexity bit/power allocation algorithm, called Reduced Complexity Algorithm (RCA), that we previously applied to the SISO case. Based on the Greedy principle, the RCA takes the interference into account to optimize the bit loading. We consider two MIMO schemes: optimum eigen beamforming and spatial multiplexing. Simulation results show the efficiency of the RCA in terms of throughput and computation cost in both cases.

*Index Terms*— Bit loading, MIMO, Interference, Power Line Communication, Greedy based approach.

### 1. INTRODUCTION

MIMO technology has been considered as a major key to increase the data rate in the next generation of broadband indoor power line communication systems. The HomePlug AV2 specification recommends the MIMO technology to achieve higher data rates as well as a larger coverage [1, 2]. MIMO-PLC is feasible since a protective earth (PE) wire is available in addition to phase (P) and neutral (N) wires. Hence, a MIMO scheme with two transmitting antennas can be applied in the context of PLC. In interference-free OFDM systems, the bit loading problem is solved by the Waterfilling algorithm [3, 4] for infinite input alphabet (referred to as continuous case in the remainder of the paper) and by Greedy-based algorithms such as bit-adding and bit-removal [5, 6] for finite input alphabet (referred to as discrete case in the remainder of the paper). The bit loading problem for SISO-OFDM systems with the presence of interference has been considered in [7, 8]. Recently, an efficient Greedybased approach has been proposed in [9] to solve the optimal bit/power allocation in SISO-PLC systems with interference resulting from an insufficient guard interval. Its achievable throughput is almost the same as the one obtained by the conventional Greedy approach. However, the complexity is significantly reduced.

In this paper, we consider the bit/power allocation problem for MIMO-Windowed OFDM PLC systems with the presence of inter-symbol and inter-carrier interference. The self inter-antenna interference is dealt with on each subcarrier. We resort to [10] to model the interference. The key point is the formulation of an equivalent bit-loading problem for a SISO-OFDM system and then the application of the reduced complexity algorithm introduced in [9] with some small modifications involving power constraints to solve it.

The paper is organized as follows. Section II describes the  $2\times 2$  MIMO-Windowed OFDM PLC system and the interference model developed in [10]. Section III is dedicated to the bit-loading optimization in a 2x2 MIMO-OFDM system with the presence of interference. After formulating the MIMO-OFDM bit-loading problem and the relation between the minimum required power and the allocated bit number, an equivalent SISO-OFDM bit-loading problem is introduced. Then we briefly remind the method proposed in [9] before applying it to solve the problem in two MIMO schemes: optimum eigen beamforming and spatial multiplexing. The simulation results and the complexity study in the context of IEEE P1901 standard are reported in Section IV. Section V is dedicated to conclusions and perspectives.

# 2. MIMO-WINDOWED OFDM PLC SYSTEM MODEL

We consider a 2x2 MIMO-OFDM system with L used subcarriers. The extension to  $n_T \times n_R$  MIMO-OFDM is straightforward. The transmission model restricted to subcarrier m is

$$\begin{bmatrix} y_1(m) \\ y_2(m) \end{bmatrix} = \begin{bmatrix} h_{11}(m) & h_{12}(m) \\ h_{21}(m) & h_{22}(m) \end{bmatrix} \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix} 
+ \begin{bmatrix} I_{11}(m) + I_{12}(m) \\ I_{21}(m) + I_{22}(m) \end{bmatrix} + \begin{bmatrix} \xi_1(m) \\ \xi_2(m) \end{bmatrix},$$
(1)

where  $y_j(m)$ ,  $x_j(m)$  and  $\xi_j(m)$  are the received signal, the transmitted signal and the complex circularly-symmetric Gaussian noise with variance  $\sigma_j^2(m)$  at antenna j on subcarrier m;  $h_{ji}(m)$  and  $I_{ji}(m)$  are the channel frequency response and interference term (resulting from an insufficient

guard interval) from antenna i to antenna j on subcarrier m. The matrix form of (1) reads

$$\mathbf{Y}(m) = \mathbf{H}(m)\mathbf{X}(m) + \mathbf{I}(m) + \Xi(m). \tag{2}$$

In SISO-PLC systems with Windowed-OFDM, the interference power on a given subcarrier depends on the signal power on all subcarriers [9, 11]. Similarly, the interference power in the MIMO system at a given receiver antenna and on a given subcarrier depends on the signal power at all antennas and on all subcarriers [10]. The covariance matrix of the noise plus interference, denoted by  $\mathbf{C}_{I+\Xi}(m)$ , equals [10]

$$\mathbf{C}_{I+\Xi}(m) = \begin{bmatrix} [\mathbf{W}_1 \mathbf{P}](m) + \sigma_1^2(m) & [\mathbf{W}_2 \mathbf{P}](m) \\ [\mathbf{W}_2^* \mathbf{P}](m) & [\mathbf{W}_3 \mathbf{P}](m) + \sigma_2^2(m) \end{bmatrix}$$
(3)

where  $\mathbf{P} = [P_1(1), ..., P_1(L), P_2(1), ..., P_2(L)]^T$  denotes the power allocation vector;  $\mathbf{W}_{\ell}$  of size  $L \times 2L$  is related to the interference (see [10] for full description) and  $[\mathbf{W}_i \mathbf{P}]$  stands for the product of  $\mathbf{W}_i$  and  $\mathbf{P}$ .

In this work, we assume that the zero-forcing detection on a subcarrier basis is used at receiver. The MIMO detection matrix denoted by  $\mathbf{A}(m)$  reads

$$\mathbf{A}(m) = \mathbf{H}^{\dagger}(m),\tag{4}$$

where † refers to the pseudo-inverse. Then, the signal of interest is

$$\mathbf{Z}(m) = \mathbf{A}(m)\mathbf{Y}(m) = \mathbf{X}(m) + \mathbf{A}(m)\left(\mathbf{I}(m) + \Xi(m)\right). (5)$$

Let  $a_{i\ell}(m)$  stand for the entry in *i*-th row and  $\ell$ -th column of matrix  $\mathbf{A}(m)$ . The post-detection signal to noise-plus-interference ratio corresponding to the *i*-th transmit antenna is denoted by  $SINR_i(m)$  and is equal to

$$SINR_i(m) = \frac{P_i(m)}{J_i(m) + N_i(m)},$$
(6)

where the interference power is

$$J_{i}(m) = \sum_{\ell=1}^{2} |a_{i\ell}(m)|^{2} [\mathbf{W}_{2\ell-1} \mathbf{P}](m) + 2\Re(a_{i2}^{*}(m)a_{i1}(m)[\mathbf{W}_{2} \mathbf{P}](m)), \quad (7)$$

and the noise power is  $N_i(m) = |a_{i1}(m)|^2 \sigma_1^2 + |a_{i2}(m)|^2 \sigma_2^2$ .

## 3. BIT LOADING FOR MIMO-PLC WITH THE PRESENCE OF INTERFERENCE

In this section, we address the problem of bit-loading for MIMO-OFDM systems assuming zero-forcing detection on a subcarrier basis and residual inter-carrier and inter-symbol interference. After formulating the optimization problem, we derive the relation between the allocated bit number and the corresponding minimum required power. We then extend to the MIMO-OFDM case the method proposed in [9] for the SISO-OFDM case.

#### 3.1. Bit-loading problem formulation

The bit-loading problem consists in finding the minimum power allocation that maximizes the achievable throughput. In the case of the 2x2 MIMO-PLC system, each subcarrier on each transmit antenna is allocated an integer number of bits, denoted by  $b_i(m)$ , which takes on values in  $\mathcal A$  specified by the standard. The optimization problem can thus be written as follows:

maximize 
$$\sum_{m=1}^{L} \sum_{i=1}^{2} b_{i}(m)$$
s.t. 
$$\sum_{m=1}^{L} \sum_{i=1}^{2} P_{i}(m) \leq P_{tot}$$

$$\sum_{i=1}^{2} P_{i}(m) \leq P_{max}(m)$$

$$b_{i}(m) \in \mathcal{A}$$

$$(8)$$

To solve (8), it is necessary to find a relation between the bit numbers  $\{b_i(m)\}$  and the corresponding minimum power allocation  $\{P_i(m)\}$ .

### 3.2. Relation between allocated power and bit loading

In the continuous case, the maximum throughput on subcarrier m at transmit antenna i is equal to  $\log_2\left(1+\frac{SINR_i(m)}{\Gamma}\right)$  where  $\Gamma$  is the SNR gap that models the practical modulation and coding scheme for a targeted symbol error rate. We consider the definition of  $\Gamma$  available in [12] i.e.,  $\Gamma=\frac{1}{3}\left[Q^{-1}(\frac{SER}{4})\right]^2$ , where SER is the target Symbol Error Rate and  $Q^{-1}(x)$  is the inverse tail probability of the standard normal distribution. The minimum power required to transmit reliably  $b_i(m)$  bits is computed from an equivalent SINR value  $\lambda_i(m)$  related to  $b_i(m)$  by

$$\lambda_i(m) = \frac{2^{b_i(m)} - 1}{\Gamma}, \ b_i(m) \in \mathcal{A}. \tag{9}$$

Using (6), we deduce the relation between allocated bit number and required power for i = 1, 2 and m = 1, 2, ..., L:

$$P_i(m) = \lambda_i(m) \Big( J_i(m) + N_i(m) \Big); \tag{10}$$

As  $J_i(m)$  is a function of the allocated power vector **P** (cf. (7)), we deduce from (9) and (10) that the number of bits allocated to a given subcarrier at a given transmit antenna is related not only to its allocated power but also to the power levels on the other subcarriers at all transmit antennas, which makes the problem complex.

Given a vector of number of bits **b** allocated to the subcarriers and on the antennas, we have to find the corresponding vector of allocated power **P** so that (10) is satisfied for all subcarriers.

(13)

By using matrix formulation, we can gather the  $2 \times L$  power constraints given by (10) into a single equation. To this end, we introduce the following notations:

$$\begin{split} & \boldsymbol{\Lambda} = \operatorname{diag} \Big( \lambda_1(1), ..., \lambda_1(L), \lambda_2(1), ..., \lambda_2(L) \Big) \\ & \boldsymbol{A}_1 = \operatorname{diag} \Big( |a_{11}(1)|^2, ..., |a_{11}(L)|^2, |a_{21}(1)|^2, ..., |a_{21}(L)|^2 \Big) \\ & \boldsymbol{A}_2 = \operatorname{diag} \Big( a_{12}^*(1) a_{11}(1), ..., a_{12}^*(L) a_{11}(L), \end{split} \tag{11}$$

$$\mathbf{A}_{3} = \operatorname{diag}\left(|a_{12}(1)|^{2}, ..., |a_{12}(L)|^{2}, |a_{22}(1)|^{2}, ..., |a_{22}(L)|^{2}\right)$$

 $a_{22}^*(1)a_{21}(1),...,a_{22}^*(L)a_{21}(L)$ 

$$\mathbf{Q} = \mathbf{A}_1 \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_1 \end{bmatrix} + 2\Re \left( \mathbf{A}_2 \begin{bmatrix} \mathbf{W}_2 \\ \mathbf{W}_2 \end{bmatrix} \right) + \mathbf{A}_3 \begin{bmatrix} \mathbf{W}_3 \\ \mathbf{W}_3 \end{bmatrix}$$
(15)

$$\mathbf{N} = [N_1(1), ..., N_1(L), N_2(1), ..., N_2(L)]^T$$
(16)

$$\mathbf{I} = \operatorname{diag}\left(\underbrace{1, 1, ..., 1}_{2L}\right) \tag{17}$$

The rewriting of  $2 \times L$  equations of (10) under matrix form yields

$$P = \Lambda \Big( QP + N \Big) \Rightarrow P = \Big( I - \Lambda Q \Big)^{-1} \Lambda N, \tag{18}$$

which gives the relation between the bit loading vector ( $\Lambda$  depends on **b**) and the required minimum power vector **P**.

### 3.3. Adaptation of the RCA algorithm [9]

In this section we describe an algorithm to solve the optimization problem (8) where the cost function satisfies (18). (18) is the same as the equation (15) in [9] (SISO-PLC bit loading problem in the presence of interference), except that the matrices are twice as big. However by doubling the number of subcarriers in the SISO-PLC problem considered in [9], we do not obtain a problem equivalent to (8). The difference comes from the power constraint  $\sum_{i=1}^2 P_i(m) \leq P_{max}(m)$ .

In the following, we apply the method adopted in [9]. The algorithm in [9] is derived from a Greedy algorithm. The conventional Greedy algorithm is a popular iterative procedure for discrete optimization problems [13]. It starts from a null bit loading vector and each iteration consists in adding one bit on the subcarrier yielding the minimum additional power increase, while satisfying all power constraints. According to (18), the power vector computation cost corresponds to a  $2L \times 2L$  matrix inversion computation cost. Thus, the complexity per iteration in the conventional Greedy algorithm is the product of the number of subcarriers that can be loaded up by the complexity of a matrix inversion. Due to its high complexity, the conventional Greedy algorithm can hardly be used in practice.

To reduce the computation cost, we have proposed in [9] the Reduced Complexity Algorithm (RCA). Its principle consists in the simplification of the cost function (which is used

**Table 1**: Computation cost comparison.

	Complexity	Number of	Number
Algorithm	per iteration	iterations	of matrix
			inversions
Z-GR	$\approx \mathcal{O}(\beta U^4)$	$N_s$	$N_s * \mathcal{O}(\beta U)$
EPA-GR	$\approx \mathcal{O}(4\beta_1 U^3)$	$N_1 << N_s$	$N_1 * \mathcal{O}(4\beta_1)$
RCA	$pprox \mathcal{O}(U^3)$	$N_K << N_s$	$N_K * \mathcal{O}(1)$

to determine the subcarriers to be loaded up) in the Greedy procedure, the substitution of the null-vector initialization by the constant power water-filling (CPWF) initialization and the efficient selection of K subcarriers to load up at every iteration. Simulations in [9] have shown that the RCA almost performs the same as the conventional Greedy algorithm in terms of throughput, while significantly reducing the complexity. For the sake of further complexity reduction, we can use the Equal Power Allocation (EPA) instead of the CPWF as initialization. With EPA, the total power  $P_{tot}$  is uniformly allocated between the subcarriers and the antennas.

The extension to the MIMO case is straightforward, the aforementioned power constraint difference will be taken into account in the iterative procedure while selecting the subcarriers to be loaded up. The complexity comparison between the conventional Greedy algorithm (referred to as Z-GR), the Greedy algorithm with EPA initialization (referred to as EPA-GR) and the reduced complexity algorithm with EPA initialization (referred to as RCA) is illustrated in Table 1 where 1  $<<\beta_1<\beta$ , U is the length of the power vector **P** and  $N_s$ ,  $N_1$ ,  $N_K$  are the total number of iterations used in the Z-GR, the EPA-GR and the RCA respectively. In SISO systems, U is equal to L, the number of active subcarriers. In 2x2 MIMO systems, U is equal to 2L. Further details about the complexity analysis can be found in [9]. We observe that the RCA reduces not only the number of iterations but also the computation cost per iteration, due to the decrease of the average number of subcarriers for which the cost function has to be evaluated in the Greedy procedure.

## 3.4. Application to the spatial multiplexing scheme

In the spatial multiplexing case, no precoding is done and then the detection matrix is  $\mathbf{A} = \mathbf{H}^{\dagger}$ .

## 3.5. Application to the optimum eigen beamforming scheme

The optimum eigen beamforming scheme makes the MIMO transmission equivalent to the superposition of multiple parallel independent SISO transmissions, which enables to increase the data rate compared to the spatial multiplexing scheme. The Singular Value Decomposition (SVD) of the channel matrix [14] enables to obtain the independent eigen

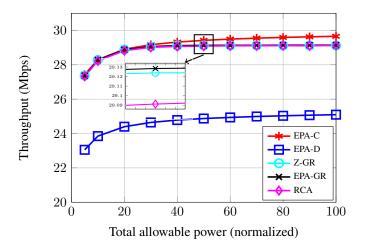
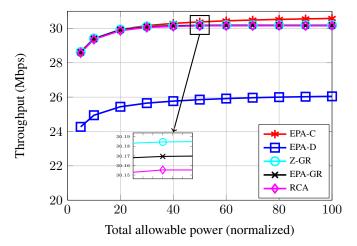


Fig. 1: Achievable throughput (spatial multiplexing).



**Fig. 2**: Achievable throughput (optimum eigen beamforming).

modes of the channel:

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H \tag{19}$$

where  $\mathbf{U}$ ,  $\mathbf{V}$  are unitary matrices, i.e.  $\mathbf{U}^{-1} = \mathbf{U}^H$  and  $\mathbf{V}^{-1} = \mathbf{V}^H$ , H refers to the Hermitian operator and  $\mathbf{D}$  is a diagonal matrix containing the singular values of  $\mathbf{H}$ .

Assuming perfect eigen beamforming with precoding matrix V, the channel matrix H is replaced by HV in (2) and the detection matrix A reads [1]

$$\mathbf{A} = (\mathbf{H}\mathbf{V})^{-1} = \mathbf{D}^{-1}\mathbf{U}^{H}.$$
 (20)

#### 4. SIMULATION RESULTS

We consider the IEEE P1901 standard as described in [15]. The simulation parameters are the following:

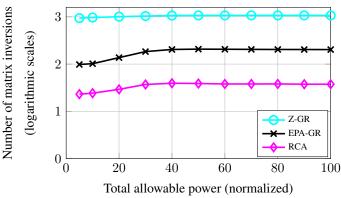


Fig. 3: Average complexity (spatial multiplexing).

- Exploitable subcarriers defined in the IEEE P1901 standard [15] with sampling rate  $f_s = 100$  MHz, frequency shift  $\Delta f = 24.414$  kHz.
- Channel model: SISO-PLC Class 2 channel of Tonello's model [16] and 2x2 MIMO-PLC (same circuit using P-N, N-PE) of Hashmat model [17].
- Noise model: 2x2 MIMO-PLC independent noise of Esmailian model [18].
- Spectral mask constraint:  $P_{max}(m) = 1$ ,  $\forall m \in \{1, 2, ..., L\}$  (normalized to  $P_0 \Delta f$  where  $P_0 = -55$  dBm (1 Hz) is the spectral mask value defined by the IEEE P1901 standard and  $\Delta f$  is the frequency shift) [15].
- $P_{tot}$  (normalized to  $P_0 \Delta f$ ) varies from 10 to 100.
- $\mathcal{A} = \{1, 2, 3, 4, 6, 8, 10, 12\}$  (bits).
- Number of channel realizations: 1000.

For the sake of simplicity, we only take into account the first 100 useful subcarriers out of 917 defined in IEEE 1901 standard in our simulation, i.e. L=100. The covariance matrix  $\mathbf{C}_{I+\Xi}$  is computed from the formula of [10] with the value of guard interval of 5.56  $\mu s$ , the minimum allowable value defined in the IEEE P1901 standard. In this case, the effect of interference on the system performance is significant. In the RCA, the number of couples (subcarrier, antenna) chosen to simultaneously increase the bit number is set to 5.

We illustrate in Figs. 1 and 2 the achieved throughput for Z-GR, EPA-GR and RCA in the case of spatial multiplexing and of optimum eigen beamforming. In the legend, 'EPA-D' corresponds to the initialization state of EPA-GR and RCA, while 'EPA-C' refers to the continuous case with equal power allocation.

In both cases, the performance of Z-GR and EPA-GR in terms of achievable throughput are almost the same. The achievable throughput of the proposed RCA is slightly degraded when compared to the Z-GR and significantly improved as compared to the EPA-D. The maximal shift of achievable throughput between RCA and the Z-GR is about 0.2% and in average, the RCA increases the achievable

**Table 2**: Throughput and run-time comparison (spatial multiplexing).

Algorithm	Throughput	Number of	Run-time
	(Mbits)	inversions	(s)
EPA-C	29.42	_	_
Z-GR	29.12	1070	17.2
EPA-GR	29.13	207	2.7
RCA	29.09	38	0.5
EPA-D	24.90	_	_

throughput by 17% w.r.t the EPA-D in both cases. In addition, we can see that the achievable throughput obtained by the RCA with optimum eigen beamforming is increased by about 4% (in average) as compared to the one of spatial multiplexing. Hence, as compared to the spatial multiplexing, the achievable throughput gain obtained by the optimum eigen beamforming is not really significant in 2x2 MIMO-PLC systems with significant interference resulting from an insufficient guard interval. The achievable throughput comparison when the total normalized power  $P_{tot}=50$  and with spatial multiplexing is shown in Table 2.

In Fig. 3, the complexity of different algorithms is illustrated by the total number of matrix inversions used to find the bit/power allocation in the case of spatial multiplexing. Note that the vertical axis is in logarithmic scale. The complexity of the algorithms in the optimum eigen beamforming case is almost the same as in the spatial multiplexing case. In both cases, the RCA has reduced the complexity by 97% as compared to the Z-GR, while achieving almost the same throughput. The run-time comparison when  $P_{tot}=50$  is also shown in Table 2. As compared to the Z-GR, the RCA only degrades the achievable throughput by 0.1%. However, the run-time is reduced by about 35 times.

#### 5. CONCLUSION

In this paper, we have extended the Reduced Complexity Algorithm (RCA) for the problem of achievable throughput maximization in SISO-PLC systems to MIMO-PLC systems in presence of interference. To this end, we have transformed the MIMO-PLC problem into an equivalent SISO-PLC problem. Simulation results have clearly shown that the reduced complexity algorithm is also efficient for the problem of achievable throughput maximization in MIMO-PLC systems in presence of interference, i.e. the achievable throughput loss is negligible and the complexity is significantly reduced as compared to the Greedy solutions. It is also shown that the RCA algorithm outperforms the EPA algorithm. Therefore, it is a good candidate to solve resource management problems for single-user Windowed-OFDM systems in the presence of significant interference for both SISO and MIMO systems. Our future work aims to solve the bit/power allocation problem for the multi-user with/without the presence of interference in SISO/MIMO-OFDM systems.

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