# EFFICIENT COOPERATIVE LOCALIZATION ALGORITHM IN LOS/NLOS ENVIRONMENTS

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## ABSTRACT

The well-known cooperative localization algorithm, 'sum-product algorithm over a wireless network' (SPAWN) has two major shortcomings, a relatively high computational complexity and a large communication load. Using the Gaussian mixture model with a model selection criterion and the sigma-point (SP) methods, we propose the SPAWN-SP to overcome these problems. The SPAWN-SP easily accommodates different localization scenarios due to its high flexibility in message representation. Furthermore, harsh LOS/NLOS environments are considered for the evaluation of cooperative localization algorithms. Our simulation results indicate that the proposed SPAWN-SP demonstrates high localization accuracy in different localization scenarios, thanks to its high flexibility in message representation.

***Index Terms*—** Cooperative localization, SPAWN, low-complexity, sigma-point methods

## 1. INTRODUCTION

For sensor network localization, a plethora of cooperative localization algorithms have been proposed, including non-Bayesian algorithms, such as [1, 2], and Bayesian ones. The promising Bayesian cooperative localization algorithm, the so-called 'sum-product algorithm over a wireless network' (SPAWN), has drawn an increasing attention [3]. However, it suffers from a high computational complexity and a large communication overhead due to the particle-based approximation of messages. Recent work showed that the parametric SPAWN, which approximates the messages using certain parametric models, requires significantly less computational and communication efforts. However, the parametric models must be specially tailored for different localization scenarios [4, 5]. In sigma point belief propagation (SPBP), the belief propagation (BP) or sum-product algorithm (SPA) is reformulated in a higher dimensional space so that the belief update procedure turns to a nonlinear filtering process, which is addressed using the sigma point filters [6].

The original contributions of this paper are as follows. By representing the messages in a new efficient way, with the aid of the sigma-point (SP) methods, we propose a new low-complexity SPAWN variant, the SPAWN-SP . The SPAWN-SP is comprehensively evaluated in terms of complexity, communication load and localization accuracy. Unlike in previous work on SPAWN, we consider mixed LOS/NLOS environments with imperfect NLOS identification.

This paper is organized as follows. Section 2 introduces the problem at hand. Section 3 briefly reviews the particle-based SPAWN and the existing parametric SPAWN. In Section 4 we propose the SPAWN-SP algorithm. The localization accuracy of the SPAWN-SP is comprehensively evaluated in Section 5. Finally, Section 6 concludes this paper.

## 2. PROBLEM FORMULATION

Consider a wireless sensor network with $N$ sensor nodes in a two-dimensional (2-D) space, although extension to the 3-D case is straightforward. There are $N_u$ nodes with unknown positions, called agents, and $N_a$ nodes with given positions, called anchors. Let $\mathcal{N}_{\text{all}} = \{1, 2, \cdots, N\}$ be the index set of all nodes and $\mathcal{N}_u = \{1, 2, \cdots, N_u\}$ be the index set of all agents. The 2-D position of node $i$ is denoted by $\mathbf{x}_i = [x_i, y_i]^T$ and it is modeled stochastically with a priori probability $p_i(\mathbf{x}_i)$ for $i \in \mathcal{N}_{\text{all}}$. Restricted by the communication range $R_c$, node $i$ can communicate with only a subset of nodes, which are called its neighbors and whose index set is denoted by $\mathcal{N}_{\rightarrow i}$.

The statistical measurement model is given by

$$z_{ji} = d_{ji} + v_{ji}, \quad j \in \mathcal{N}_{\rightarrow i}, i \in \mathcal{N}_{\text{all}}, \tag{1}$$

where $z_{ji}$ is the distance measurement obtained at node $i$, $d_{ji} = ||\mathbf{x}_j - \mathbf{x}_i||$ is the Euclidean distance between two nodes and $v_{ji}$ is the measurement error. A collection of all measurements is denoted by a vector $\mathbf{z}$. We assume that the LOS and NLOS measurement error follows the statistical models $p_{\text{L}}(v; \boldsymbol{\beta}_{\text{L}})$ with the parameter vector $\boldsymbol{\beta}_{\text{L}}$ and $p_{\text{NL}}(v; \boldsymbol{\beta}_{\text{NL}})$ with the parameter vector $\boldsymbol{\beta}_{\text{NL}}$, respectively. Using NLOS identification techniques, for instance from [7], we assume that NLOS identification results of all measurements are available.

With the two assumptions that all measurements are statistically independent and the a priori probabilities of all sensors are independent, the joint posterior distribution $p(\mathbf{x}_1, \cdots, \mathbf{x}_N|\mathbf{z})$ is written as

$$p(\mathbf{x}_1, \cdots, \mathbf{x}_N|\mathbf{z}) \propto p(\mathbf{z}|\mathbf{x}_1, \cdots, \mathbf{x}_N)\, p(\mathbf{x}_1, \cdots, \mathbf{x}_N) = \prod_{i=1}^{N} p_i(\mathbf{x}_i) \prod_{j\in\mathcal{N}_{\to i}} p(z_{ji}|\mathbf{x}_i, \mathbf{x}_j)\,. \tag{2}$$

The goal is to find the marginal posterior distribution $p(\mathbf{x}_i|\mathbf{z})$ of each agent position and ultimately give a point estimate.

## 3. REVISITING COOPERATIVE LOCALIZATION ALGORITHMS

### 3.1. The Classical (particle-based) SPAWN

The marginal posterior distribution $p(\mathbf{x}_i|\mathbf{z})$ could be obtained by integrating $p(\mathbf{x}_1, \cdots, \mathbf{x}_N|\mathbf{z})$, however at a prohibitively high computational cost. Alternatively, using the sum-product algorithm (SPA), the marginalization is largely facilitated, giving rise to the so-called 'SPA over a wireless network' (SPAWN) [3]. The gist of the SPAWN is to iteratively update belief messages according to the update procedure as follows:

$$I_{ji}^{\eta}(\mathbf{x}_i) = \int p(z_{ji}|\mathbf{x}_i, \mathbf{x}_j) B_j^{\eta}(\mathbf{x}_j) d\mathbf{x}_j \tag{3}$$

$$B_i^{(\eta+1)}(\mathbf{x}_i) \propto p_i(\mathbf{x}_i) \prod_{j\in\mathcal{N}_{\to i}} I_{ji}^{\eta}(\mathbf{x}_i), \tag{4}$$

where the superscript $\eta$ is the iteration index, $p(z_{ji}|\mathbf{x}_i, \mathbf{x}_j)$ is a likelihood function and $B_j^{\eta}(\mathbf{x}_j)$ and $I_{ji}^{\eta}(\mathbf{x}_i)$ are the belief message and internal message, respectively. The internal message $I_{ji}(\mathbf{x}_i)$ is maintained only inside agent $i$ and contributes to the update of $B_i(\mathbf{x}_i)$. Note that $B_i(\mathbf{x}_i)$ indicates uncertainty about $\mathbf{x}_i$. After several iterations, it approaches the marginal distribution $p(\mathbf{x}_i|\mathbf{z})$ under certain conditions [3].

The nonlinear relationship in $p(z_{ji}|\mathbf{x}_i, \mathbf{x}_j)$ and the non-Gaussian uncertainty of $B_j(\mathbf{x}_j)$ make the analytical evaluation of Eq. (3) and Eq. (4) infeasible. Representing the messages based on particles, enables the update procedure, giving rise to the particle-based SPAWN. We choose node $i$ to illustrate the update procedure at the $\eta^{\text{th}}$ iteration. Node $i$ starts with broadcasting its belief message $\{\mathbf{x}_i^{r,\eta}, w_i^{r,\eta}\}_{r=1}^{R}$, where $\mathbf{x}_i^{r,\eta}$ is the particle and $w_i^{r,\eta}$ is the corresponding weight. Once $B_j^{\eta}(\mathbf{x}_j)$ is received, $\{\mathbf{x}_{ji}^{r,\eta}, w_{ji}^{r,\eta}\}_{r=1}^{nR}$ ($n$ is a scaling factor) are generated to approximate $I_{ji}^{\eta}(\mathbf{x}_i)$ as detailed in [5]. Next, the particle-based approximation of $B_i^{(\eta+1)}(\mathbf{x}_i)$ is obtained by drawing $\left\{\mathbf{x}_i^{r,(\eta+1)}\right\}_{r=1}^{nR}$ from a proposal distribution $q^{\eta}(\mathbf{x}_i)$, e.g., the sum of internal messages, and subsequently computing the weights

$$w_i^{r,(\eta+1)} \propto \frac{p_i(\mathbf{x}_i^{r,(\eta+1)}) \prod_{j\in\mathcal{N}_{\to i}} I_{ji}^{\eta}(\mathbf{x}_i^{r,(\eta+1)})}{q^{\eta}\left(\mathbf{x}_i^{r,(\eta+1)}\right)} \tag{5}$$

$$q^{\eta}(\mathbf{x}_i^{r,(\eta+1)}) = \sum_{j\in\mathcal{N}_{\to i}} I_{ji}^{\eta}(\mathbf{x}_i^{r,(\eta+1)}), \tag{6}$$

where $I_{ji}^{\eta}(\mathbf{x}_i)$ approximates to

$$I_{ji}^{\eta}(\mathbf{x}_i) \simeq \sum_{r=1}^{nR} w_{ji}^{r,\eta} \mathcal{N}\left(\mathbf{x}_i; \mathbf{x}_{ji}^{r,\eta}, \mathbf{H}\right), \tag{7}$$

where $\mathbf{H}$ is an appropriately chosen covariance matrix and $\mathcal{N}\left(\mathbf{x}_i; \mathbf{x}_{ji}^{r,\eta}, \mathbf{H}\right)$ denotes the Gaussian distribution with mean $\mathbf{x}_{ji}^{r,\eta}$ and covariance matrix $\mathbf{H}$. The analytical approximation of $I_{ji}^{\eta}(\mathbf{x}_i)$ in Eq. (7) is visualized in Fig. 1a, and Figs. 1b–1d depict other approximations, which will be explained later. Finally, a further resampling step can be conducted to achieve equally weighted samples $\left\{\mathbf{x}_i^{r,(\eta+1)}\right\}_{r=1}^{R}$ and $2R$ real numbers are required to represent $B_i^{(\eta+1)}(\mathbf{x}_i)$. In Table 1 gives the complexity and communication requirement based on one agent at one iteration step are listed. The computation of the weights using Eq. (5) and Eq. (7) is the most computationally demanding part, requiring $\mathcal{O}(R^2)$ operations; the communication requirement is $2R$ real numbers.

### 3.2. The Parametric SPAWN

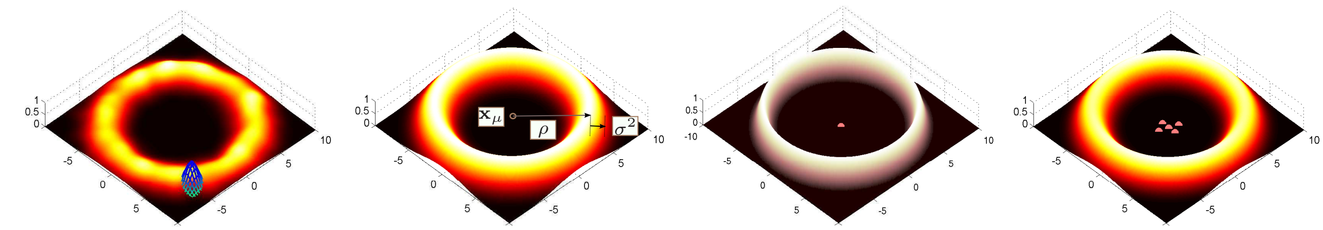
With the aim to reducing complexity and the communication load, the parametric SPAWN has been proposed. The work in [5] demonstrates that the belief message $B_i^{\eta}(\mathbf{x}_i)$ can be approximated by a mixture of $K$ Gaussian distributions,

$$B_i^{\eta}(\mathbf{x}_i) \simeq \sum_{k=1}^{K} \alpha_i^{k,\eta} \mathcal{N}\left(\mathbf{x}_i; \boldsymbol{\mu}_i^{k,\eta}, \boldsymbol{\Sigma}_i^{k,\eta}\right), \tag{8}$$

where $\alpha_i^{k,\eta}$ is the mixing component. Accordingly, the communication overhead reduces to $6K-1$ real numbers, where $K$ is negligible as compared to $R$. For an efficient computation of the weights, according to Eq. (5), analytical approximation of the internal message is highly desirable. With the assumption of a Gaussian distributed measurement error, the following parametric model for the internal message was proposed in [4],

$$I_{ji}^{\eta}(\mathbf{x}_i) \simeq \sum_{k=1}^{K} \alpha_{ji}^{k,\eta}\, \mathcal{C}\left(\mathbf{x}_i; \rho_{ji}^{k,\eta}, (\mathbf{x}_\mu)_{ji}^{k,\eta}, (\sigma^2)_{ji}^{k,\eta}\right), \tag{9}$$

where $\mathcal{C}(\mathbf{x}; \rho, \mathbf{x}_\mu, \sigma^2)$ is a specially designed function, depicted in Fig. 1b. More details about the $\mathcal{C}$ function can be found in [4]. Instead of Eq. (7), the parametric representation Eq. (9) is utilized for the calculation of the weights in Eq. (5), which requires $\mathcal{O}(R)$ operations.




(a) (b) (c) (d)

**Fig. 1**: Visualization of the analytical approximations of $I_{ji}(\mathbf{x}_i)$ in the special case of $B_j(\mathbf{x}_j)$ being unimodal, namely $K = 1$. (a) shows the analytical approximation Eq. (7), summation of multiple Gaussian kernels, where the blue net represents one Gaussian kernel. (b) depicts the parametric approximation Eq. (9) and gives a simple graphical explanation about the parameters $\{\rho, \mathbf{x}_\mu, \sigma^2\}$ of $\mathcal{C}(\mathbf{x}; \rho, \mathbf{x}_\mu, \sigma^2)$. (c) shows the likelihood function $p(z_{ji}|\mathbf{x}_i, \mathbf{s}_j^r)$, where $\mathbf{x}_i$ is the argument and the sigma point $\mathbf{s}_j^r$, depicted as the red point, is a known variable. (d) visualizes the analytical approximation Eq. (13) in the SPAWN-SP, that summation of several $p(z_{ji}|\mathbf{x}_i, \mathbf{s}_j^r)$. As the number of particles goes to infinity, Eq. (7) achieves the most accurate approximation. Among the other two analytical approximations, Eq. (13) provides more flexibility in representing messages of different shapes, i.e., symmetric or asymmetric, since the likelihood function is preserved in Eq. (13); while Eq. (9) is a symmetric parametric model.

## 4. THE SPAWN-SP

Despite the reduction of complexity and communication overhead, two problems of the existing parametric SPAWN remain to be solved. First, a fixed number of Gaussian components in Eq. (8) fails to achieve a good balance of the communication load and the accuracy of representing the belief messages. We propose to choose an appropriate number of Gaussian components individually for each belief message, making use of the greedy expectation maximization (EM) algorithm [8]. Second, the parametric model for the internal messages requires to be specially tailored to different localization scenarios, otherwise, a model mismatch may lead to localization performance degradation. In our work, we develop, with the aid of sigma-point methods, a novel analytical approximation comprised of the original form of the likelihood function. The proposed algorithm is highly flexible to measurement error distributions.

The belief message $B_i^\eta(\mathbf{x}_i)$ in Eq. (8) should be rewritten, with $K_i^\eta$ instead of $K$, as

$$B_i^\eta(\mathbf{x}_i) \simeq \sum_{k=1}^{K_i^\eta} \alpha_i^{k,\eta} \mathcal{N}\left(\mathbf{x}_i; \boldsymbol{\mu}_i^{k,\eta}, \boldsymbol{\Sigma}_i^{k,\eta}\right), \tag{10}$$

where $K_i^\eta$ can be determined according to a model selection criterion, based on a sequence of mixture parameters computed from the greedy EM [8]. Subsequently, we have the internal message with the following representation

$$I_{ji}^\eta(\mathbf{x}_i) = \sum_{k=1}^{K_j^\eta} \alpha_j^{k,\eta} \int p\left(z_{ji}|\mathbf{x}_i, \mathbf{x}_j\right) \mathcal{N}\left(\mathbf{x}_j; \boldsymbol{\mu}_j^{k,\eta}, \boldsymbol{\Sigma}_j^{k,\eta}\right) d\mathbf{x}_j. \tag{11}$$

It is apparent from Eq. (11) that the integrand is of the special form: *nonlinear function* $\times$ *Gaussian distribution function*. Such an integral can be effectively approximated by sigma-point methods, such as the Unscented transform [9]. The principle is to deterministically choose a small set of sigma points and then approximate the integral using the weighted summation of the nonlinear function at those sigma points [9]. For the integral $\mathcal{G}_{ji}^{k,\eta}(\mathbf{x}_i)$, which is the short-hand notation of the integral explicitly shown in Eq. (11), a small set of weighted sigma points is determined and denoted by $\left\{\mathbf{s}_j^{k,r,\eta}, u_j^{k,r,\eta}\right\}_{r=1}^{R_{\text{sp}}}$, where $\mathbf{s}_j^{k,r,\eta}$ is the sigma point, $u_j^{k,r,\eta}$ is the weight, and $R_{\text{sp}}$ is negligible as compared to $R$. Accordingly, the analytical approximation of $\mathcal{G}_{ji}^{k,\eta}(\mathbf{x}_i)$ is obtained as

$$\mathcal{G}_{ji}^{k,\eta}(\mathbf{x}_i) \simeq \sum_{r=1}^{R_{\text{sp}}} u_j^{k,r,\eta} p\left(z_{ji}|\mathbf{x}_i, \mathbf{s}_j^{k,r,\eta}\right). \tag{12}$$

By inserting Eq. (12) into Eq. (11), we yield the final analytical approximation of the internal messages

$$I_{ji}^\eta(\mathbf{x}_i) \simeq \sum_{k=1}^{K_j^\eta} \alpha_j^{k,\eta} \sum_{r=1}^{R_{\text{sp}}} u_j^{k,r,\eta} p\left(z_{ji}|\mathbf{x}_i, \mathbf{s}_j^{k,r,\eta}\right). \tag{13}$$

A visualization of this analytical approximation is given in Fig. 1c and Fig. 1d. Note that a special case of Eq. (13) is

$$I_{ji}^\eta(\mathbf{x}_i) \simeq \sum_{r=1}^{R} w_j^\eta p\left(z_{ji}|\mathbf{x}_i, \mathbf{x}_j^{r,\eta}\right), \tag{14}$$

when the whole set of particles of $B_j^\eta(\mathbf{x}_j)$ is used [10], instead of $K_j^\eta$ sets of sigma points. For the calculation of the weights, Eq. (13) is used instead of Eq. (7) or Eq. (9), requiring operations of the order $\mathcal{O}(R)$. We name the proposed algorithm as SPAWN-SP and the complete algorithm is outlined in Algorithm 1. The communication overhead and complexity of the SPAWN-SP are given in Table 1, where $\bar{K} = \frac{1}{N_{\text{ite}}} \frac{1}{N} \sum_{\eta=1}^{N_{\text{ite}}} \sum_{i=1}^{N} K_i^\eta$ and $N_{\text{ite}}$ is the number of iterations.

**Algorithm 1** SPAWN-SP

**Initialize:**
$B_i^1(\mathbf{x}_i) = p_i(\mathbf{x}_i),\, i \in \mathcal{N}_{\text{all}}$ .
**for** $\eta = 1, \ldots, N_{\text{ite}}$ **do**
  **for** all nodes in parallel, e.g., node $i$ **do**
    Broadcast $\left\{\alpha_i^{k,\eta}, \boldsymbol{\mu}_i^{k,\eta}, \boldsymbol{\Sigma}_i^{k,\eta}\right\}_{k=1}^{K_i^\eta}$ .
    Receive belief messages from neighbors.
    Draw $nR$ particles from the proposal distribution.
    For each neighbor's belief message, generate weighted sigma points.
    Compute the weights $\left\{w_i^{r,(\eta+1)}\right\}_{r=1}^{nR}$ using Eq. (5) and Eq. (13) .
    Refine the parameter set $\left\{\alpha_i^{k,(\eta+1)}, \boldsymbol{\mu}_i^{k,(\eta+1)}, \boldsymbol{\Sigma}_i^{k,(\eta+1)}\right\}_{k=1}^{K_i^{(\eta+1)}}$ .
  **end for**
**end for**

Note that the SPAWN-SP and the SPBP are two totally different algorithms. In the SPAWN-SP, the SP plays a role in facilitating the analytical representation of the internal messages. In the SPBP, the belief update is reformulated into a nonlinear filtering process, which is addressed using the SP filters. However, an unrealistic prerequisite in the SPBP is that the belief messages are characterized using their first two moments, which are surely insufficient to characterize the multimodal messages.

Two options for the likelihood function $p\left(z_{ji}|\mathbf{x}_i, \mathbf{x}_j\right)$ are considered. The NLOS identification result $\hat{H}_{ji}$ has binary values 0 and 1 for LOS and NLOS, respectively. The most straightforward alternative reads

$$p\left(z_{ji}|\mathbf{x}_i, \mathbf{x}_j\right) = \begin{cases} p_{\text{L}}\left(z_{ji} - d_{ji}; \boldsymbol{\beta}_{\text{L}}\right) & \text{if } \hat{H}_{ji} = 0, \\ p_{\text{NL}}\left(z_{ji} - d_{ji}; \boldsymbol{\beta}_{\text{NL}}\right) & \text{otherwise.} \end{cases} \tag{15}$$

The second alternative [11] consists of both hypotheses

$$\begin{aligned} p\left(z_{ji}|\mathbf{x}_i, \mathbf{x}_j\right) = & \Pr\left(\text{L}|\hat{H}_{ji}\right) p_{\text{L}}\left(z_{ji} - d_{ji}; \boldsymbol{\beta}_{\text{L}}\right) + \\ & \Pr\left(\text{NL}|\hat{H}_{ji}\right) p_{\text{NL}}\left(z_{ji} - d_{ji}; \boldsymbol{\beta}_{\text{NL}}\right), \end{aligned} \tag{16}$$

where $\Pr\left(\text{L}|\hat{H}_{ji}\right)$ is the probability of $z_{ji}$ being a LOS measurement conditional on the identification result $\hat{H}_{ji}$.

## 5. SIMULATION RESULTS

### 5.1. Simulation setup

The localization accuracy of the SPAWN-SP is extensively evaluated in comparison with the particle-based SPAWN and the parametric SPAWN. The SPBP is not considered as it requires rather accurate initialization to achieve comparable accuracy as compared to other SPAWN variants. For each simulation, 100 Monte Carlo trials are conducted. The sensor network is defined over a 40 m $\times$ 40 m area and contains 6 anchors and 10 agents. A fixed uniform deployment is defined for these anchors. The agents' positions are randomly chosen for each trial, according to a uniform distribution. A collection of other parameters is listed in Table 2.

| **Algorithms** | **Complexity** | **Communication Load** |
|---|---|---|
| Particle-based SPAWN | $\mathcal{O}(R^2)$ | $2R$ |
| Parametric SPAWN | $\mathcal{O}(R)$ | $6K-1$ |
| SPAWN-SP | $\mathcal{O}(R)$ | $6\bar{K}-1$ |

**Table 1**: Analysis on complexity and communication load.

| **Parameters** | **Values** | **Description** |
|---|---|---|
| $R$ | 500 | Number of particles for belief messages |
| $n$ | 2 | Scaling factor |
| $R_{\text{sp}}$ | 5 | Number of sigma points |
| K | 3 | Mixture components |
| $N_{\text{ite}}$ | 15 | Number of iterations |

**Table 2**: Parameters for simulations.

### 5.2. Simulation results

First, the impact of the two likelihood functions Eq. (15) and Eq. (16) on the localization accuracy is investigated. Two statistical models of measurement errors, namely $p_{\text{L}}$ and $p_{\text{NL}}$, are set as $\mathcal{N}(0, 0.2^2)$ and $\mathcal{N}(\mu_{\text{NLOS}}, 3.2^2)$, respectively. Additionally, $R_c$ is set to 20 m. The localization RMSEs are depicted in Fig. 2, where, the 'SPAWN-SP-hard-10' in the legend means the SPAWN-SP with Eq. (15) in the case of a misidentification rate of $10\%$. It is apparent from Fig. 2, that the SPAWN-SP-soft consistently outperforms the SPAWN-SP-hard over $\mu_{\text{NLOS}}$ and a range of misidentification rates, revealing the benefit provided by Eq. (16) thanks to its two hypotheses. As the misidentification rate goes from $10\%$ up to $15\%$, the advantageous performance of the SPAWN-SP-soft over the SPAWN-SP-hard becomes more apparent.

In the following simulations, Eq. (16) is chosen for all SPAWN variants. Next, the proposed algorithm is evaluated over different communication ranges $R_c$ with $p_{\text{L}}$ and $p_{\text{NL}}$ set as $\mathcal{N}(0, 0.2^2)$ and $\mathcal{N}(4.4, 3.2^2)$, respectively [11]. As depicted in Fig. 3, the localization RMSEs of three algorithms monotonically go down as $R_c$ increases. This result is quite logical, since increasing the communication range results in more distance measurements and accordingly the localization performance is improved. The SPAWN-SP slightly outperforms the parametric SPAWN and the particle-based SPAWN for $R_c$ being larger than 20 m. For $R_c$ smaller than 20 m, the parametric SPAWN is slightly superior to the others, since a good match between its parametric model Eq. (9) and the true internal message exists in this case. For increased $R_c$, the symmetric model Eq. (9) deviates from the true internal message. Namely, in the case of Gaussian distributed noise

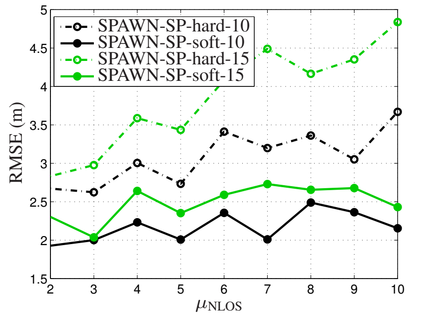



**Fig. 2**: Effect of two likelihood functions on the SPAWN-SP over different $\mu_{\mathrm{NLOS}}$.

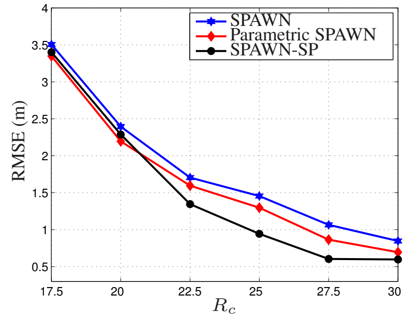



**Fig. 3**: Comparison of algorithms over communication ranges $R_c$.

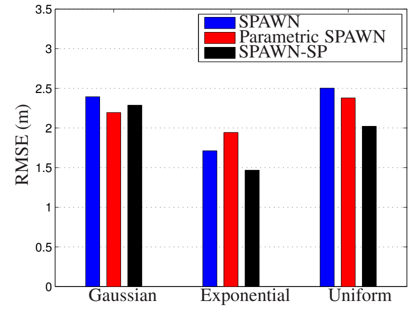



**Fig. 4**: Comparison of algorithms over different NLOS distributions.

and small distance measurements, the analytical model Eq. (9) achieves a good match. The inferior performance of the particle-based SPAWN is caused by insufficient particles.

In the last simulation, three algorithms are compared in three different scenarios, where the NLOS error follows the Gaussian distribution, the exponential distribution or the uniform distribution. The same parameters are chosen for the LOS error and the NLOS Gaussian distributed error. The $R_c$ is set to 20 m. Let the mean of the exponential distribution be $1/0.38$ and we choose $\mathcal{U}\,[0, 11)$ as the uniform distribution (Fig. 6(e) in [11]). In Fig. 4, it is observed that the SPAWN-SP outperforms the other two competitors by far in the exponential and uniform cases, while it performs slightly worse than the existing parametric SPAWN in the Gaussian case for the reason given in the second simulation. The superiority of the SPAWN-SP over the existing parametric SPAWN in the non-Gaussian cases is attributed to the suitability of Eq. (13) in characterizing the internal messages. This result verifies the high flexibility offered by the proposed SPAWN-SP, and the potential performance degradation of the existing parametric SPAWN due to a strong model mismatch. Regarding the convergence speed, they all reach the converged results after 5 iterations. Note again that the particle-based SPAWN can improve its performance by enlarging the internal messages' particle number, at an increasing computational cost.

## 6. CONCLUSIONS

We proposed a novel cooperative localization algorithm, the SPAWN-SP, which requires less computational and communication cost than the classical particle-based SPAWN. As compared to the existing parametric SPAWN, the SPAWN-SP requires similar computational and communication cost, but achieves higher localization accuracy in different LOS/NLOS scenarios, owing to its flexibility in message representation.

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