# An Enhanced Spectrum Sensing Algorithm with Maximum Ratio Combination of Spectral Correlation

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Abstract—In cognitive radio networks, the task of spectrum sensing is required to be reliable at low signal-to-noise ratios (SNRs). Spectral correlation is an effective approach to satisfy the requirement. The algorithms based on statistic spectral correlation profiles are a good method as shown in some previous works. In this paper, we propose an algorithm with maximum ratio combination for the profiles to enhance the method. We construct a formula of statistic test and describe an implementation for our algorithm in practice. Extensive simulations are carried out to verify the performance of algorithms. As a result, the proposed algorithm outperforms the existing algorithms with a neglectful cost of additional complexity.

## I. Introduction

The concept of cognitive radio (CR) was proposed by Joseph Mitola III in 1999 [1] by which a radio node can be aware of the surrounding radio environment. For the awareness, spectrum sensing is one main task which is to recognize the presence of signals with noise. The research on spectrum sensing has been an interested field for two decades. Spectrum sensing should be reliable at low SNRs. Regulators encourage the research and recommended requirements for spectrum sensing such as: the probability of detection (Pd) under the constraint of a constant false-alarm rate (CFAR) for some given SNRs. In the regulation of Federal Communications Commission of USA [2] it is required that Pd is higher than 90% with CFAR lower than 10% at a low signal-level, e.g., -116dBm for digital TV (normally corresponding to SNR = -21dB).

Spectral correlation is considered as a good approach [3] for spectrum sensing to realize the aforementioned goal. Gardner is a pioneer in the research on spectral correlation. The theory of spectral correlation was well investigated in his works [4]–[6]. As pointed out by Gardner, many man-made signals contain components which vary periodically due to the underlying mechanism of signal manipulations. Some *hidden periodicity* [4] can be exploited by some techniques of spectral analysis, e.g., a quadratic time-invariant (QTI) transformation for signals. They are second-order periodicity. The second-order periodicity with (cyclic) frequency is equivalent to spectral lines of the output from QTI transformation appearing in a specific period. It is therefore called as spectral correlation. The spectral correlation of noise is significantly different from that

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of desired man-made signals. For noise, the spectral correlation approaches to zeros. Meanwhile, the spectral correlation is non-zero for desired signals even at low SNRs. The spectral correlation therefore is applicable in modulation classification and signal detection. In CR, this approach helps to detect the desired signal in a low SNR regime as the one specified by relevant regulators.

For desired signals, their spectral correlation is related with the parameters of the signals: the baud rate and the carrier frequency. Sutton analyzed extensively in [7] the effect of the parameters on spectral correlation in a plane of cyclic frequency,  $\alpha$ , and spectral frequency, f. A sum of correlation could be computed with the whole  $\alpha f$  plane for a statistic test of detection. This method is called as the conventional algorithm. However, the high burden of computation for sensing nodes is due to high number of correlation peaks in the plane. To reduce the complexity, Fehske proposed an algorithm in [8] which selects the maximum correlation peaks along the  $\alpha$  axis into an  $\alpha$  profile. Basically, this algorithm computes the correlation for a half of peaks in the plane. Its complexity therefore is significantly decreased comparing to the conventional algorithm. For a further enhancement, Wu and Eric et al. [9], [10] proposed an algorithm based on the  $\alpha f$  profile which is the maximum peaks in the both axes of  $\alpha f$  plane. They proved that the  $\alpha f$ -profile-based algorithm has significantly low-complexity and outperforms those of the  $\alpha$ -profile-based and the conventional algorithm.

In our work, we consider not only the confident regions of spectral correlation peaks in the  $\alpha f$  plane but also the amplitude of the peaks. We propose an enhanced algorithm which uses the peaking amplitudes of the  $\alpha f$  profile as weighting coefficients. The proposed algorithm outperforms the  $\alpha f$  profile algorithm. An additional complexity in our algorithm compared to the  $\alpha f$  profile algorithm, is the multiplication between the amplitude of peaks and the weighting coefficients.

The rest of paper is organized as follows. The background of spectral correlation is presented in Section II. In Section III, the new algorithm is mentioned. The simulation results then show performance of the algorithm. Finally, conclusions are drawn in Section V.

# II. BACKGROUND OF SPECTRAL CORRELATION

Second-order periodicity of a signal is examined by the cyclic autocorrelation function (CAF) which is a kind of QTI

transformation. For an envelop signal  $y\left(t\right)$ , CAF is defined as [4]:

$$R_{y}\left(\tau,\alpha\right) = \lim_{\Delta t \to \infty} \int_{-\Delta t/2}^{\Delta t/2} y\left(t + \tau/2\right) y^{*}\left(t - \tau/2\right) e^{-j2\pi\alpha t} dt,$$
(1)

where  $\tau$  and  $\alpha$  denotes time lag and cyclic frequency, respectively. CAF exhibits the second-order periodicity in time domain with cyclic frequency  $\alpha$ . With the above form, CAF is considered as a generalization of conventional autocorrelation. It reduces to conventional autocorrelation at  $\alpha=0$ . Meanwhile, CAF presents the correlation between two components  $y\left(t+\tau/2\right)e^{-j\pi\alpha t}$  and  $y\left(t-\tau/2\right)e^{j\pi\alpha t}$ . This is also called spectral correlation.

In frequency domain, CAF is transformed by Fourier operation into spectral correlation function (SCF):

$$S_y^{\alpha}(f) = \int_{-\infty}^{\infty} R_y(\tau, \alpha) e^{-j2\pi f t} dt.$$
 (2)

To estimate the spectral correlation, a spectrally smoothed cyclic periodogram is defined in [4] as

$$S_{y_{\Delta t}}^{\alpha}\left(t,f\right) = \frac{1}{\Delta t} Y_{\Delta t}\left(t,f+\frac{\alpha}{2}\right) Y_{\Delta t}^{*}\left(t,f-\frac{\alpha}{2}\right), \quad (3)$$

where

$$Y_{\Delta t}(t, f) = \int_{t-\Delta t/2}^{t+\Delta t/2} y(v) e^{-j2\pi f v} dv.$$

This equation shows that SCF is equivalent to the correlation between a spectral line and its shifted version with a frequency  $\alpha$ . Thus, a SCF is approximated by the periodogram as:

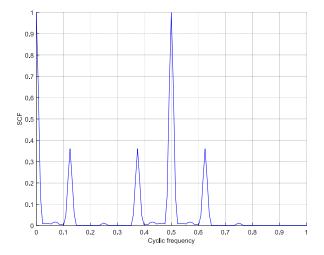
$$\widehat{S}_{y}^{\alpha}\left(f\right) = \lim_{\Delta f \to 0} \lim_{\Delta t \to \infty} \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} S_{y_{\Delta t}}^{\alpha}\left(t, u\right) du. \quad (4)$$

Parameter  $\Delta t$  and  $\Delta f$  are temporal resolution and spectral resolution [7], respectively. These two parameters are so chosen that their product is much greater than unity:  $\Delta f \Delta t \gg 1$ . Temporal resolution  $\Delta t$  normally is determined by the baud rate of signal. It can be a multiple integer of the symbol duration. Spectral resolution  $\Delta f$  meanwhile is calculated for a reliable  $\hat{S}_y^\alpha(f)$  as analyzed in [11], by which we can observe the spectral correlation peaks of confident regions in the  $\alpha f$  plane.

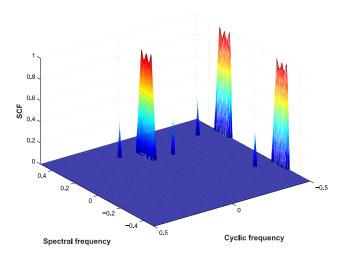
## III. PROPOSED ALGORITHM

## A. Profile of spectral correlation

For cellular networks, i.e. the uplinks of 4G networks, data modulation can be binary phase-shift keying (BPSK) or higher quadrature amplitude modulation (QAM) due to the density of users. The  $\alpha f$  profile varies with modulation types. The profile of SCF for BPSK and 16QAM are shown in Fig. 1 and Fig. 2, respectively. For BPSK, SCF is symmetric with respect to  $\alpha$  and f axes because BPSK symbol is symmetric



(a)  $\alpha$  profile



(b)  $\alpha f$  profile with coefficients

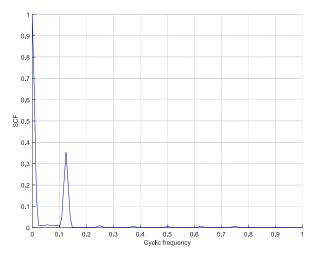
Fig. 1: SCF of BPSK.

data:  $\{+1,-1\}$ . When  $\alpha=\{0,0.5\}$ , SCF is high due to carrier frequency. Other peaks appear due to the pulse-shape filter of transmitter:  $\alpha=\{0.125,0.375,0.75\}$ . The SCF is nonsymmetric for high-order modulation such as 16QAM. There are only high peaks at  $\alpha=0$ , equivalent to conventional power spectral density, and low peaks at  $\alpha=0.125$ .

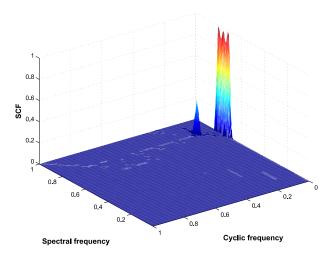
In realistic scenarios,  $\alpha$  and f profile are computed when the parameters of the underlying signals at the transmitter are known. The baud rate and carrier frequency can be looked up from a radio environment map (REM) as described in [12], [13].

## B. Statistic test

The  $\alpha f$  profile is selected as in [9]. The profile contains the position cyclic frequency, spectral frequency, and the amplitudes of the peaks. Considering the advantages of maximum ratio combination (MRC), we use the amplitudes of some selected peaks as the weighting coefficients. This



(a)  $\alpha$  profile



(b)  $\alpha f$  profile with coefficients

Fig. 2: SCF of high order modulation: 16QAM.

combination can give further performance gain compared to existing algorithms in which all coefficients are equal to 1. The statistic test of the new algorithm is as follows

$$T_{proposed} = \left\| \sum_{a,k} c_{a,k} \widehat{S}_{y}^{\alpha}\left(f\right) \right\| / \left( \sum_{t} \|y\left(t\right)\| \times \sum_{a,k} c_{a,k} \right), \quad (5)$$

where  $c_{a,k}$  are coefficients of the spectral peaks of selected confident regions at cyclic and spectral frequencies with the indices a and k, respectively.

In the denominator, the component  $\sum \|y(t)\|$  is to normalize the signal power. Hence, the computation for predefined thresholds does not depend on noise estimation. It results in that the proposed algorithm is completely intensive to noise-uncertainty phenomenon [14]. The numerator is a sum of complex-amplitudes of selected peaks. For convenient comparison, we use subscripts  $\alpha f$  and  $\alpha f co$  to denote the algorithms based on  $\alpha f$  profile without/with the weighted

coefficients, respectively. In previous works [8]–[10], there is no explicit formula of statistic test. Additionally, we validate algorithms with other forms. Thus, we use subscripts abs and complex to indicate the two different statistic forms with the numerator as  $\sum \|(.)\|$  and  $\|\sum (.)\|$ , respectively. The proposed algorithm is equivalent to  $\alpha fco_{complex}$ .

As shown in Eq. (5), the additional complexity of the proposed algorithm is the multiplication operation for the coefficients and SCF peaks compared to the  $\alpha f$  algorithms. Therefore, the additional complexity can be neglected.

# C. Flowchart of the proposed algorithm

The flowchart of the new algorithm are described as follows Step 1 - Measure the average of SCF:

The average of SCF can be statically measured by collaborative sensing nodes or be computed by a neutral network with the knowledge of baud-rate and carrier frequency stored in a REM. The sensing node acquires  $\alpha f$  profile from the SCF before it computes a statistic test. In this step, thresholds are also computed as the expected values of CFAR.

Step 2 - Compute the SCF of the received signal: The received signal is transformed into frequency domain by FFT. Spectral lines are selected to compute spectral correlation peaks as a part of SCF corresponding to the spectral and cyclic frequency in the predefined  $\alpha f$  profile.

## Step 3 - Combine for the statistic test:

The selected peaks are scaled up with the weighting coefficients as defined by the averaged SCF. Next, the statistic test is combined as given in Eq. (5). The test is compared with a predefined threshold to decide whether or not the desired signal appears.

#### IV. SIMULATION RESULTS

The proposed algorithm is simulated with the two modulation types: BPSK, and 16QAM. The main simulation parameters are listed as in Table I. CFAR is sensitive to the pre-defined thresholds. The number of simulation trials for threshold computation is therefore sufficiently high to compute the thresholds. The accuracy of threshold computation leads to the exact comparison for the algorithms. In simulations, the performance of algorithms are verified including: our proposed algorithm as in Eq. (5),  $\alpha f co_{abs}$ ,  $\alpha f_{abs}$ , and  $\alpha f_{complex}$ .

TABLE I: Simulation parameters

Parameter	Values
Modulation	BPSK, 16QAM
FFT size	128
Sampling frequency	1.92MHz
Over-sampling ratio	8
Channel model	Additive white Gaussian noise
Threshold	1%
Simulation trials	CFAR: 6000, Pd: 1200

The proposed algorithm outperforms  $\alpha f$  algorithms for different modulations as shown in Fig. 3. Nevertheless, there is a low performance gain between  $\alpha f co_{abs}$  and  $\alpha f_{abs}$ . The reason is that the summation of scaled peaks with the complex SCF (in the case of the proposed algorithm) eliminates noise

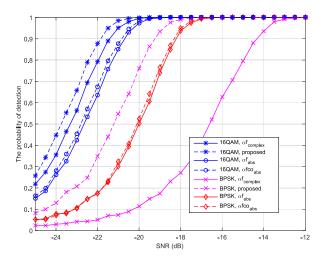


Fig. 3: The probability of detection with CFAR: 1%, and observation time: 5ms.

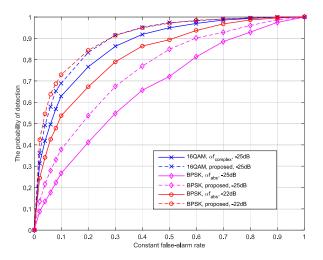


Fig. 4: The receiver operating characteristic (ROC) with observation time: 5ms.

components more effective than that with the absolute SCF (in the case of  $\alpha fco_{abs}$  algorithm). For BPSK modulation, the performance gain of the proposed algorithm is 1dB and 5dB when comparing with  $\alpha f_{abs}$  and  $\alpha f_{complex}$ , respectively. With a summation of the real-part, af algorithms has a higher performance of 4dB than that with a summation of the complex-part. The performance gain of  $\alpha f_{abs}$  compared to  $\alpha f_{complex}$  can be explained by the reflective symmetry of the SCF as shown in Fig. 1b. This symmetry appears only if the SCF is computed with the real-part of the envelop signal. For 16QAM, meanwhile, the SCF with the real-part is no difference to the complex form. The performance of the proposed algorithm is about 1dB and 0.5dB higher than that of  $\alpha f_{abs}$  and  $\alpha f_{complex}$ , respectively.

The curves in Fig. 4 present comparisons among the algorithms at very low SNRs. The new algorithm has better performance than existing algorithms,  $\alpha f$ . For example,

 $false\ alarm: 10\%$  with BPSK the performance gain is 12% and 19% at -25dB and -22dB, respectively. With 16QAM, the performance gain is 6% at -25dB.

From the simulation results we can conclude that: (1)  $\alpha fco$  algorithm outperforms  $\alpha f$  algorithm. The performance gain depends on the  $\alpha f$  profile; (2) for BPSK, we should use the absolute SCF for  $\alpha f$  algorithms; for higher-order data modulation, i.e. QAM, we should use complex SCF for both  $\alpha f$  and  $\alpha fco$  algorithms.

## V. CONCLUSION

In this paper, we proposed an enhanced spectrum sensing which is based on maximum ratio combination of spectral correlation. The new algorithm outperforms the existing algorithms with a marginal increase in the complexity. The proposed algorithm can be applied to the detection of primary signals in cellular networks.

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