RECURSIVE TOTAL LEAST-SQUARES ESTIMATION OF FREQUENCY IN THREE-PHASE POWER SYSTEMS

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ABSTRACT

We propose an adaptive algorithm for estimating the frequency of a three-phase power system from its noisy voltage readings. We consider a second-order autoregressive linear predictive model for the noiseless complex-valued $\alpha\beta$ signal of the system to relate the system frequency to the phase voltages. We use this model and the noisy voltage data to calculate a total least-square (TLS) estimate of the system frequency by employing the inverse power method in a recursive manner. Simulation results show that the proposed algorithm, called recursive TLS (RTLS), outperforms the recursive least-squares (RLS) and the bias-compensated RLS (BCRLS) algorithms in estimating the frequency of both balanced and unbalanced three-phase power systems. Unlike BCRLS, RTLS does not require the prior knowledge of the noise variance.

***Index Terms*—**Adaptive signal processing, frequency estimation, inverse power method, linear predictive modeling, total least-squares.

1. INTRODUCTION

In electric power grids, deviation of the system frequency from its nominal value represents an imbalance between load and generation. Therefore, variations of the system frequency should be closely watched. Most protection-and-control applications in electric power systems require accurate and fast estimation of the system frequency. An erroneous estimate of the frequency may eventually result in a catastrophic grid failure due to inadequate or delayed load shedding [1]-[5].

The Clarke's transformation applied to the voltages of a three-phase power system produces a complex-valued signal (known as the $\alpha\beta$ signal) that incorporates the information of the three phases. In many applications, the $\alpha\beta$ signal can be considered as a faithful representative for a three-phase system [6]. The phase voltages are digitized at the measurement points by sampling at a fixed interval and quantizing the samples. Therefore, in practice, the observed voltage data and consequently the $\alpha\beta$ signal are contaminated with noise/error.

From a signal-processing point of view, the noisy samples of the $\alpha\beta$ signal and the sampling rate comprise the available data while the amplitudes of the three phase voltages, the initial phase angle, and the system frequency are the unknown parameters. A plethora of techniques have been developed to extract these parameters from the observable data.

When the main objective is to estimate the system frequency, a second-order autoregressive (AR2) linear predictive model for time evolution of the noiseless $\alpha\beta$ signal can be employed [7]-[11]. This model, which we will simply call AR2, linearly relates three consecutive noiseless samples of the $\alpha\beta$ signal via a single real-valued parameter that is equal to the cosine of the multiplication of the system angular frequency and the sampling interval. Thus, the system frequency can be estimated by identifying the parameter of the AR2 model from the noisy observations of the $\alpha\beta$ signal while being safely oblivious to the values of the phase voltage amplitudes and the initial phase angle. In other words, since the parameter of the AR2 model depends only on the system frequency and the sampling interval, any frequency estimator built on this model is virtually insensitive to the balance state of the three-phase system. Relative values of the voltage amplitudes of three phases determine the balance state.

Since the $\alpha\beta$ signal is observed with noise, a reliable frequency estimation technique based on the AR2 model should minimize the effect of noise. In [9], [10], the least-squares (LS) method has been used for this purpose. A recursive LS (RLS) frequency estimator has also been proposed in [11]. The LS-based approaches are best suited to counter the effect of noise at the output of a linear model. However, as in the AR2 model, the input of the model is also subject to observational noise, the LS-based frequency estimators are biased. One way to eliminate the estimation bias is to evaluate the bias separately and subtract it from the biased estimate [12]. However, evaluation of the bias usually requires prior knowledge of the noise variance or an extra procedure for estimating the noise variance. Alternatively, the total least-squares (TLS) estimation technique can be utilized to compensate for the noise at both

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input and output of the AR2 model. A TLS estimator can eliminate the estimation bias induced by the input noise without performing any explicit bias calculation [13]-[14]. The TLS technique has recently been utilized to estimate the frequency of three-phase power systems based on a first-order autoregressive widely-linear predictive model for the noiseless $\alpha\beta$ signal [15], [16].

In this paper, we derive a recursive TLS (RTLS) algorithm for adaptive frequency estimation of three-phase power systems. We use the AR2 model for the $\alpha\beta$ signal and compute a TLS estimate of the model parameter by implementing a single iteration of the inverse power method [17] at each time instant. We verify the effectiveness of the proposed RTLS algorithm in estimating the frequency of three-phase power systems through simulated experiments.

2. ALGORITHM DERIVATION

The phase voltages of a three-phase power system, sampled at the interval of $\tau \in \mathbb{R}$, are expressed as

$$\hat{v}_{a,n} = V_a \cos(2\pi f \tau n + \theta),$$

$$\hat{v}_{b,n} = V_b \cos\left(2\pi f \tau n + \theta - \tfrac{2\pi}{3}\right),$$

and

$$\hat{v}_{c,n} = V_c \cos\left(2\pi f \tau n + \theta + \tfrac{2\pi}{3}\right)$$

where $f \in \mathbb{R}$ is the system frequency, V_a, V_b, and $V_c \in \mathbb{R}$ are the peak values, $\theta \in \mathbb{R}$ is a constant initial phase angle, and $n \in \mathbb{N}$ is the time index. In practice, the phase voltages are observed with noise. The noisy observations are expressed as

$$v_{a,n} = \hat{v}_{a,n} + \eta_{a,n},$$

$$v_{b,n} = \hat{v}_{b,n} + \eta_{b,n},$$

and

$$v_{c,n} = \hat{v}_{c,n} + \eta_{c,n}$$

where $\eta_{a,n}$, $\eta_{b,n}$, and $\eta_{c,n} \in \mathbb{R}$ denote the additive noise at the respective phases. We assume that the noises are i.i.d. with zero mean and fixed variance.

Application of the Clarke's transformation to the observed noisy voltages of the three phases is formulated as

$$\begin{bmatrix} \hat{v}_{\alpha,n} \\ \hat{v}_{\beta,n} \end{bmatrix} + \begin{bmatrix} \eta_{\alpha,n} \\ \eta_{\beta,n} \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \left(\begin{bmatrix} \hat{v}_{a,n} \\ \hat{v}_{b,n} \\ \hat{v}_{c,n} \end{bmatrix} + \begin{bmatrix} \eta_{a,n} \\ \eta_{b,n} \\ \eta_{c,n} \end{bmatrix} \right).$$

This transformation yields a complex-valued voltage signal (known as the $\alpha\beta$ signal) that can represent the three-phase power system [6]. The noiseless $\alpha\beta$ signal is calculated as

$$\begin{aligned} \hat{v}_n &= \hat{v}_{\alpha,n} + j\hat{v}_{\beta,n} \\ &= (A + jB)\cos(2\pi f \tau n + \theta) + (B + jC)\sin(2\pi f \tau n + \theta) \end{aligned}$$

where

$$A = \tfrac{\sqrt{2}}{\sqrt{3}} V_a + \tfrac{1}{2\sqrt{6}} (V_b + V_c),$$

$$B = -\tfrac{1}{2\sqrt{2}} (V_b - V_c),$$

and

$$C = \tfrac{\sqrt{3}}{2\sqrt{2}} (V_b + V_c).$$

The noisy $\alpha\beta$ signal is given by

$$v_n = \hat{v}_n + \eta_n$$

where the additive complex noise is related to the noise at the individual phases via

$$\begin{aligned} \eta_n &= \eta_{\alpha,n} + j\eta_{\beta,n} \\ &= \frac{\sqrt{2}}{\sqrt{3}}\left(\eta_{a,n} - \frac{\eta_{b,n}}{2} - \frac{\eta_{c,n}}{2}\right) + \frac{j}{\sqrt{2}}\left(\eta_{b,n} - \eta_{c,n}\right). \end{aligned}$$

We denote the variance of the absolute value of η_n by σ^2, i.e., $\sigma^2 = E[|\eta_n|^2]$.

It is known that the time evolution of the noiseless $\alpha\beta$ signal, $\hat{v}_n$, can be described via a second-order autoregressive (AR2) linear predictive model as

$$\frac{1}{2}(\hat{v}_{n-2} + \hat{v}_n) = h\hat{v}_{n-1} \tag{1}$$

where h is the model parameter expressed as

$$h = \cos(2\pi f \tau).$$

This model relates the system frequency to three consecutive noiseless voltage samples of the three phases in a linear manner. However, the measured samples are noisy. Hence, in order to identify the model parameter h and subsequently the system frequency, a reliable linear estimation technique need be employed that can minimize the effect of noise.

Since both input and output in the AR2 model, (1), are observed with noise, we utilize the TLS estimation technique to identify h. For this purpose, we define the input and output data vectors as

$$\mathbf{x}_n = \mathbf{\Lambda} \begin{bmatrix} v_0 \\ v_1 \\ \cdots \\ v_{n-2} \\ v_{n-1} \end{bmatrix} \text{ and } \mathbf{y}_n = \frac{\mathbf{\Lambda}}{2} \begin{bmatrix} v_{-1} + v_1 \\ v_0 + v_2 \\ \cdots \\ v_{n-3} + v_{n-1} \\ v_{n-2} + v_n \end{bmatrix},$$

respectively, where

$$\mathbf{\Lambda} = \operatorname{diag}\left\{\sqrt{\lambda^{n-1}}, \sqrt{\lambda^{n-2}}, \ldots, \sqrt{\lambda}, 1\right\}$$

is an exponential weighting matrix and $0 \ll \lambda \leq 1$ is the forgetting factor. We compute the TLS estimate of h at time instant n, denoted by w_n, such that it fits the input data vector, $\mathbf{x}_n$, to the output data vector, $\mathbf{y}_n$, by incurring minimum perturbation in the data, i.e., it holds that

$$(\mathbf{x}_n + \boldsymbol{\varepsilon}_n) w_n = \mathbf{y}_n + \boldsymbol{\delta}_n$$

where $\boldsymbol{\varepsilon}_n \in \mathbb{C}^{n\times 1}$ and $\boldsymbol{\delta}_n \in \mathbb{C}^{n\times 1}$ are the minimum input and output perturbations, respectively. According to the analysis of [14], the TLS estimate is given by

$$w_n = -\frac{z_{1,n}}{\gamma z_{2,n}}$$

where $\mathbf{z}_n = \left[z_{1,n}, z_{2,n}\right]^T$ is the eigenvector corresponding to the smallest eigenvalue of the augmented and weighted data covariance matrix

$$\boldsymbol{\Psi}_n = \begin{bmatrix} \mathbf{x}_n^H \\ \gamma \mathbf{y}_n^H \end{bmatrix} [\mathbf{x}_n, \gamma \mathbf{y}_n].$$

The weight γ accounts for the disparity in the variance of the noise at input and output, denoted by σ_i^2 and σ_o^2, respectively, and is calculated as

$$\gamma = \sqrt{\frac{\sigma_i^2}{\sigma_o^2}} = \sqrt{2}.$$

The matrix $\boldsymbol{\Psi}_n$ can be written as

$$\boldsymbol{\Psi}_n = \begin{bmatrix} r_n & \sqrt{2} p_n \\ \sqrt{2} p_n^* & 2 s_n \end{bmatrix}$$

where

$$\begin{aligned} r_n &= \mathbf{x}_n^H \mathbf{x}_n \\ &= \sum_{i=1}^{n} \lambda^{n-i} v_{i-1}^* v_{i-1} \\ &= \lambda r_{n-1} + |v_{n-1}|^2, \\ p_n &= \mathbf{x}_n^H \mathbf{y}_n \\ &= \frac{1}{2} \sum_{i=1}^{n} \lambda^{n-i} v_{i-1}^* (v_{i-2} + v_i) \\ &= \lambda p_{n-1} + \frac{1}{2} v_{n-1}^* (v_{n-2} + v_n), \end{aligned}$$

and

$$\begin{aligned} s_n &= \mathbf{y}_n^H \mathbf{y}_n \\ &= \frac{1}{4} \sum_{i=1}^{n} \lambda^{n-i} (v_{i-2}^* + v_i^*)(v_{i-2} + v_i) \\ &= \lambda s_{n-1} + \frac{1}{4} |v_{n-2} + v_n|^2. \end{aligned}$$

The vector $\mathbf{z}_n$ is normally found using the eigenvalue decomposition of $\boldsymbol{\Psi}_n$. A computationally more efficient alternative is to update $\mathbf{z}_n$ adaptively by executing a single iteration of the inverse power method [17] at each time instant through the following recursion:

$$\mathbf{z}_n = \boldsymbol{\Psi}_n^{-1} \mathbf{z}_{n-1}. \tag{2}$$

Multiplying both sides of (2) by $\boldsymbol{\Psi}_n$ and dividing by $\sqrt{2} z_{2,n-1} z_{2,n}$ give

$$\boldsymbol{\Psi}_n \begin{bmatrix} w_n \\ -1/\sqrt{2} \end{bmatrix} = \frac{z_{2,n-1}}{z_{2,n}} \begin{bmatrix} w_{n-1} \\ -1/\sqrt{2} \end{bmatrix}$$

or equivalently

$$r_n w_n - p_n = \frac{z_{2,n-1}}{z_{2,n}} w_{n-1} \tag{3}$$

and

$$p_n^* w_n - s_n = -\frac{z_{2,n-1}}{2 z_{2,n}}. \tag{4}$$

Substituting (4) into (3) and solving it for w_n yield a recursive TLS (RTLS) estimate of h as

$$w_n = \frac{p_n + 2 s_n w_{n-1}}{r_n + 2 p_n^* w_{n-1}}. \tag{5}$$

Having calculated w_n, we can estimate the system frequency via

$$f_n = \frac{1}{2\pi\tau} \cos^{-1}(w_n).$$

3. SIMULATIONS

We compare the frequency estimation performance of the RLS algorithm, i.e.,

$$w_n = \frac{p_n}{r_n},$$

the bias-compensated RLS (BCRLS) algorithm, i.e.,

$$w_n = \frac{p_n}{r_n} + \frac{\sigma^2}{(1-\lambda) r_n} w_{n-1},$$

and the RTLS algorithm, (5), for a three-phase power system where $f = 50$ Hz, $\tau = 2$ ms, and $\sigma_a^2 = \sigma_b^2 = \sigma_c^2 = \sigma^2/2$. We also set $\lambda = 0.999$.

In Fig. 1, we depict the estimated frequency using different algorithms when $\sigma^2 = 0.01$ and the system undergoes several voltage sags making it progressively more unbalanced as shown in Fig. 2. The system is balanced during the initial 0.25 seconds. Then, the voltage of phase *c* drops by 50%. After 0.25 seconds, the voltage of phase *a*, and after further 0.25 seconds, the voltage of phase *b* reduce to zero. Zero or almost zero phase voltages can occur in the aftermath of short-circuit to ground or combined asymmetric sags. It is observed from Fig. 1 that all the considered algorithms converge almost equally fast.

In Fig. 3, we plot the estimated frequency using different algorithms when $\sigma^2 = 0.01$ and a sinusoidal oscillation occurs in the frequency of a balanced system after 0.33 seconds. The peak and the maximum change rate for the oscillation are 1 Hz and 2 Hz/s, respectively.

In Figs. 4-7, we plot the steady-state bias and root-mean-square error, defined as

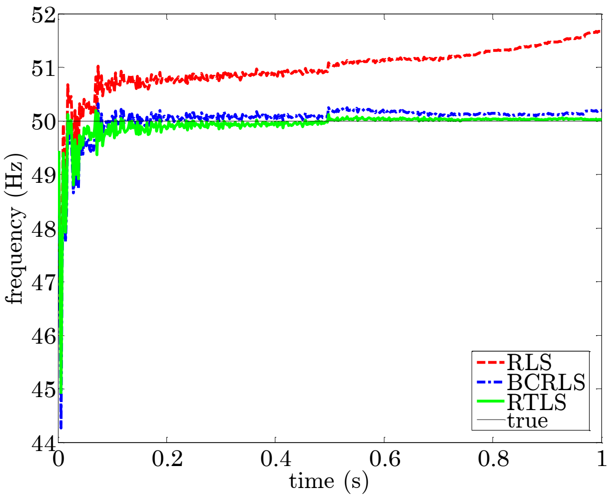

Fig. 1. Estimated frequency of the system of Fig. 2.

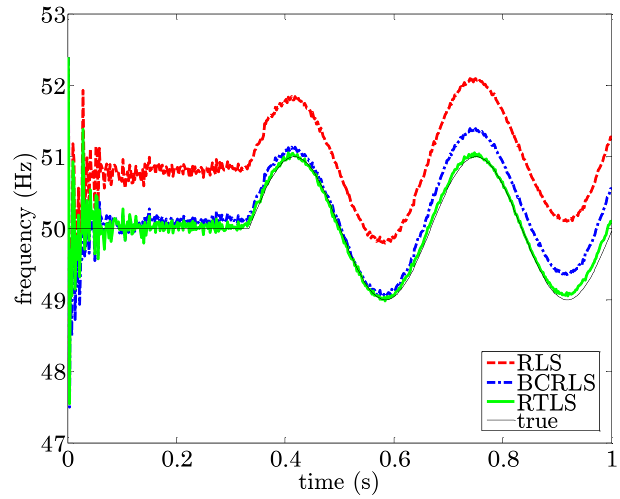

Fig. 3. Estimated and tracked frequency.

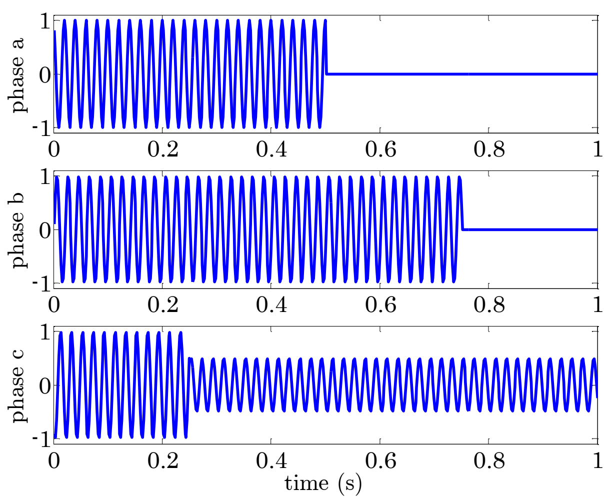

Fig. 2. Voltages of the three phases in the considered system.

$$\lim_{n\to\infty} |E[f_n] - f| \quad \text{and} \quad \lim_{n\to\infty} \sqrt{E[(f_n - f)^2]},$$

respectively, against the signal-to-noise ratio (SNR), which is considered to be $1/\sigma^2$, for different algorithms. In Figs. 4 and 5, the system is balanced while, in Figs. 6 and 7, short circuit between phase a and ground has made the system unbalanced. We run the algorithms for 4 seconds to reach the steady state. We evaluate the expectations by taking the ensemble average over 10^4 independent trials and the steady-state values by averaging over the last 0.1 seconds.

4. CONCLUSION

We proposed an adaptive algorithm for estimating the frequency of three-phase power systems. To this end, we utilized an intrinsic second-order autoregressive linear predictive model for the noiseless $\alpha\beta$ signal of the system whose parameter is equal to the cosine of the system frequency. Then, we found a TLS estimate for the model's parameter using the noisy observations of the $\alpha\beta$ signal and employing the inverse power method in a recursive manner. The resultant recursive TLS (RTLS) algorithm compensates for noise at both input and output of the model. We showed through simulation examples that the RTLS algorithm

- achieves considerably lower frequency estimation bias and root mean-square error compared with the RLS algorithm for both balanced and unbalanced three-phase power systems;
- is more efficacious than the bias-compensated RLS algorithm in eliminating the estimation bias while bypassing the need for prior knowledge of the noise variance;
- has a good capability to track the changes in the system frequency.

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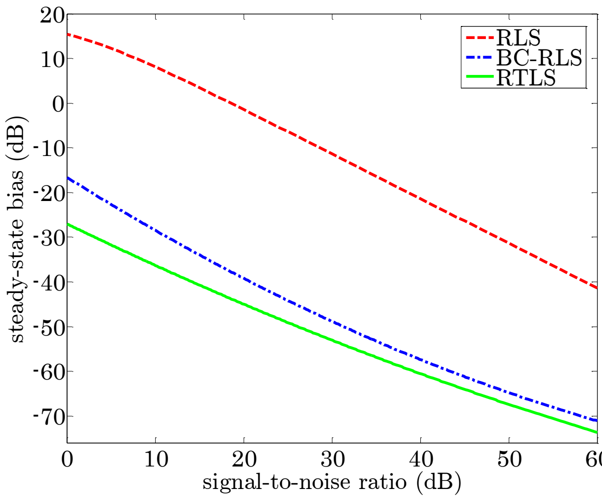

Fig. 4. Steady-state frequency estimation bias versus SNR when the system is balanced.

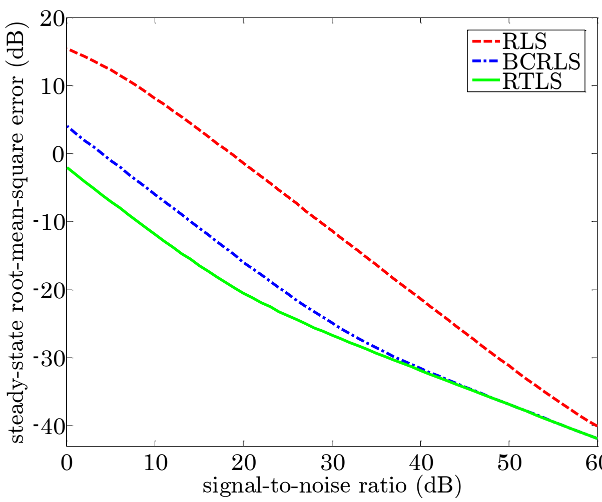

Fig. 5. Steady-state frequency estimation root-mean-square error versus SNR when the system is balanced.

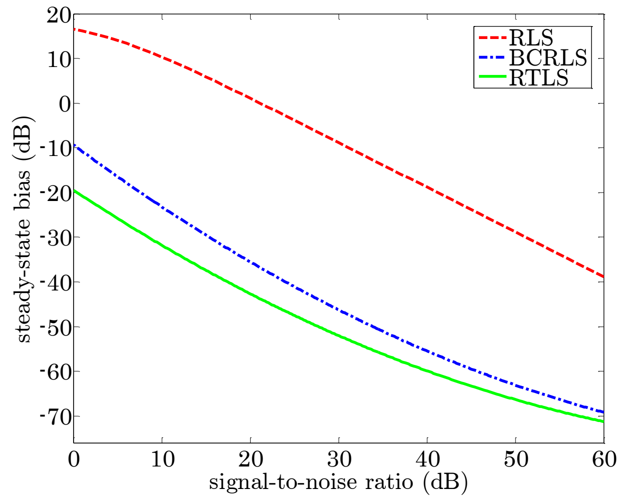

Fig. 6. Steady-state frequency estimation bias versus SNR when the system is unbalanced due to a phase-to-ground fault.

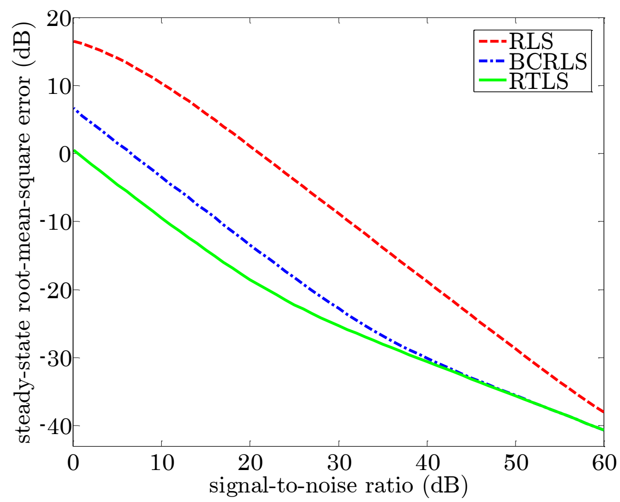

Fig. 7. Steady-state frequency estimation root-mean-square error versus SNR when the system is unbalanced due to a phase-to-ground fault.

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