OPTIMAL OFDM PULSE DESIGN, ANALYSIS AND IMPLEMENTATION OVER DOUBLY DISPERSIVE CHANNEL

Raouia Ayadi(1), *Inès Kammoun*(2), *Mohamed Siala*(1)

(1) Sup'com, ElGhazala, 2083, Ariana-Tunisia

(2) LETI Department, ENIS, BPW 3038 Sfax - Tunisia

Email: raouia.ayadi@supcom.rnu.tn, ines.kammoun@isecs.rnu.tn, mohamed.siala@supcom.rnu.tn

ABSTRACT

This paper considers optimization and implementation of the pulse shape for Orthogonal Frequency Division Multiplexing (OFDM) systems over doubly dispersive channel. The proposed pulse shape is expressed as a linear combination of the most localized Hermite waveforms. In our previous works, we have proposed pulses optimization based on an exact expression of the Signal to Interference Ratio (SIR). In order to accelerate the optimization procedure, we propose an approximate expression of the SIR using Taylor series of interference and desired signal mean powers. For a performance evaluation of the optimized OFDM system, we propose an efficient implementation of both the modulator and the demodulator of the system and an efficient method to find out the resulting simulated SIR. Simulations results, obtained for different values of the dispersion factor, show that the SIR computed through system level simulation matches the maximum achievable SIR obtained through numerical optimization.

Index Terms— OFDM, Hermite waveforms, exact SIR, approximate SIR, simulated SIR.

1. INTRODUCTION

The conventional OFDM system uses the rectangular filter to maintain maximum spectral efficiency. However, this filter is bad frequency localized and leads to Inter-Carrier Interference (ICI) over time-selective channels. The design of time-frequency well localized pulses for OFDM systems is an attractive research topic [1–5]. In [1], the prototype pulse of the maximizing Signal to Interference plus Noise Ratio receiver is proposed for Hexagonal Multicarrier Transmission over doubly dispersive channel. Jung et *al.*, in [2], presented a new method for the transmit/receive pulses design by applying mathematical results on Weyl-Heisenberg frames to the multicarrier context. In [3], Haas and Belfiore proposed to use the Hermite waveforms for OFDM systems. These functions form an orthonormal base and present a good time-frequency localization. In [4], we optimized the pulse shape for OFDM systems in order to reduce Inter-Symbol Interference (ISI) and ICI over time-frequency dispersive channels. In [5], the optimization of the transmit and receive pulses is carried out for BFDM systems. For OFDM and BFDM systems, the optimal pulses are expressed as linear combinations of the most localized Hermite Waveforms. In [4] and [5], the optimization of the pulse shapes is based on the exact expression of the SIR.

In order to accelerate the optimization procedure, we propose, in this paper, an approximation of the SIR using taylor series, truncated at the second order, of both interference and desired signal mean powers. Thereafter, we propose pulse shape optimization for OFDM system with the obtained approximate expression of the SIR for different values of the dispersion factors. Another aim of this paper is to provide an efficient implementation and a realistic performance. of the optimized OFDM system. For this aim, we simulate the multicarrier transmission system. At first, we propose an efficient implementation, using the Fast Fourier Transform (FFT), of the modulator and the demodulator in OFDM systems. Thereafter, we propose a new method to determine the mean power of the interference and that of the desired signal to find out the value of the simulated SIR.

This paper is organized as follows. In section II, we introduce the system model. Section III provides an approximation of the SIR and the optimization of the pulse shape based on maximizing the approximate SIR. We describe our simplified approach to implement the modulator and the demodulator in section IV. In section V, we present our method for the simulated SIR derivation. Section VI is devoted to simulation results. The conclusion is given in section VII.

2. SYSTEM MODEL

Transmitter and receiver models. We consider a baseband model of an OFDM system with K sub-carriers regularly spaced by F in frequency. In practice, the value of K is a finite number but great enough to be considered next as an infinite number for simplicity reason. In this case, the transmitted signal is expressed by

$$s(t) = \sum_{mn} a_{mn}\varphi_{mn}(t). \tag{1}$$

Here, a_{mn}, $m, n \in Z$, denote the i.i.d transmitted symbol of zero mean and mean transmitted energy E. The symbol a_{mn} is transmitted at time mT using sub-carrier nF, where T denotes the OFDM symbol duration.
The elementary signals $\varphi_{mn}(t)$ are defined by

$$\varphi_{mn}(t) = \varphi(t - mT)e^{j2\pi nFt}, \tag{2}$$

where $\varphi(t)$ is an elementary pulse that must guarantee quasi-orthogonality with its neighboring time-frequency shifted versions to avoid ICI and ISI and a good time-frequency localization to preserve the quasi-orthogonality with distant functions even over dispersive channels.
While crossing a time varying channel $\mathbb{H}$ with AWGN, the received signal $r(t)$ can be expressed as

$$r(t) = (\mathbb{H}s)(t) = \int_{-\infty}^{+\infty} s(t-\tau)h(\tau,t)d\tau + b(t), \tag{3}$$

where $h(\tau, t)$ is the impulse response of the channel at time t and $b(t)$ is the additive white Gaussian noise with power spectral density σ_b^2. The decision variable

$$\hat{a}_{mn} =< \varphi(t), r(t) >= \int_{-\infty}^{+\infty} r(t)\varphi_{mn}^*(t)dt, \tag{4}$$

on the transmitted symbol a_{mn} is obtained by projecting the received signal $r(t)$ on the base of elementary signals $\varphi_{mn}(t)$.
Channel model. The channel $\mathbb{H}$ is assumed to satisfy the Wide-Sense Stationary Uncorrelated Scattering proprety. We assume that it is characterized by a Doppler power spectrum density obeying to the Jake's model and an exponentially decreasing Power Delay Profile (PDP). Adopting the symmetry of the 2-D isotropic scattering environment, the impulse response for a single path, according to Jake's model, is written in the following form [6]

$$h(t) = \sqrt{\frac{8}{N_1}} \sum_{k=0}^{N_1/4-1} c_k \exp(j\phi_k)\cos(2\pi\nu_k t + \varsigma_k), \tag{5}$$

where $\nu_k = f_d\cos(\theta_k)$, $\phi_k = (\omega_k + \omega_{K-k-1})/2$ and $\varsigma_k = (\omega_k - \omega_{K-k-1})/2$ are uniformly distributed on $[-\pi, \pi]$. We note that ϕ_k and ς_k are independent. We note also c_k, $\theta_k = \frac{\pi}{N_1} + k\frac{2\pi}{N_1}$ and ω_k are respectively the complex Gaussian random gain, the angle and the random initial phase of the k^{th} incoming wave at the mobile.
Without loss of generality, we consider a channel of unit average power. Thus, the scattering function of the doubly dispersive channel is given by

$$S_{\mathbb{H}}(\tau,\nu) = \frac{2}{\pi B_d T_m} \frac{\exp(-\tau/T_m)}{\sqrt{1-(2\nu/B_d)^2}}, \tag{6}$$

where T_m and $B_d = 2f_D$ are respectively the delay root mean square and maximum Doppler spreads. Notice that $\tau \geq 0$ and $-B_d/2 < \nu < B_d/2$ are respectively the delay and Doppler spreads introduced by the channel.

3. OPTIMIZATION OF THE PULSE SHAPE BASED ON THE SIR APPROXIMATE EXPRESSION

As in [4], we express the OFDM pulses as linear combinations of the most localized Hermite Waveforms as follows

$$\varphi(t) = \frac{1}{\sqrt{\sum_{k=0}^{N-1} |\alpha_{2k}|^2}} \sum_{k=0}^{N-1} \alpha_{2k} u_{2k}(t), \tag{7}$$

where N is a finite integer fixing the number of most localized combined Hermite waveforms.
In [4], our approach of optimization consisted in maximizing the exact expression of the SIR

$$\varphi^{opt}(t) = \max_{\varphi(t)} \text{SIR}, \tag{8}$$

where the exact SIR is expressed by [4]

$$\begin{aligned} \text{SIR} &= \frac{\sigma_D^2}{\sigma_I^2} = \frac{E.\iint |A_\varphi(\tau,\nu)|^2 S_{\mathbb{H}}(\tau,\nu)d\tau d\nu}{E.\iint \Gamma_\varphi(\tau,\nu) S_{\mathbb{H}}(\tau,\nu)d\tau d\nu} \\ &= \frac{\sum\limits_{0\leq k,l,k',l'\leq N-1} \alpha_{2k}\alpha_{2l}^*\alpha_{2k'}^*\alpha_{2l'}\mathbf{S}(k,l,k',l')}{\sum\limits_{0\leq k,l,k',l'\leq N-1} \alpha_{2k}\alpha_{2l}^*\alpha_{2k'}^*\alpha_{2l'}\mathbf{I}(k,l,k',l')}, \end{aligned} \tag{9}$$

with $\mathbf{S}(k,l,k',l') = \int\int A_{2k,2l}(\tau,\nu)A^*_{2k',2l'}(\tau,\nu)S_H(\tau,\nu)d\tau d\nu$, $\mathbf{I}(k,l,k',l') = \int\int \Gamma_{2k,2l}(\tau,\nu)S_H(\tau,\nu)d\tau d\nu$, $\Gamma_\varphi(\tau,\nu) = \sum_{(m,n)\neq(0,0)} |A_\varphi(\tau + mT, \nu + nF)|^2$ and $A_\varphi(\tau,\nu)$ denotes the ambiguity function that measures the autocorrelation between two successive shifts of $\varphi(t)$ in time of τ and in frequency of ν.
The maximisation of the exact SIR, in [4], is based on the Arrow-Hurwicz Algorithm which employs a Lagrangian dual function to find out the optimal coefficients of the pulse shape in the Hermite orthonormal base. In oder to accelerate the optimization procedure, we undertake the pulse shape optimization with an approximate expression of the SIR, instead of the exact one.

3.1. Approximation of the SIR

Now, we derive an approximate expression of the SIR as a function of the delay spread and of the Doppler shift using taylor series, truncated at the second order, of both interference and desired signal mean powers. Using the second-order taylor series, we can approximate $|A_\varphi(x,y)|^2$ as follows

$$\begin{aligned} &|A_\varphi(x,y)|^2 \approx |A_\varphi(x_0,y_0)|^2 + \frac{\partial|A_\varphi(x,y)|^2}{\partial x}]_{(x_0,y_0)}(x-x_0) \\ &+ \frac{\partial|A_\varphi(x,y)|^2}{\partial y}]_{(x_0,y_0)}(y-y_0) + \frac{1}{2}\frac{\partial^2|A_\varphi(x,y)|^2}{\partial x^2}]_{(x_0,y_0)}(x-x_0)^2 \\ &+ \frac{1}{2}\frac{\partial^2|A_\varphi(x,y)|^2}{\partial y^2}]_{(x_0,y_0)}(y-y_0)^2 + \frac{1}{2}\frac{\partial|A_\varphi(x,y)|^2}{\partial x\partial y}]_{(x_0,y_0)} \\ &(x-x_0)(y-y_0), \end{aligned} \tag{10}$$

where $x_0 = mT$ and $y_0 = nF$ (respectively $x_0 = 0$ and $y_0 = 0$) to approximate the interference mean power (respectively to approximate the desired signal mean power).

By substituting equation (10) in the expressions of ${\sigma_I}^2$ and ${\sigma_D}^2$ and taking into account the symmetry of the Doppler power spectral density of the Jake's model, we easily obtain the following approximate expressions

$$\begin{aligned}{\sigma_I}^2 \approx E. \Big(&\textstyle\sum_{(m,n)\neq(0,0)} \\ &|A_\varphi(mT,nF)|^2 + T_m^2\zeta_\varphi(mT,nF) + \tfrac{f_D^2}{4}\xi_\varphi(mT,nF)\Big).\end{aligned} \tag{11}$$

and

$${\sigma_D}^2 \approx E.\left(|A_\varphi(0,0)|^2 + T_m^2\zeta_\varphi(0,0) + \frac{f_D^2}{4}\xi_\varphi(0,0)\right). \tag{12}$$

Here $\zeta_\varphi(x,y) = 8\pi^2\int_{-\infty}^{+\infty} f^2|\varphi(f+y/2)|^2|\varphi(f-y/2)|^2 df$ and $\xi_\varphi(x,y) = 8\pi^2\int_{-\infty}^{+\infty} t^2|\varphi(t+x/2)|^2|\varphi(t-x/2)|^2 dt$.

3.2. Optimization of the pulse shape

Since we write the pulse as a linear combination of the most localized Hermite functions, the approximate SIR criterion can be expressed as follows

$$\mathrm{SIR} = \frac{\sum\limits_{0\le k,l,k',l'\le N-1} \alpha_{2k}\alpha_{2l}^*\alpha_{2k'}^*\alpha_{2l'}\mathbf{S}_{k,l,k',l'}(0,0)}{\sum\limits_{0\le k,l,k',l'\le N-1} \alpha_{2k}\alpha_{2l}^*\alpha_{2k'}^*\alpha_{2l'}.(\sum\limits_{(m,n)\neq(0,0)}\mathbf{S}_{k,l,k',l'}(mT,nF))}, \tag{13}$$

where

$\mathbf{S}_{k,l,k',l'}(mT,nF) = A_{2k,2l}(mT,nF)A^*_{2k',2l'}(mT,nF) + 8\pi^2T_m^2.\int f^2u_{2k}(f+nF/2)u^*_{2k'}(f+nF/2)u^*_{2l}(f-nF/2)u_{2l'}(f-nF/2)df + 8\pi^2\frac{f_D^2}{4}\int t^2u_{2k}(t+mT/2)u^*_{2k'}(t+mT/2)u^*_{2l}(t-mT/2)u_{2l'}(t-mT/2)dt$, for any m and n.

Let $\mathbf{u}^T = [\alpha_0, ..., \alpha_{N-1}]$. As a function of $\mathbf{u}$, the SIR can be expressed as

$$g(\mathbf{u}) = \frac{(\mathbf{u}\otimes\mathbf{u})^\dagger\mathbf{A}(\mathbf{u}\otimes\mathbf{u})}{(\mathbf{u}\otimes\mathbf{u})^\dagger\mathbf{B}(\mathbf{u}\otimes\mathbf{u})}, \tag{14}$$

where $\otimes$ is the Kronecker tensor product, $\mathbf{A}$ is a $N^2\times N^2$ Hermitian positive-definite matrix of elements $\mathbf{S}_{k,l,k',l'}(0,0)$ and $\mathbf{B}$ is a $N^2\times N^2$ Hermitian positive-definite matrix of elements $\sum_{(m,n)\neq(0,0)}\mathbf{S}_{k,l,k',l'}(mT,nF)$.

The optimization procedure is based on iterative method using the gradient algorithm to find the optimal coefficients of pulse $\varphi(t)$ that maximize the approximate SIR.

The iterative algorithm is composed of two steps: an initialization step and an iterative one. In the first step, we initialize the vector $\mathbf{u}^0$. Then, the algorithm inductively reestimates $\mathbf{u}$ as follows

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \mu^{(k)}\nabla_{\mathbf{u}}g(\mathbf{u}^{(k)}) \tag{15}$$

with $\mu^{(k)}$ is the optimal step size respectively associated to $\mathbf{u}$. At the k^{th} iteration, the functions $\Upsilon(\mu) = \mathbf{u}^{(k)} + \mu\nabla_{\mathbf{u}}g(\mathbf{u}^{(k)})$ is the rational function respectively in μ. Then, the optimal step size is obtained as follows

$$\mu^{(k)} = \mathrm{argmax}_\mu\Upsilon(\mu). \tag{16}$$

For the complexity evaluation, we note that a mathematical number of operations in terms of additions and multiplications to make for every iteration (for each m and n). Every iteration requires by square (each k, l, k' and l') approximately $40 * n_{step_{\Delta_\tau}} * n_{step_{\Delta_\nu}}$ operations in the exact case and $37 + 40 * n_{step_{\Delta_f}} + 40 * n_{step_{\Delta_t}}$ operations in the approximate case. Thus, we note that our approximate expression leads to a reduction of about 49% the number of mathematical operations with respect to the exact expression.

4. SIMPLIFIED MODULATOR AND DEMODULATOR IMPLEMENTATION

We note that the complexity of the modulator and demodulator implementation is even more important when the number of subcarriers is large. In this paper, we propose a simplified and efficient modulator/demodulator implementation, by using IFFT in the transmitter and FFT in the receiver, to reduce the hardware complexity. This simplification reduces also the manufacturing costs and the consumption of terminals which is very interesting for OEMs.

4.1. Simplified modulator implementation

Times domain samples of the OFDM signal are obtained by sampling the baseband transmitted signal $s(t)$ given in (1) at time instants $t = qT_s$, where $T_s = 1/KF$ is the sampling period. Then, we have

$$\begin{aligned} s(qT_s) &= \sum_m(\sum_n a_{mn}e^{j2\pi nF(qT_s-mT)})\varphi(qT_s-mT) \\ &= \sum_m(\sum_n a_{mn}e^{j2\pi nq/K}e^{-j2\pi nmFT})\varphi(qT_s-mT) \\ &= \sum_m\left\{{\scriptstyle\mathrm{IFFT}}(a_{mn}e^{-j2\pi nmFT})\right\}_{\mathrm{mod}(q,K)}\varphi(qT_s-mT), \end{aligned} \tag{17}$$

with IFFT(.) is the Inverse Fast Fourier Transform and mod(.,.) is the modulo division operator.

In order to reduce the implementation complexity of the system, we assume that the OFDM symbol duration T is an integer multiple of T_s and $T = PT_s$, with $P = TFK$.

The generation of the sampled signal given in equation (18) is illustrated in Figure 1 and described as follows

Preparotory framework: the optimized pulse $\varphi(t)$ is truncated to a MT duration and the other values are set to zero. We note that M is chosen great enough to neglect the lowest values of $\varphi(t)$. Then, the optimized pulse is sampled at time instants qT_s. The obtained samples are saved in a MP-length memory. Moreover, we initialize a cyclic register with MP zeros and we point at instant 0.

Signal generation: For the generation of the sampled signal given in (18), we need M steps described in the following:

1. In the first step, we apply the IFFT on the K symbols $\{a_{0,n}, n = 0, ..., K-1\}$. Thereafter, we duppli-

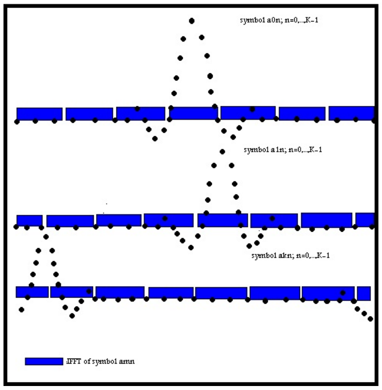

Fig. 1. Modulator implementation steps

cate the IFFT, of period $1/F$, MP/K times. Then, we multiply the dupplicated IFFT with the samples of the optimized pulse, saved in the memory. The result of the multiplication is then transferred to the register. We transmit the P first samples corresponding to the first T seconds of the register and empty its content after its transmission. The pointer is advanced by P samples to point at T seconds.

2. In the $(m+1)^{th}$ step, we take into account the IFFT symbol of $a_{m,n}$, $n = 0, ..., K-1$ in the sampled signal generation. For this aim, we multiply the IFFT dupplicated MP/K times with the pulse samples saved in the memory. The resut is circularly shifted with mP samples and added with the register content. Then, we transmit the P samples from the pointer. We move the pointer by P samples to point at instant $(m+1)T$ seconds.

4.2. Simplified Demodulator implementation

At the receiver, the optimized pulse $\varphi(t)$ is also truncated to MT duration and the other values are set to zero. We sample the truncated pulse at the sampling rate $1/T_s$ and save in a MP-length memory. We initialize a cyclic register with MP zeros and we point at instant 0. We collect the MP successive samples corresponding to the received signal between the instant 0 and instant MT seconds. We multiply the register content with the memory, sum over all periods of $1/F$ length and apply the FFT to recover the symbols $\{a_{0,n}, n = 0, ..., K-1\}$. We move the pointer to instant T seconds. Then, we replace the P first samples of the register with P new samples corresponding to the received signal between MT and $(M+1)T$ time instants. We multiply the register from the pointer indication with the memory, sum over all segments of $1/F$ duration and apply the FFT to recover the symbols $\{a_{1,n}, n = 0, ..., K-1\}$. The process is repeated in the same manner to recover the all transmited symbols.

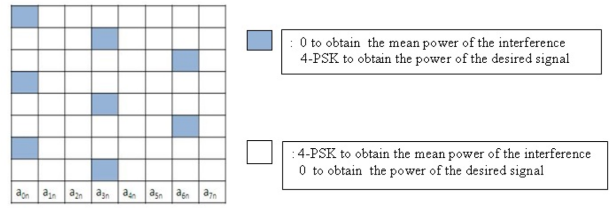

Fig. 2. Layout of the time-frequency plane for SIR evaluation

5. SIR OBTAINED THROUGH SYSTEM LEVEL SIMULATION

In this section, we propose an efficient method to determine the mean powers of the interference and desired signals. With these mean powers, we evaluate the simulated SIR. Our method is described as follows:

Mean power of the interference: Our idea is to generate OFDM signals of finite length. For each signal, we insert some zeros and compute the sum of the square of the decision variables on these zeros in order to compute the mean power of the interference. Without loss of generality, we fill the time-frequency plane with 4-PSK modulated symbols. The symbols wich are used for the interference evaluation are set to zero. They are chosen far apart so that we neglect the non accounted interference that could have been generated if one of these symbols is not set to zero but used to send 4-PSK data. We fill by the maximum of zeros in the frequency and time domains as shown in Figure 2 and transmit the symbols over our multicarrier system described in the last section. This method to determine the mean power of the interference is considered accurate since the ambiguity of the optimized pulse decreases exponentially.

Desired signal power: We fill the time-frequency plane with zeros. Without loss of generality, the symbols wich are used to compute the desired signal power are replaced with 4-PSK modulated symbols. As shown in Figure 2, these symbols are chosen so that they do not interfere with their neighbouring ones in the time-frequency domain. We transmit the symbols over our optimized multicarrier system and compute the sum of the square of the decision variables on 4-PSK data to find the desired signal power.

6. NUMERICAL AND SIMULATION RESULTS

Simulations are realized for a highly time and frequency dispersive channel characterized by the channel scattering function obeying (6). The optimization process is carried out for discrete lattice densities $\zeta = 1/FT$ ranging from 0.5 to 1.

In Figure 3, we present the evolution of the exact SIR and of the approximate SIR with respect to FT for OFDM systems operating over doubly dispersive noiseless channels with dispersion factor $\vartheta = B_D T_m = 10^{-2}$. Figure 3 shows that the approximate SIR follows closely the exact one for any order of combination. Also, notice that exact SIR optimization takes much more time than approximate SIR optimization. Then, we can conclude that our approximation of the SIR allows a low complexity pulses design with imperciptibe loss in the performance. Moreover, Figure 3 shows that the values of the SIR for $N = 11$ and $N = 13$ are the same. Next, we consider that the optimal pulses are reached as the combinations of the 11 most localized Hermite waveforms.

In Figure 4, we compare the numerically optimized SIR with the SIR computed through optimized OFDM system level simulation for different values of the dispersion factor ($\vartheta = 10^{-2}$, $\vartheta = 10^{-3}$ and $\vartheta = 10^{-4}$). Since $P = TFK$ must be an integer and $TF \geq 1$, we have $TF \in \{1, 1+\frac{1}{K}, 1+\frac{2}{K}, ...\}$. So, in order to have many possible values of TF between 1 and 2, we should consider a great number of subcarriers. In our simulations, we consider $K = 128$ and three values of TF: $TF = 1.2656$, $TF = 1.5625$ and $TF = 1.8906$. For the considered values of TF, we compare in Figure 4 the optimized SIR obtained through numerical optimization with the SIR computed through system level simulation. Figure 4 shows that the SIR achieved through system level simulation is roughly equal to the SIR obtained through pulses numerical maximization.

7. CONCLUSION

In this paper, an optimized OFDM pulse is proposed and implemented for its realistic performance evaluation. The optimization of the pulse shape is undertaken with an approximate expression of the SIR instead of the exact one, in order to accelerate the optimization procedure. We show that the SIR optimized using the approximate expression can achieve approximately the same value of the SIR optimized using the exact one. For the optimized OFDM system simulation, we proposed efficient modulator and demodulator implementations. Moreover, we proposed a simple and efficient method to determine the mean power of the interference and the desired signal power in order to deduce a mean value of the SIR resulting from the system simulation. Our simulation results showed that the SIR obtained through system level simulation matches the numerically optimized SIR.

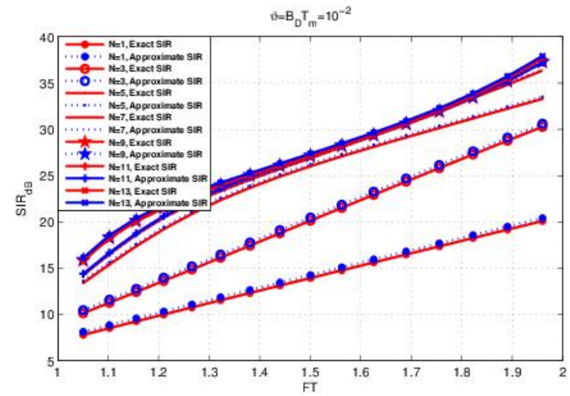

Fig. 3. SIR versus FT for the noiseless channel with different values of N and a dispersion factor $\vartheta = 10^{-2}$ in OFDM systems.

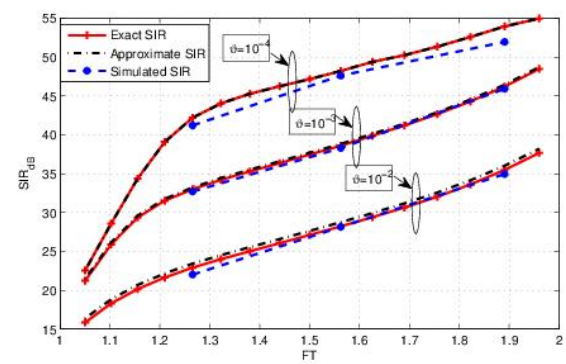

Fig. 4. Comparision of the numerically optimized SIR with the SIR computed through system level simulation for different values ϑ of the dispersion factor.

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