SINGLE-CARRIER EQUALIZATION AND DISTRIBUTED BEAMFORMING FOR ASYNCHRONOUS TWO-WAY RELAY NETWORKS

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ABSTRACT

In this paper, we consider a single-carrier communication scheme, where two transceivers exchange information with the help of multiple relays. The propagation delay in each relaying path is assumed to be different from those of the other paths. As such, the end-to-end channel is frequency selective, and hence, it produces inter-symbol-interference (ISI) at the two transceivers. The simple amplify-and-forward scheme is used at the relays and channel equalization is employed at both transceivers to combat ISI. We minimize the mean square error (MSE) of the total estimated received signals at the both transceivers, subject to a total power budget constraint, by optimizing the channel equalizers, the relay beamforming weights, and the transceivers' powers. We show that our proposed approach leads to a relay selection method which transforms the end-to-end channel into a frequency flat channel. We also present a semi-closed-form solution for the optimal relay beamforming weight.

Index Terms— Two-way relay network, single-carrier equalization, distributed beamforming

1. INTRODUCTION

In a wireless two-way relay network, two transceivers communicate with each other with the help of one or more intermediate relays. There are different relaying approaches among which, because of its simplicity, amplify-and-forward technique is of particular interest and has been widely studied in the literature [1–4]. In almost all these publications, the authors consider perfectly time-synchronized relays and transceivers. However, since the propagation delay of each relaying path can be different from that of the other paths, the signals transmitted by the relays arrive at the transceivers at different times, thereby leading to the frequency selectivity of the end-to-end channel. Therefore, even if the relay-transceiver channels are frequency flat, ISI is inevitable at the transceivers.

There are two different approaches to combat such an ISI. The first approach is based on multi-carrier communication, where all the nodes of the network are equipped with orthogonal frequency division multiplexing (OFDM) transmission and reception to transform the end-to-end frequency selective channel into multiple parallel flat fading channels [5, 6]. In the second approach, which is a single-carrier scheme, block or symbol channel equalization is used at the nodes of the network to eliminate ISI [7–10].

In this work, we consider a single-carrier scheme in an asynchronous two-way relay network and develop our system model for such a scheme. The relays are assumed to simply amplify their received signals, while the burden of equalization is on the shoulders of two *block channel equalizers* implemented at the two transceivers. We optimally obtain the transceivers' powers and the relay beamforming weight vector as well as the block equalizers in order to minimize the total MSE of the estimated received signals at both transceivers. We prove that such an optimization problem leads to a relay selection scheme, where only the relays contributing to one tap of the end-to-end channel are turned on and the remaining relays have to be turned off. Moreover, we present a semi-closed-form solution to find the optimum relay weights for such a relay selection method.

Notations: The statistical expectation is represented as $\mathrm{E}\{\cdot\}$ and $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. Complex conjugate, transpose, and Hermitian transpose are denoted as $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$, respectively. We represent the $N \times N$ identity matrix and the $M \times N$ all-zero matrix as $\mathbf{I}_N$ and $\mathbf{0}_{M\times N}$, respectively. We let $\mathrm{diag}(\mathbf{a})$ stand for a diagonal matrix whose diagonal entries are the elements of the vector $\mathbf{a}$.

2. SIGNAL MODEL

We consider a two-way relay network which includes two single-antenna transceivers which communicate with each other with the help of L single-antenna relay nodes. For the sake of simplicity, the relays are assumed to simply amplify and forward their received signals. We use block transmission to send a vector of $N_s \times 1$ symbols. As it is shown in Fig. 1, the transmitted symbols go through the serial-to-parallel block represented as "S/P". The ith transmitted block from Transceiver p is represented as $\mathbf{s}_p(i) = \left[\begin{array}{ccc} s_p[iN_s] & \cdots & s_p[iN_s+N_s-1] \end{array}\right]^T$, for $p \in \{1, 2\}$. Due to the frequency selectivity of the channel, inter-block-interference (IBI) arises between successive transmitted blocks which renders the received signal dependent on both

$\mathbf{s}_p(i)$ and $\mathbf{s}_p(i-1)$. In order to mitigate the IBI (thereby preventing the consecutive blocks from overlapping with each other), a cyclic prefix insertion matrix, represented as $\mathbf{T}_{\mathrm{cp}}$, is pre-multiplied by $\mathbf{s}_p(i)$ to transmit the data in blocks with the size $N_t = N + N_s$. Hence, the ith transmitted block is defined as

$$\overline{\mathbf{s}}_p(i) \triangleq \mathbf{T}_{\mathrm{cp}}\mathbf{s}_p(i) = \big[s_p[(i+1)N_s - N] \ \cdots \\ s_p[(i+1)N_s - 1] \ \ s_p[iN_s] \ \ \cdots \ \ s_p[(i+1)N_s - 1]\big]^T.$$

Here, N is the length of the vector of the equivalent discrete-time end-to-end channel taps and $\mathbf{T}_{\mathrm{cp}} = [\mathbf{I}_{\mathrm{cp}}^T \ \ \mathbf{I}_{N_s}^T]^T$ is an $N_t \times N_s$ matrix which is the concatenation of the last N rows of the identity matrix $\mathbf{I}_{N_s}$ (denoted as $\mathbf{I}_{\mathrm{cp}}$), and the identity matrix $\mathbf{I}_{N_s}$. Then, using the parallel-to-serial block denoted as "P/S", the serial signal goes through the channel. After the self-interference-cancelation block, denoted as "SIC", the first N entries of the received block are simply discarded by pre-multiplying it with the cyclic removal matrix $\mathbf{R}_{\mathrm{cp}} \triangleq [\mathbf{0}_{N_s \times N} \ \ \mathbf{I}_{N_s}]$. The burden of channel equalization is on the shoulders of the block equalizers, which are implemented at Transceivers 1 and 2 and are denoted as $\mathbf{F}_{r1}$ and $\mathbf{F}_{r2}$, respectively.

Assuming reciprocal and frequency flat channels between each transceiver and each relay, the 2×2 matrix $\mathbf{H}[n] = \begin{bmatrix} h_{11}[n] & h_{12}[n] \\ h_{21}[n] & h_{22}[n] \end{bmatrix}$ represents the end-to-end channel between the two transceivers. Note that for $p \in \{1,2\}$, $h_{pp}[n]$ represents the linear time-invariant (LTI) channel corresponding to the so-called self-interference signal, which is transmitted by Transceiver p and is relayed back to the same transceiver. Let T_s be the sampling time and $\tau_{l_{pq}}$ be the propagation delay of the signal path from Transceiver p to Transceiver q which is going through the lth relay, for $p, q \in \{1,2\}$. Therefore, the impulse response of the end-to-end channel from Transceiver p to Transceiver q, can be written as

$$h_{pq}[n] = \sum_{l=1}^{L} b_{l_{pq}} \delta[n - \check{n}_l] \quad \text{for} \quad p, q \in \{1,2\} \tag{1}$$

where $\check{n}_l$ is an integer number satisfying $(\check{n}_l - 1)T_s \leq \tau_{l_{pq}} \leq \check{n}_l T_s$, whereas $b_{l_{pq}} \triangleq w_l g_{lp} g_{lq}$ is the total amplification factor applied to the signal going through the lth relay, w_l is the complex weight of the lth relay, and g_{lp} is the flat fading channel coefficient between Transceiver p and the lth relay, for $p, q \in \{1,2\}$. The vector of channel impulse response coefficients can be represented as $\mathbf{h}_{pq} = \begin{bmatrix} h_{pq}[0] & h_{pq}[1] & \cdots & h_{pq}[N-1] \end{bmatrix}^T$. The contribution of different relays to different taps of $h_{pq}[\cdot]$ is shown by $N \times L$ matrix $\mathbf{B}_{pq}$, whose (n,l)th element is defined as

$$B_{pq}(n,l) \triangleq \begin{cases} g_{lp} g_{lq}, & (n-1)T_{\mathrm{s}} \leq \tau_{l_{pq}} \leq nT_{\mathrm{s}} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Denoting the vector of complex relay weights as $\mathbf{w} \triangleq [w_1, \ldots, w_L]^T$, the vector of channel impulse response coefficients can be represented as

$$\mathbf{h}_{pq} = \mathbf{B}_{pq}\mathbf{w}. \tag{3}$$

The reciprocity of the end-to-end channel implies that $\mathbf{h}_{12} = \mathbf{h}_{21} \triangleq \mathbf{h} \triangleq \begin{bmatrix} h[0] & h[1] & \cdots & h[N-1] \end{bmatrix}^T$. We assume that the propagation delay between the lth relay and Transceiver q, denoted as τ'_{lq}, satisfies $(n'_l - 1)T_s \leq \tau'_{lq} \leq n'_l T_s$. Let $\upsilon_l[n]$ represent the temporally and spatially zero-mean white Gaussian noise at the lth relay with variance of σ^2. This noise is multiplied by w_l, is delayed by n'_{lq}, and arrives at the block equalizers. Hence, the relay noise received at Transceiver q can be written as

$$\xi_q[n] = \sum_{l=1}^{L} w_l g_{lq} \upsilon_l[n - n'_{lq}] = \boldsymbol{\upsilon}_{n,q}^T \mathbf{G}_q \mathbf{w} \tag{4}$$

where $\boldsymbol{\upsilon}_{n,q} \triangleq \begin{bmatrix} \upsilon_1[n - n'_{1q}] & \cdots & \upsilon_L[n - n'_{Lq}] \end{bmatrix}^T$ and $\mathrm{G}_q = \mathrm{diag}\{g_{lq}, g_{2q}, \cdots, g_{Lq}\}$, for $q \in \{1,2\}$. Considering $\gamma'_q[n]$ as the vector of noise added to the signal at Transceiver q and before the serial-to-parallel block, the total noise vector received at Transceiver q and before the cyclic removal matrix can be written as

$$\overline{\boldsymbol{\gamma}}_q(i) = \overline{\boldsymbol{\xi}}_q(i) + \overline{\boldsymbol{\gamma}}'(i) = \overline{\boldsymbol{\Upsilon}}_q(i)\mathbf{G}_q\mathbf{w} + \overline{\boldsymbol{\gamma}}'_q(i) \tag{5}$$

where $\overline{\boldsymbol{\gamma}}_q(i) \triangleq \begin{bmatrix} \gamma_q[iN_t] & \cdots & \gamma_q[iN_t + N_t - 1] \end{bmatrix}^T$, $\overline{\boldsymbol{\xi}}_q(i) \triangleq \begin{bmatrix} \xi_q[iN_t] & \cdots & \xi_q[iN_t + N_t - 1] \end{bmatrix}^T$, $\overline{\boldsymbol{\gamma}}'_q(i) \triangleq \begin{bmatrix} \gamma'_q[iN_t] & \cdots & \gamma'_q[iN_t + N_t - 1] \end{bmatrix}^T$, and

$$\overline{\boldsymbol{\Upsilon}}_q(i) \triangleq \begin{bmatrix} \boldsymbol{\upsilon}_{iN_t,q} & \boldsymbol{\upsilon}_{iN_t+1,q} & \cdots & \boldsymbol{\upsilon}_{(iN_t+N_t-1),q} \end{bmatrix}^T$$

is an $N_t \times L$ matrix. Assuming $\mathrm{E}\{|s_p[k]|^2\} = 1$ for $p \in \{1,2\}$, we can write the ith transmitted block received at the output of self-interference-cancelation block at Transceiver q as

$$\overline{\mathbf{r}}_q(i) = \sqrt{P_p}\mathbf{H}_0\overline{\mathbf{s}}_p(i) + \sqrt{P_p}\mathbf{H}_1\overline{\mathbf{s}}_p(i-1) + \overline{\boldsymbol{\gamma}}_q(i) \tag{6}$$

where P_p is the transmit power of Transceiver p, and the following definitions are used:

$$\mathbf{H}_0 \triangleq \begin{bmatrix} h[0] & 0 & 0 & \cdots & 0 \\ \vdots & h[0] & 0 & \cdots & 0 \\ h[N-1] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & h[N-1] & \cdots & h[0] \end{bmatrix} \tag{7}$$

$$\mathbf{H}_1 \triangleq \begin{bmatrix} 0 & \cdots & h[N-1] & \cdots & h[1] \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & h[N-1] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}. \tag{8}$$

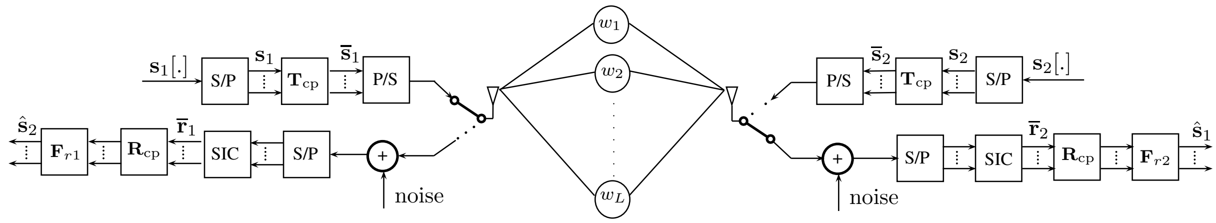

Fig. 1. System model

After removing the cyclic prefix and discarding the first N entries from the received block, the received block can be written as

$$\mathbf{r}_q(i) = \mathbf{R}_{\rm cp}\overline{\mathbf{r}}_q(i) = \sqrt{P_p}\widetilde{\mathbf{H}}\mathbf{s}_p(i) + \widetilde{\boldsymbol{\gamma}}_q(i) \tag{9}$$

where $\widetilde{\boldsymbol{\gamma}}_q(i) \triangleq \mathbf{R}_{\rm cp}\overline{\boldsymbol{\gamma}}_q(i)$ and $\widetilde{\mathbf{H}} \triangleq \mathbf{R}_{\rm cp}\mathbf{H}_0\mathbf{T}_{\rm cp}$ is an $N_s \times N_s$ circulant matrix with its (k,l)th entry given by $\tilde{h}[(k-l) \mod N_s]$, where $\tilde{h}[\cdot]$ is obtained by zero padding $h[\cdot]$ with $(N_s - N)$ zeros. It can be verified that $\mathbf{R}_{\rm cp}\mathbf{H}_1 = \mathbf{0}$, and therefore, the IBI-inducing matrix $\mathbf{H}_1$ is eliminated. Hence, the estimated transmitted signal block at Transceiver q can be represented as

$$\hat{\mathbf{s}}_p(i) \triangleq \mathbf{F}_{rq}\mathbf{r}_q(i) = \sqrt{P_p}\mathbf{F}_{rq}\widetilde{\mathbf{H}}\mathbf{s}_p(i) + \mathbf{F}_{rq}\widetilde{\boldsymbol{\gamma}}_q(i) \tag{10}$$

where $\hat{\mathbf{s}}_p(i)$ is an $N_s \times 1$ vector. Considering $\boldsymbol{\upsilon}_l(i)$ as the noise added to the ith signal block at the lth relay, the total relayed signal at the lth relay can be represented as

$$\bar{\mathbf{x}}_l(i) = w_l\left[\sqrt{P_1}g_{l1}\bar{\mathbf{s}}_1(i) + \sqrt{P_2}g_{l2}\bar{\mathbf{s}}_2(i) + \boldsymbol{\upsilon}_l(i)\right] \tag{11}$$

where $\overline{\mathbf{x}}_l(i) \triangleq \left[\begin{array}{ccc} \overline{x}_l[iN_t] & \cdots & \overline{x}_l[iN_t + N_t - 1] \end{array}\right]^T$, while $\overline{x}_l[t]$ is the signal transmitted by the lth relay at time t, and $\boldsymbol{\upsilon}_l(i) \triangleq \left[\begin{array}{ccc} \upsilon_l[iN_t] & \cdots & \upsilon_l[iN_t + N_t - 1] \end{array}\right]^T$ is the ith block of measurement noise of the lth relay. The average transmit power of the lth, $\widetilde{P}_l = \frac{1}{N_s}\mathrm{E}\left\{\mathbf{x}_l^H(i)\mathbf{x}_l(i)\right\}$ relay is calculated as[1]

$$\widetilde{P}_l = |w_l|^2\left(|g_{l1}|^2P_1 + |g_{l2}|^2P_2 + \sigma^2\right) \tag{12}$$

and $P_{total} = P_1 + P_2 + \sum_{l=1}^{L}\widetilde{P}_l$ is the total consumed power.

3. PROPOSED DESIGN APPROACH

In this section, we aim to optimally obtain the block channel equalizers at both transceivers as well as the relay weight vector and the transceivers' powers in order to minimize the sum-MSE of the estimated received signals at both transceiver. The vector of signal estimation error at Transceiver q can be written as $\mathbf{e}_q(i) = \hat{\mathbf{s}}_p(i) - \mathbf{s}_p(i) = \mathbf{F}_{rq}\mathbf{r}_q(i) - \mathbf{s}_p(i)$. We now formulate our design problem as

$$\min_{\substack{P_1\geq 0\\ P_2\geq 0}} \min_{\mathbf{w}} \min_{\mathbf{F}_{r1},\mathbf{F}_{r2}} \sum_{q=1}^{2}\mathrm{E}\left\{\|\mathbf{e}_q(i)\|^2\right\} \text{ s.t. } P_{total} \leq P_{max} \tag{13}$$

where P_{max} is the total available power in the network. Assuming that $\mathrm{E}\{\mathbf{s}_p(i)\} = \mathbf{0}$ and that $\mathbf{s}_p(i)$ and $\widetilde{\boldsymbol{\gamma}}_q(i)$ are statistically independent, we can write

$$\begin{aligned}\mathrm{E}\left\{\mathbf{e}_q^H(i)\mathbf{e}_q(i)\right\} &= \mathrm{tr}\left(\mathbf{F}_{rq}\mathbf{R}_q\mathbf{F}_{rq}^H\right)\\ &- \sqrt{P_p}\ \mathrm{tr}\left(\mathbf{F}_{rq}\widetilde{\mathbf{H}} + \widetilde{\mathbf{H}}^H\mathbf{F}_{rq}^H\right) + N_s\end{aligned} \tag{14}$$

where

$$\begin{aligned}\mathbf{R}_q &\triangleq \mathrm{E}\left\{\mathbf{r}_q(i)\mathbf{r}_q^H(i)\right\} = P_p\widetilde{\mathbf{H}}\mathrm{E}\left\{\mathbf{s}_p(i)\mathbf{s}_p^H(i)\right\}\widetilde{\mathbf{H}}^H +\\ &\mathrm{E}\left\{\widetilde{\boldsymbol{\gamma}}_q(i)\widetilde{\boldsymbol{\gamma}}_q^H(i)\right\} = P_p\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^H + \sigma^2\left(||\mathbf{G}_q\mathbf{w}||^2 + 1\right)\mathbf{I}_{N_s}\end{aligned}$$

is the correlation matrix of the received block. The optimal value of $\mathbf{F}_{r1}$ can be obtained by taking the derivative of (14) with respect to $\mathbf{F}_{r1}$ and equating it to zero. This yields $\mathbf{F}_{r1}^{opt} = \sqrt{P_2}\widetilde{\mathbf{H}}^H\mathbf{R}_1^{-1}$. Similarly, we can obtain $\mathbf{F}_{r2}^{opt} = \sqrt{P_1}\widetilde{\mathbf{H}}^H\mathbf{R}_2^{-1}$ and write

$$\begin{aligned}\lambda(\mathbf{w}, P_1, P_2) &\triangleq \min_{\mathbf{F}_{r1},\mathbf{F}_{r2}} \sum_{q=1}^{2}\mathrm{E}\left\{\mathbf{e}_q^H(i)\mathbf{e}_q(i)\right\}\\ &= 2N_s - tr\left(\widetilde{\mathbf{H}}^H(P_2\mathbf{R}_1^{-1} + P_1\mathbf{R}_2^{-1})\widetilde{\mathbf{H}}\right).\end{aligned} \tag{15}$$

The $N_s \times N_s$ circulant matrix $\widetilde{\mathbf{H}}$ can be diagonalized by pre- and post-multiplying it with N_s-point DFT and IDFT matrices as represented below

$$\mathbf{F}\widetilde{\mathbf{H}}\mathbf{F}^{-1} = \mathbf{D}_H \triangleq \mathrm{diag}\{H(e^{j0}), \cdots, H(e^{j\frac{2\pi(N_s-1)}{N_s}})\} \tag{16}$$

where $H(e^{j2\pi f}) = \sum_{n=0}^{N-1} h[n]exp(-j2\pi fn)$ is the frequency response of the LTI channel at the normalized frequency f, and the elements of the DFT matrix are defined as $F(k,n) = N_s^{-\frac{1}{2}}\exp(-j2\pi kn/N_s)$. Using (16), we deduce that $\widetilde{\mathbf{H}} =$

[1] Here, we assume that the transmission length is much longer than the difference between the times of arrivals of the transceiver's signals at the relay.

$\mathbf{F}^{-1}\mathbf{D}_H\mathbf{F}$, and hence, rewrite (15) as

$$\begin{aligned}\lambda(\mathbf{w},P_1,P_2) &= 2N_s \\ &- P_2\mathrm{tr}\left[(P_2\mathbf{I}+\sigma^2\left(\|\mathbf{G}_1\mathbf{w}\|^2+1\right)\mathbf{D}_H^{-2})^{-1}\right] \\ &- P_1\mathrm{tr}\left[(P_1\mathbf{I}+\sigma^2\left(\|\mathbf{G}_2\mathbf{w}\|^2+1\right)\mathbf{D}_H^{-2})^{-1}\right]. \quad (17)\end{aligned}$$

Let $\mathbf{f}_k \triangleq \frac{1}{\sqrt{N_s}}\left[\begin{array}{cccc} 1 & e^{j\frac{2\pi(k-1)}{N_s}} & \cdots & e^{j\frac{2(N-1)(k-1)\pi}{N_s}}\end{array}\right]^T$, we can write $\mathbf{D}_H = \mathrm{diag}\{\sqrt{N_s}\mathbf{f}_1^H\mathbf{B}\mathbf{w},\cdots,\sqrt{N_s}\mathbf{f}_{N_s}^H\mathbf{B}\mathbf{w}\}$. Therefore, $\lambda(\mathbf{w},P_1,P_2)$ can be written as

$$\lambda(\mathbf{w},P_1,P_2) = \sum_{q=1}^{2} N_s - P_p\ \mathrm{tr}\left[\left(\mathrm{diag}\{P_p+\frac{\sigma^2\left(\|\mathbf{G}_q\mathbf{w}\|^2+1\right)}{N_s|\mathbf{f}_k^H\mathbf{B}\mathbf{w}|^2}\}_{k=1}^{k=N_s}\right)^{-1}\right]. \quad (18)$$

Using (18) and the fact that the trace of a diagonal matrix can be written as the sum of its diagonal entries, we can rewrite (13) as

$$\min_{\substack{P_1\geq 0\\ P_2\geq 0}} \min_{\mathbf{w}} \sum_{q=1}^{2}\sum_{k=1}^{N_s}\frac{1}{\phi_{k,q}(\mathbf{w})} \text{ s.t. } P_{total}\leq P_{max} \quad (19)$$

where $\phi_{k,q}(\mathbf{w}) = 1+\frac{P_pN_s|\mathbf{f}_k^H\mathbf{B}\mathbf{w}|^2}{\sigma^2(\|\mathbf{G}_q\mathbf{w}\|^2+1)}$. In [11], it has been shown that optimization problem (19) can be written as

$$\begin{aligned}&\min_{\substack{P_1\geq 0\\ P_2\geq 0}} \min_{\mathbf{w}} \sum_{q=1}^{2}\frac{N_s}{\frac{P_p\mathbf{w}^H\mathbf{B}^H\mathbf{B}\mathbf{w}}{\sigma^2(\|\mathbf{G}_q\mathbf{w}\|^2+1)}+1} \\ &\text{subject to } \sum_{l=1}^{L}|w_l|^2\left(|g_{l1}|^2P_1+|g_{l2}|^2P_2+\sigma^2\right) \\ &\qquad + P_1+P_2\leq P_{max} \text{ and } \mathbf{w}\in\mathcal{U}_n. \quad (20)\end{aligned}$$

where $\mathcal{U}_n$ is the set of relay weight vectors such that only the nth tap of the channel is non-zero and the remaining taps are zero [6]. Indeed, the constraint $\mathbf{w}\in\mathcal{U}_n$, implies that only relays which contribute to the nth tap of the end-to-end channel are turned on and the remaining of the relays have to turned off. Let $\mathbf{w}_n$ represent any vector of relay weights corresponding to the nth tap of the channel. Hence, if $\mathbf{w}\in\mathcal{U}_n$ then, $\mathbf{w}^H\mathbf{B}^H\mathbf{B}\mathbf{w} = \mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n$, where $\mathbf{b}_n^H$ captures the non-zero entries of the $(n+1)$th row of $\mathbf{B}$. In order to minimize the error, we need to find the index of the tap which yields the minimum value for the objective function. Therefore, the optimization problem (20) can be written as

$$\begin{aligned}&\min_{0\leq n\leq N-1}\min_{\substack{P_1\geq 0\\ P_2\geq 0}}\min_{\mathbf{w}_n} \sum_{q=1}^{2}\frac{N_s}{\psi_{n,q}(\mathbf{w})} \quad \text{for } p\neq q \\ &\text{subject to } P_1+P_2+P_1\|\mathbf{G}_1^{(n)}\mathbf{w}_n\|^2+P_2\|\mathbf{G}_2^{(n)}\mathbf{w}_n\|^2 \\ &\qquad +\sigma^2\mathbf{w}_n^H\mathbf{w}_n\leq P_{max} \quad (21)\end{aligned}$$

where $\psi_{n,q}(\mathbf{w}) \triangleq \left(\frac{P_p\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n}{\sigma^2\left(\|\mathbf{G}_q^{(n)}\mathbf{w}_n\|^2+1\right)}+1\right)$ and $\mathbf{G}_q^{(n)}$, for $q\in\{1,2\}$, is a diagonal matrix whose diagonal entries are a subset of those diagonal entries of $\mathbf{G}_q$ which correspond to the relays that contribute to the nth tap of the end-to-end channel impulse response. Without loss of optimality, we can assume that $\psi_{n,1}(\mathbf{w}) = \psi_{n,2}(\mathbf{w})$. Otherwise, if for example, $\psi_{n,1}(\mathbf{w}) > \psi_{n,2}(\mathbf{w})$ we can reduce the optimal value of P_2 to make $\psi_{n,1}(\mathbf{w})$ and $\psi_{n,2}(\mathbf{w})$ equal, thereby contradicting optimality. Therefore, the optimization problem (21) can be written as

$$\begin{aligned}&\min_{0\leq n\leq N-1}\min_{\substack{P_1\geq 0\\ P_2\geq 0}}\min_{\mathbf{w}_n} \frac{2N_s}{\frac{P_2\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n}{\sigma^2\left(\|\mathbf{G}_1^{(n)}\mathbf{w}_n\|^2+1\right)}+1} \\ &\text{subject to } P_1\left(1+\|\mathbf{G}_1^{(n)}\mathbf{w}_n\|^2\right)+P_2\left(1+\|\mathbf{G}_2^{(n)}\mathbf{w}n\|^2\right) \\ &\qquad +\sigma^2\mathbf{w}_n^H\mathbf{w}_n\leq P_{max} \\ &\quad P_1\left(\|\mathbf{G}_1^{(n)}\mathbf{w}_n\|^2+1\right) = P_2\left(\|\mathbf{G}_2^{(n)}\mathbf{w}_n\|^2+1\right). \quad (22)\end{aligned}$$

In [11], it is shown that the constrained optimization problem (22) can be written as

$$\begin{aligned}&\max_{0\leq n\leq N-1}\max_{\mathbf{w}_n}\frac{(P_{max}-\sigma^2\mathbf{w}_n^H\mathbf{w}_n)\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n}{2\sigma^2\left(\|\mathbf{G}_1^{(n)}\mathbf{w}_n\|^2+1\right)\left(\|\mathbf{G}_2^{(n)}\mathbf{w}_n\|^2+1\right)} \\ &\text{subject to } \quad \mathbf{w}_n^H\mathbf{w}_n\leq\frac{P_{max}}{\sigma^2}. \quad (23)\end{aligned}$$

In [12], a semi-closed-form solution to (23) is given as

$$\mathbf{w}_n^o(\mu_n) = \sigma^2\kappa(\mu_n)\sqrt{2\nu_n}\left(2\mu_n\mathbf{Q}_1^{(n)}+2\nu_n\mathbf{Q}_2^{(n)}+\sigma^2\mathbf{I}\right)^{-1}\mathbf{b}_n \quad (24)$$

where $\mathbf{Q}_q^{(n)} = \mathbf{G}_q^{(n)H}\mathbf{G}_q^{(n)}$, $\nu_n \triangleq 0.5P_{\max}/\sigma^2-\mu_n$, and $\kappa(\mu_n)$ is defined as

$$\begin{aligned}\kappa(\mu_n) \triangleq& \sigma^{-1}(\mathbf{b}_n^H(\sigma^2\mathbf{I}+2\mu_n\mathbf{Q}_1^{(n)}) \\ &\times(\sigma^2\mathbf{I}+2\mu_n\mathbf{Q}_1^{(n)}+2\nu_n\mathbf{Q}_2^{(n)})^{-2}\mathbf{b}_n)^{-\frac{1}{2}} \quad (25)\end{aligned}$$

and μ_n is the *unique* solution to the following equation:

$$\begin{aligned}&\sigma^2(P_{\max}/\sigma^2-4\mu_n)\mathbf{b}_n^H \\ &\times(2\mu_n\mathbf{Q}_1^{(n)}+(P_{\max}/\sigma^2-2\mu_n)\mathbf{Q}_2^{(n)}+\mathbf{I})^{-1}\mathbf{b}_n- \\ &\mu_n(P_{\max}/\sigma^2-2\mu_n)\mathbf{b}_n^H \\ &\times(2\mu_n\mathbf{Q}_1^{(n)}+(P_{\max}/\sigma^2-2\mu_n)\mathbf{Q}_2^{(n)}+\mathbf{I})^{-2} \\ &\times(2\mathbf{Q}_1^{(n)}-2\mathbf{Q}_2^{(n)})\mathbf{b}_n = 0 \quad (26)\end{aligned}$$

which satisfies $0\leq\mu_n\leq 0.5P_{\max}/\sigma^2$. A simple bisection method can be used to solve (26) and to find the value of μ_n in the interval $[0 \quad 0.5P_{\max}/\sigma^2]$. The optimal value of n is determined by evaluating the objective function for $\mathbf{w}_n$ for $n=0,1,\cdots,N-1$ and choosing the value of n which leads to the maximum objective function. In other words, the

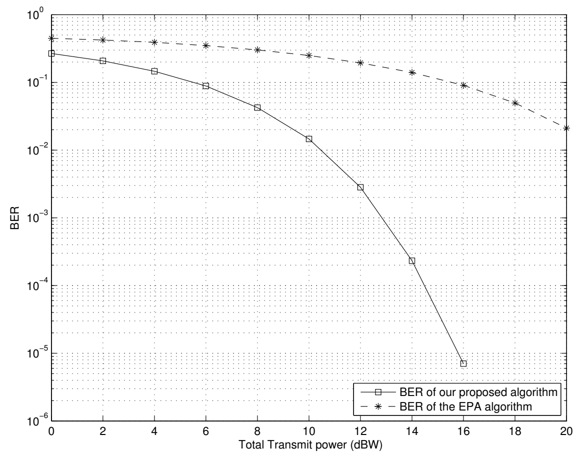

Fig. 2. Bit error rate versus total transmit power

set of relays which contribute to one tap of the finite impulse response channel with the smallest total MSE, is selected and the remaining relays have to be turned off. In [2], it is shown that the solution to (24) requires all participating relays to use half of the total available transmit power.

4. SIMULATION RESULTS

We consider an asynchronous bi-directional relay network with $L = 8$ single-antenna relays. The transceivers are assumed to transmit signal blocks of $N_s = 64$ symbols with a cyclic prefix length of $N = 16$. The zero-mean Gaussian white noise has a variance $\sigma^2 = 1$. The flat fading channel coefficients are zero-mean independent and identically distributed (i.i.d) complex Gaussian random variables with unit variance. For sampling time $T_s = 1$, we model the propagation delay of each relay path as a uniformly distributed random variable uniformly distributed in the interval $[0, 8T_s]$.

Assuming QPSK modulation, in Fig. 2 we show the end-to-end BER curves versus the total transmit power P_{max} for our proposed algorithm and compare it with the one for equal power allocation (EPA) method where the total transmit power is equally allocated to the all nodes of the network. As it is illustrated in this figure, our proposed approach yields a better performance in comparison with the EPA scheme. That is for a given total transmit power, our proposed algorithm yields less error compared to the EPA method. Moreover, for transmit powers higher than 10 dB, in our proposed algorithm, the bit error rate decreases drastically.

5. CONCLUSION

In this paper, we considered two transceivers which aim to communicate using an amplify-and-forward relay network with different propagation delays in different relaying paths. Since the end-to-end channel is frequency selective, we used cyclic insertion and removal matrices at the transceivers to combat IBI. The channel equalizers, the relay beamformer weights and the transceivers' powers were optimized to minimize the MSE of the total estimated transmitted signals under a total power budget constraint. We proved that at the optimum, single-carrier equalization leads to a relay selection scheme and presented a semi-closed-form solution for the optimal beamformer.

6. REFERENCES

[1] M. Zaeri-Amirnai and S. ShahbazPanahi, "Performance comparison in amplify-and-forward bi-directional network beamforming," *IEEE Transactions, Signal Processing*, vol. revised, Sep. 2011.

[2] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 58, pp. 1238–1250, Mar. 2010.

[3] R. Vaze and R. W. Heath, "Optimal amplify and forward strategy for two-way relay channel with multiple relays," in *Proc. IEEE ITW 2009*, Volos, Greece, Jun. 12–10, 2009, pp. 181–185.

[4] Y. Zhao, R. Adve, and T. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 3114 –3123, Aug. 2007.

[5] F. Gao, R. Zhang, and Y.-C. Liang, "Channel estimation for ofdm modulated two-way relay networks," *IEEE Transactions on Signal Processing*, vol. 57, pp. 4443 –4455, Nov 2009.

[6] R. Vahidnia and S. Shahbazpanahi, "Multi-carrier asynchronous bi-directional relay networks: Joint subcarrier power allocation and network beamforming," *To be appeared, IEEE Trans. Wireless Communication.*, 2013.

[7] A. Schad, H. Chen, A. Gershman, and S. Shahbazpanahi, "Filter-and-forward multiple peer-to-peer beamforming in relay networkswith frequency selective channels," in *Proc. ICASSP'10*, 2010, pp. 3246–3249.

[8] Y. wen Liang, A. Ikhlef, W. Gerstacker, and R. Schober, "Cooperative filter-and-forward beamforming for frequency-selective channels with equalization," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 228 –239, Jan 2011.

[9] M. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1999 – 2004, Nov. 2004.

[10] P. Wu and R. Schober, "Cooperative beamforming for single-carrier frequency-domain equalization systems with multiple relays," *IEEE Trans. Wireless Communication.*, vol. 11, no. 6, pp. 2276–2286, 2012.

[11] R. Vahidnia and S. Shahbazpanahi, "Single-carrier equalization for asynchronous two-way relay networks," *Submitted to IEEE Trans. Signal Proc*, 2013.

[12] S. Shahbazpanahi and M. Dong, "Achievable rate region and sum-rate maximization for network beamforming for bi-directional relay networks," in *Proc. ICASSP'10*, Mar. 2010, pp. 2510–2513.