MULTI-SENSOR QUICKEST DETECTION BY EXPLOITING RADIO CORRELATION

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ABSTRACT

In this paper, we present a novel quickest detection scheme to sequentially detect the emergence of an event with the help of multiple sensors. In the proposed scheme, we exploit the fact that the observations of the spatially proximal sensor nodes are highly correlated due to correlated shadowing effects. However, we will see that when it comes to infer the spatial correlation, the estimate of the spatially structured covariance matrix is not feasible as the proposed quickest detection scheme is a recursive method and operates with single sample. Hence, we propose to model the spatial covariance structure by using the a-priori information about the locations of the sensors. Moreover, the proposed scheme also takes into account a scenario where only a subset of sensors are affected by the event's signal. Therefore, it takes into account both the exploitation of the spatial structure and the selection of the subset of sensors in the process of detection. Both analytical and numerical results are developed for the mean detection delay, showing important advantages.

***Index Terms*—** Spatial Correlation, CUSUM, Wireless Sensor Network, Likelihood ratio.

1. INTRODUCTION

Most of the detection approaches in wireless sensor network (WSN) are block-based in the sense that the sensor nodes take a block of samples to decide on the activity state of an event. The main goal of these block-based detection schemes is to maximize the detection probability subject to constraints on the false alarm probability. However, while detecting abrupt changes due to an event, other than the probability of detection, the WSN should be able to detect the changes as quickly as possible. For example in cognitive radio, the secondary users need to detect and quit the frequency band as quickly as possible if the corresponding primary radio emerges [1]. In such dynamic applications, the multi-sensor should continuously observe the region to detect the event quickly as possible based on the change in the received observations. Therefore, a detection framework is required that minimizes the detection delay for a certain level of false alarm.

The above discussion leads us to the concept of multi-sensor sequential change detection [2, 3]. A detailed discussion on the multi-sensor sequential (quickest) detection can be found in [1, 4, 5]. In sequential change detection methods, the well-known Page's cumulative sum (CUSUM) algorithm has been shown to be optimal in the sense of minimizing the detection delay while maintaining an acceptable level of false alarm [1, 2]. Moreover, CUSUM algorithm has been previously adopted for the collaborative spectrum sensing by assuming that the observations from multiple sensors are independent and identically distributed both in time as well as in space [1, 3, 6]. However, while observing the signal emitted from a single event or due to correlated shadowing effects the observations of spatially proximal sensors are highly correlated with the degree of correlation increasing with decreasing inter-node separation [7, 8]. To the best of our knowledge the previous work on multi-sensor quickest detection either ignores the presence of the mutual correlation in the received observations or consider it as a deleterious effect [9]. However, this spatial correlation is a feature that can be used for detection since the noise processes at different nodes can be safely assumed statistically independent.

Taking into account the above discussions, we formulate the Multivariate Cumulative SUM(MCUSUM) algorithm by exploiting the spatial correlation structure. Consequently, the proposed scheme also considers the fact that due to the limited coverage of the signal emitted from a weak intensity event, just a subset of sensors (in the form of a spatial cluster) are able to receive power levels enough for reliable detection [10]. While considering the spatial correlation, it is required that the information regarding the spatially structured covariance matrix should be available. The MCUSUM is a recursive method that operates with single sample, hence, the estimation of the unknown covariance matrix is not feasible [2]. In order to overcome this hurdle we propose to model the spatial covariance structure by using the a-priori spatial information based on the known sensor-to-sensor distances. Similarly, to observe the change point and ignore noisy observations from the unaffected sensors (i.e. sensors that receive the event signal with negligible power), the proposed detection method se-

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lects observations of the affected sensors(subset). Finally, to asses the performance of the proposed scheme, we analyze the expression of the mean detection delay by adopting the asymptotic results given in [11]. Simulations results are included to verify these analytical expressions.

The remaining paper is organized as follows. The problem formulation and the proposed system model are covered in Section 2 and 3, respectively. The proposed collaborative quickest detection scheme is explained in section 4. The performance analysis is presented in Section 5 and numerical results are shown in Section 6. Finally, conclusions are drawn in Section 7.

2. PROBLEM FORMULATION AND ASSUMPTIONS

We consider an infrastructure-based WSN where in order to observe the event, K sensors continuously measure their total received power (in dBm). They normalize the measurements by subtracting the the respective mean noise powers(in dBm, assumed to be known) as in [12], and relay the normalized measurements $\{x_i\}_{i=1}^{K}$ (in dBm) to the fusion center. Under $\mathcal{H}_0$, the power at the output of the i-th local energy detector is simply the sum of noise and interference powers. Consequently, due to the effect of the interferences, the total noise power can be modeled as a independent log-normally distributed random variable with variance $\sigma_{n_i}^2$ (assumed to be known) [12]. Hence, under the null hypothesis $\{x_i\}_{i=1}^{K}$ (in dBm) are independent Gaussian distributed random variables with zero means and variances $\left\{\sigma_{n_i}^2\right\}_{i=1}^{K}$. Under $\mathcal{H}_1$, the total power at i-th sensor is the sum of the event signal power(modeled with log-normal random variable having variance $\sigma_{s_i}^2$, that quantifies shadow fading) and the noise power[1]. Given this, the total received power under $\mathcal{H}_1$ is equal to the sum of two independent log-normally distributed random variables and the sum can be approximated as log-normal random variable [13]. Based on this information, given $\mathcal{H}_1$, the measurements $\{x_i\}_{i=1}^{K}$ (in dBm) are assumed to be Gaussian distributed with means $\mu_i \triangleq \mathrm{E}\left[10\log_{10}\left(1+\mathrm{SNR}_i\right)\right]$ and covariances $\sigma_{s_i}\sigma_{s_j}\rho_{i,j} + \sigma_{n_i}^2$ where $\rho_{i,j}$ quantifies correlation of shadowing between sensor i and j [12]. Having said this, at time instant n, the observation vector received at the fusion center can be represented as $\mathbf{x}(n) \triangleq \left[\begin{array}{cccc} x_1(n) & x_2(n) & \cdots & x_K(n) \end{array}\right]^T$ and the signal model can be written as:

$$\begin{aligned} \mathcal{H}_0 :& \mathbf{x}(n) = \mathbf{w}(n), \\ \mathcal{H}_1 :& \mathbf{x}(n) = \mathbf{s}(n) + \mathbf{w}(n), \end{aligned} \tag{1}$$

where $\mathbf{s}(n) \sim \mathcal{N}\left(\boldsymbol{\mu}_s(n), \boldsymbol{\Sigma}_s\right)$ with i-th element of the mean vector $\boldsymbol{\mu}_s(n)$ is μ_i and i,j-th element of $\boldsymbol{\Sigma}_s$ is $\sigma_{s_i}\sigma_{s_j}\rho_{i,j}$. Similarly, $\mathbf{w}(n) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_0)$ contains noise with $\boldsymbol{\Sigma}_0 = \mathrm{diag}\left\{\begin{array}{cccc} \sigma_{n_1}^2 & \sigma_{n_2}^2 & \cdots & \sigma_{n_K}^2 \end{array}\right\}$. Hence, under $\mathcal{H}_0$, $\mathbf{x}(n) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_0)$ and under $\mathcal{H}_1$, $\mathbf{x}(n) \sim \mathcal{N}(\boldsymbol{\mu}_s(n), \boldsymbol{\Sigma}_1)$ with $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_s + \boldsymbol{\Sigma}_0$.

[1] We are implicitly assuming that the noise includes interference and modeled as a independent log-normally distributed.

3. SELECTION OF ACTIVE SENSORS

We assume that the event will appear at an unknown random position and the signal power emitted by the event decays isotropically and only a subset of sensors are affected due to it [10]. This observation allows us to distinguish between the so-called *active* (affected) and *inactive* (unaffected) sensors, respectively. For an improved detection performance intuition suggests that the detection rule should rely on the observations of the $L \leq K$ active sensors, thus discarding the observations from the rest of inactive sensors. Keeping this in mind we are in front of a detection problem where it is convenient to use a rank-reduced version of the signal model in (1). To do so, we need to select the observations of the L most relevant sensors. Herein, to simplify the mathematical exposition, we assume that the received signal vector $\mathbf{x}(n)$ is already ordered such that the first L samples of $\mathbf{x}(n)$ correspond to the L active sensors and the remaining L^c samples consist of only noise correspond to the inactive sensors [14]. We believe that in practice, this assumption can be easily relaxed. For example the signal samples (in dBm) in $\mathbf{x}(n)$ can be sorted by ordering the vector $\mathbf{x}(n)$ in descending order. Further details about such a process can be found in [9, 14]. Similarly, another way to relax the assumption is that the local sensors weight themselves locally based on their received powers and the fusion center sort the observations based the provided weights. Adding further, the sensors can even perform local selection by comparing their received powers with some predefined threshold and the qualified sensors declare themselves as active sensors. Keeping these facts into considerations the rank-reduced signal model can be written as:

$$\begin{aligned} &\mathcal{H}_0 : \mathbf{x}(n) = \mathbf{w}(n), \\ &\mathcal{H}_1 : \mathbf{x}(n) = \mathbf{s}_L(n) + \mathbf{w}(n), \end{aligned} \tag{2}$$

where $\mathbf{s}_L(n) = \mathbf{H}_L\mathbf{s}(n)$ is an ordered vector that has the first L non-zero signal elements corresponding to the received signals at the L active sensors and

$$\mathbf{H}_L = \left[\begin{array}{cc} \mathbf{I}_L & \mathbf{0}_{L\times(K-L)} \\ \mathbf{0}_{(K-L)\times L} & \mathbf{0}_{(K-L)\times(K-L)} \end{array}\right] \tag{3}$$

Hence, we have $\mathbf{s}_L(n) = \mathcal{N}\left(\mathbf{H}_L\boldsymbol{\mu}_s(n), \mathbf{H}_L\boldsymbol{\Sigma}_s\mathbf{H}_L^T\right)$. For simplicity and without loss of generality, we assume that under $\mathcal{H}_0$ the variables in $\mathbf{w}(n)$ are I.I.D, therefore $\sigma_{n_i}^2 = \sigma_{n_j}^2 = \sigma_0^2$ such that $\mathbf{x}(n) = \mathcal{N}\left(\mathbf{0}, \sigma_0^2\mathbf{I}\right)$. Then under $\mathcal{H}_1$ we have $\mathbf{x}(n) \sim \mathcal{N}\left(\mathbf{H}_L\boldsymbol{\mu}_s(n), \mathbf{H}_L\boldsymbol{\Sigma}_s\mathbf{H}_L + \sigma_0^2\mathbf{I}\right)$. The detection problem (2) says that due to the emergence of an event signal, changes occur both in the mean vector and covariance structure. Therefore, the quickest detection technique should be capable of simultaneously monitoring the mean as well as the covariance matrix. Furthermore, the proposed signal model also takes into account the rank-reduction of the signal covariance matrix by introducing $\mathbf{H}_L$ in (2). Therefore, in addition to the spatial information, the rank-reduced signal model (2) will indeed allow us to benefit from an equivalent signal to noise ratio gain due to selection of the active sensors.

4. PROPOSED QUICKEST DETECTION APPROACH

In this section we present the proposed quickest detection scheme. In order to proceed we define a time $n \triangleq N$ as the the discrete-time at which the change (occurs at time N_d) is detected. If $N > N_d$ then the detection delay is $\delta = N - N_d$. Similarly, if $N < N_d$, a false alarm has occurred with the average time between the false alarm is being $T_{FA} = \mathrm{E}_0\left[N\right]$, where E_0 denotes the average time taken before the change occurs (i.e. event emerges). Similarly, the worst case detection delay is defined as:

$$\mathrm{E}_1\left[N\right] = \underset{N_d \geq 1}{sup}\left(esssup\,\mathrm{E}\left[N - N_d \mid N \geq N_d, \{\mathbf{x}\left(n\right)\}_{n=1}^{N_d}\right]\right), \tag{4}$$

where $esssup$ denotes essential supremum. Under the Lorden's criterion, the objective is to find the stopping rule that minimizes the worst-case delay while maintaining the time between the false alarms larger than a certain threshold, $T_{FA} \geq \lambda$ [1]. An alternative approach is to minimize the average detection delay,

$$T_{\Delta} = \sup_{N_d \geq 1} \mathrm{E}_{N_d}\left[N - N_d \mid N \geq N_d\right], \tag{5}$$

which is asymptotically equivalent to the worst case delay [1]. Now, the algorithm that finds the minimum T_{Δ} in this problem is the Page's CUSUM Test (PCT) [1, 6, 3]. The stopping time of the PCT is determined as:

$$T\left(q\right) = \inf\left\{n : \mathcal{C}_n \geq q\right\}, \tag{6}$$

where q is a pre-determined threshold [2]. The cumulative statistic $\mathcal{C}_n$ can be recursively calculated for $n \geq 1$ as follow:

$$\mathcal{C}_n = \max\left(\mathcal{C}_{n-1}, 0\right) + \mathcal{L}_n\left(\mathbf{x}\right), \tag{7}$$

where $\mathcal{C}_0 = 0$, $\mathcal{L}_n\left(\mathbf{x}\right) \triangleq \log_e \frac{f_1(\mathbf{x}(n))}{f_0(\mathbf{x}(n))}$ and $f_h\left(\mathbf{x}\left(n\right)\right)$ with $h = 0, 1$ are likelihood functions under the alternate hypotheses. Observing (7) we can see that the CUSUM algorithm finds the first n for which $C_n \geq q > 0$.

In the proposed MCUSUM based on the signal model (2), not only N_d but also L is unknown, hence, it involves a nested likelihood ratio. Taking into effect these consideration, the Log-likelihood ratio statistic to be used in (7) is given as:

$$\mathcal{L}_n(\mathbf{x}) = \max_{1 \leq L \leq K} \sum_{l=1}^{L} \mathrm{Log}\left\{\mathcal{D}_l \frac{\exp\left(-\frac{1}{2}\tilde{\mathbf{x}}^T(n)\boldsymbol{\Phi}_l\tilde{\mathbf{x}}(n)\right)}{\exp\left(-\frac{1}{2}\mathbf{x}^T(n)\boldsymbol{\Sigma}_0^{-1}\mathbf{x}(n)\right)}\right\}, \tag{8}$$

where $\mathcal{D}_l = \sqrt{\det\boldsymbol{\Sigma}_0}/\sqrt{\det\boldsymbol{\Sigma}_{l,1}}$, $\boldsymbol{\Phi}_l = \boldsymbol{\Sigma}_{l,1}^{-1}$ with $\boldsymbol{\Sigma}_{l,1} = \mathbf{H}_l\boldsymbol{\Sigma}_s\mathbf{H}_l^T + \sigma_0^2\mathbf{I}$, and $\tilde{\mathbf{x}}(n) = \mathbf{x}(n) - \boldsymbol{\mu}_{l,s}(n)$ with $\boldsymbol{\mu}_{l,s}(n) = \mathbf{H}_l\boldsymbol{\mu}_s(n)$.

Now the recursive test (7) needs input in the form of $\mathcal{L}_n\left(\mathbf{x}\right)$ (8) at every time instant n, the estimation of the unknown covariance matrix $\boldsymbol{\Sigma}_{l,1}$ is not feasible. Therefore, we present a mechanism to find the unknown covariance matrix $\boldsymbol{\Sigma}_{l,1}$ by assuming locations of nodes are a-priori known [12]. In order to do so, first we assume the shadowing effects at different sensors have similar variances as $\sigma_{s_i}^2 = \sigma_{s_j}^2 = \sigma_s^2$, then we can write $\mathbf{H}_l\boldsymbol{\Sigma}_s\mathbf{H}_l = \sigma_s^2\mathbf{H}_l\boldsymbol{\Psi}\mathbf{H}_l^T$ with $\boldsymbol{\Psi}$ as the correlation matrix having elements $\rho_{i,j}$ for $i, j = 1, 2, ...K$. Similarly, the shadowing parameter σ_s^2 is an experimentally obtained parameter that is dependent on the propagation environment and it is assumed to be known [12, 15]. Morover, in (8) the matrix $\boldsymbol{\Phi}_l$ can be considered as a concentration matrix or a precision matrix. The elements of the precision matrix can be interpreted in terms of partial correlations and partial variances. Partial correlation measures the degree of association between two random variables. In our problem, we model this association with the help of correlation model in [15] by exploiting the fact that nodes locations are known. Hence, the correlation between i^{th} and j^{th} sensor can be modeled as $\rho_{i,j} = e^{-ad_{i,j}}$ [12, 15]. It is mentioned in [15], that for urban areas, at 1700 MHz $a \approx 0.12$ and for suburban areas at 900MHz $a \approx 0.002$. Moreover, in (8) we can write $\tilde{\mathbf{x}}^T(n)\boldsymbol{\Phi}_l\tilde{\mathbf{x}}(n) = \mathrm{Tr}[\boldsymbol{\Phi}_l\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^T(n)]$. Keeping all these into effect, we can deduce that the statistic (8) indeed matches the expected covariance matrix $\boldsymbol{\Sigma}_{l,1}$ with the received covariance matrix $\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^T(n)$ as an additional detection metric jointly with the signal energy. In other words, it takes into account both radio correlation and the reliability of information by selecting the active sensors.

Finally, the proposed MCUSUM statistic is achieved by putting $\mathcal{L}_n(\mathbf{x})$ from (8) in (7). The proposed quickest detection can be interpreted as a two dimensional MCUSUM as it operates with "max" operation both in the space and time. At every received observation vector it selects the active sensors and exploits the spatial structure of the received observations by matching the received covariance matrix with the modeled one, as explained in the previous paragraph.

5. PERFORMANCE ANALYSIS

The performance of the quickest detector is normally defined by T_{FA} and T_{Δ} [3, 11]. For a particular detector larger T_{FA} and smaller T_{Δ} means better detection performance. Consequently, to explain the advantages of sensor selection and exploitation of spatial structure, herein, we assume the case where the active sensors have been perfectly identified. Keeping this in mind, based on the well known result by Lorden [11], the expressions for the asymptotic T_{FA} and T_{Δ} are given as [3]:

$$T_{FA} = \mathrm{E}_0\left[N\right] \geq e^q \text{ as } q \to \infty \tag{9}$$

and

$$T_{\Delta} = \mathrm{E}_1\left[N\right] \sim \frac{q}{\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right)} \text{ as } q \to \infty, \tag{10}$$

where for a specific L, $f_{\mathcal{H}_1}\left(\mathbf{x}\right)$ and $f_{\mathcal{H}_0}\left(\mathbf{x}\right)$ are the likelihood functions of the signal model (2) under $\mathcal{H}_1$ and under $\mathcal{H}_0$, respectively. Similarly, $\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) = \mathrm{E}_1\left[\mathcal{L}_n\left(\mathbf{x}\right)\right]$ is the

Kullback-Leibler Divergence(KLD) between the two densities. Note that $\mathrm{E}_1\left[\mathcal{L}_n\left(\mathbf{x}\right)\right]$ is always positive and $\mathrm{E}_0\left[\mathcal{L}_n\left(\mathbf{x}\right)\right]$ is always negative [2]. The asymptotic results from Lorden's approach can be interpreted as for a constant T_{FA} or q, T_{Δ} is inversely proportional to the KLD. It means that the higher is the value of $\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right)$, the smaller will be T_{Δ}. The KLD between the two densities is defined as:

$$\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) = \int f_{\mathcal{H}_1}\left(\mathbf{x}\right)\log\frac{f_{\mathcal{H}_1}\left(\mathbf{x}\right)}{f_{\mathcal{H}_0}\left(\mathbf{x}\right)}d\mathbf{x}. \tag{11}$$

In order to analyze $\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right)$ of the proposed scheme, let us suppose the exact number of the active sensors is L', then the solution to the integral (11) is [16, Chapter 10]:

$$\begin{aligned}\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) &= \frac{1}{2}\log\frac{\det\left(\mathbf{\Sigma}_0\right)}{\det\left(\mathbf{\Sigma}_{L',1}\right)} + \frac{1}{2}\mathrm{Tr}\left(\mathbf{\Sigma}_{L',1}\mathbf{\Sigma}_0^{-1}\right)\\ &-\frac{1}{2}\mathrm{Tr}\left(\mathbf{\Sigma}_{L',1}\mathbf{\Sigma}_{L,1}^{-1}\right) + \frac{1}{2}\mathrm{Tr}\left(\mathbf{\Sigma}_0^{-1}\boldsymbol{\mu}_{L',s}(n)\boldsymbol{\mu}_{L',s}^T(n)\right),\end{aligned} \tag{12}$$

where $\mathbf{\Sigma}_{L',1} = \mathbf{H}_{L'}\mathbf{\Sigma}_s\mathbf{H}_{L'}^T + \sigma_0^2\mathbf{I}$ and $\mathbf{\Sigma}_{L,1} = \mathbf{H}_L\mathbf{\Sigma}_s\mathbf{H}_L^T + \sigma_0^2\mathbf{I}$. Similarly, based on the assumption that the number of active sensors L' is perfectly known, we have the following two cases.

Case 1. $L = L'$*: Number of active sensors are perfectly selected.*

$$\begin{aligned}\mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) &= \frac{1}{2}\log\frac{\det\left(\mathbf{\Sigma}_0\right)}{\det\left(\mathbf{\Sigma}_{L',1}\right)} - \frac{K}{2}\\ &+ \frac{L'}{2}\frac{\sigma_s^2}{\sigma_0^2} + \left(\frac{1}{2\sigma_0^2}\sum_{i=1}^{L'}\mu_{i,s}^2\right).\end{aligned} \tag{13}$$

Case 2. $L = K$*, Active sensors are not selected*

$$\begin{aligned}\mathcal{D}^{'}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) &= \frac{1}{2}\log\frac{\det\left(\mathbf{\Sigma}_0\right)}{\det\left(\mathbf{\Sigma}_{K,1}\right)} + \frac{L'}{2}\frac{\sigma_s^2}{\sigma_0^2} -\\ &\frac{1}{2}\mathrm{Tr}\left(\mathbf{\Sigma}_{L',1}\mathbf{\Sigma}_{K,1}^{-1}\right) + \left(\frac{1}{2\sigma_0^2}\sum_{i=1}^{L'}\mu_{i,s}^2\right),\end{aligned} \tag{14}$$

where $\mathbf{\Sigma}_{K,1} = \mathbf{\Sigma}_s + \sigma_0^2\mathbf{I}$.

Lemma 1. $\mathcal{D}^{'}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) \leq \mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right)$ *and equality prevails when* $L' = K$.

Proof. To compare (13) and (14) and analyze the impact of only sensor selection, we suppose $\mathbf{\Sigma}_s = \sigma_s^2\mathbf{I}$ in both of the equations. Therefore, $\mathrm{Tr}\left(\left(\mathbf{H}_{L'}\mathbf{\Sigma}_s\mathbf{H}_{L'}^T + \sigma_0^2\mathbf{I}\right)\mathbf{\Sigma}_{K,1}^{-1}\right) = L'\sigma_s^2/\left(\sigma_s^2+\sigma_0^2\right) + K\sigma_0^2/\left(\sigma_s^2+\sigma_0^2\right)$, $\log\left\{\det\left(\mathbf{\Sigma}_0\right)/\det\left(\mathbf{\Sigma}_{K,1}\right)\right\} = K\log\left(\frac{\sigma_0^2}{\sigma_s^2+\sigma_0^2}\right)$ and $\log\left\{\det\left(\mathbf{\Sigma}_0\right)/\det\left(\mathbf{H}_{L'}\mathbf{\Sigma}_s\mathbf{H}_{L'}^T + \sigma_0^2\mathbf{I}\right)\right\} = L'\log\left(\frac{\sigma_0^2}{\sigma_s^2+\sigma_0^2}\right)$. Putting these values in (13) and (14) we can easily deduce that $\mathcal{D}^{'}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right) \leq \mathcal{D}\left(f_{\mathcal{H}_1} \parallel f_{\mathcal{H}_0}\right)$ for the case $K \geq L'$. □

By using the results of this lemma and the Lorden's asymptotic approximation (10), it is clear that by using sensor selection jointly with a-prior spatial information indeed enhances the performance of the MCUSUM.

6. NUMERICAL RESULTS

For the analysis we use $K = 30$ sensors uniformly distributed in a square field. The event appears at a random location that emits power P0 = 3dB. To comply with the discussions in Section 3, we set the coverage of the event's signal such that an unknown subset of K sensors receive non-zero powers from the event. Consequently, we fix $\sigma_0^2 = 2$, $\sigma_s^2 = 1.5$. The behaviour of the adaptive multivariate CUSUM statistic $\mathcal{C}_n$ for the three schemes shown in Figure 1. These results show the change in the log-likelihood ratios for the three schemes before and after the emergence of the event. For all three cases the curves of MCUSUM statistics show sudden increase right after the change in the distributions occurs at $n = 50$. We can also see that for a certain value of q, the algorithm that uses jointly spatial information and sensor selection has the quickest response to the change created by the event's signal. In Figure 2, with the help of monte-carlo simulations we present the normalized histograms of the detection delays in responding to the change occurs. These histograms also shows that the proposed sequential detection scheme based on the spatial information performs better. The results in Figure 3 confirm the above numerical results where we plot the average detection delay versus the changing value of the threshold q. As asymptotically T_{FA} is exponential function of the threshold q, increasing q also means increase in T_{FA}. In Figure 3, we can see that increase in q (or T_{FA}) results in increase in the average detection delay times of the three schemes. Similarly, it can be seen that the increase in the response time of the proposed detection scheme is less than the scheme that assume no spatial structure. Moreover, using observations of only active sensors further increase the performance.

7. CONCLUSION

We have proposed the multivariate CUSUM based detector for the collaborative detection with the help of multi-sensor. The proposed detector exploits the spatial correlation structure and selects the important sensors in the detection process. The proposed scheme has been analyzed analytically based on the asymptotic mean detection delay. Simulation results have been drawn to verify these analytical expressions. The results show that comparatively the proposed scheme consistently performs better.

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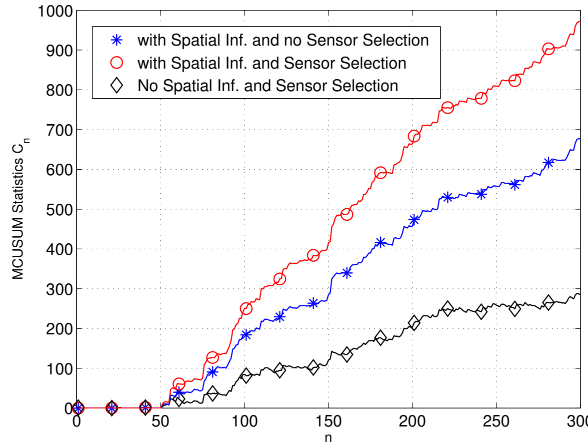

Fig. 1. Typical behaviour of the MCUSUM statistic $\mathcal{C}_n$

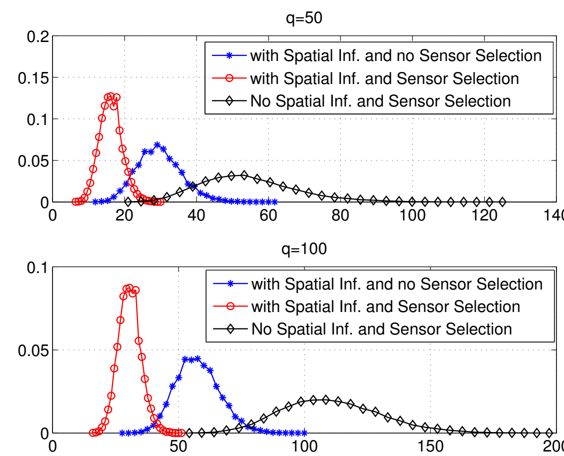

Fig. 2. Normalized histograms of 20000 points of the detection delay for threshold q = 50 and q=100

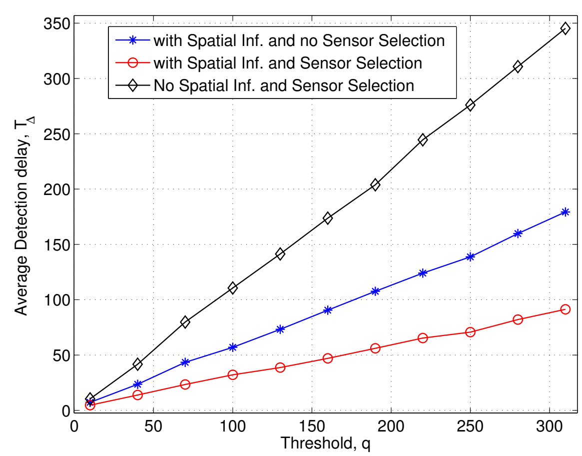

Fig. 3. Average Detection delay time T_Δ vs threshold q