COMPRESSIVE AND NON-COMPRESSIVE RELIABLE WIDEBAND SPECTRUM SENSING AT SUB-NYQUIST RATES

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ABSTRACT

A significant challenge of cognitive radio (CR) is to monitor wide ranges of the radio spectrum and unveil vacant spectrum subbands, whilst operating at affordable data acquisition rates. In this paper, we evaluate a number of wideband spectrum sensing techniques that alleviate the data acquisition rate bottleneck by operating at remarkably low sub-Nyquist rates; thus dubbed sub-Nyquist spectrum sensing. These methods were independently developed and are rarely (if at all) compared. Among other deductions, the non-compressive sub-Nyquist approach is shown to have numerous benefits and delivers competitive detection performance compared with techniques based on the compressive sensing methodology.

***Index Terms*—** Cognitive Radio, Compressive Sensing, Nonuniform Sampling, Spectral Analysis.

1. INTRODUCTION

With the proliferation of communications technology and the growing demand for wireless services, the RF spectrum is a natural resource that is becoming increasingly scarce. This is due to the current governmental allocation policies that stipulate a licensed Primary User (PU) has the exclusive right of using a certain spectrum band or range. Motivated by studies showing that the spectrum is often heavily underutilised at any given time-instant or geographic location (average occupancy as low as 10%), cognitive radio presents a paradigm shift aimed at alleviating the scarce radio spectrum limitation by opportunistically using vacant spectral bands (a.k.a. *spectral holes*) without interfering with PU(s) [1-7]. Therefore, a key functionality of the CR is spectrum sensing, which entails unveiling the unused spectral subbands prior to the opportunistic access.

One of the main challenges faced by CR is the ability to digitally achieve spectral awareness over wide frequency ranges, i.e. wideband spectrum sensing. This involves the simultaneously sensing of a large number of spectral bands harnessing more spectrum opportunities and delivering higher opportunistic network throughput [1-7]. Classical Digital Signal Processing (DSP) imposes sampling the incoming signal at rates exceeding the Nyquist rate, which is twice the total monitored bandwidth regardless of the spectrum occupancy (no *a priori* knowledge of the spectral activity is assumed). Otherwise, the aliasing phenomenon would render the detection of the active subbands or PU(s) unattainable. Such rates can be prohibitively high if the overseen bandwidth is considerably/ultra wide.

Wideband spectrum sensing has lately received notable attention leading to the emergence of novel approaches to drastically reduce the operational sampling rates to well below Nyquist [1, 2]. Such techniques relax the stringent data acquisition requirements and lead to more efficient uses of the sensing module resources such as data-storage and power. The majority of the sub-Nyquist wideband spectrum sensing methods are motivated by the Compressive Sensing (CS) methodology [4-8]. However, a sub-Nyquist paradigm referred to as Digital Alias-free Signal Processing (DASP) preceded CS [9]. It mitigates the spectral aliasing phenomenon by utilising random Nonuniform Sampling (NUS). Sensing techniques based on DASP were proposed in [10, 11] for various NUS schemes. Both DASP and CS are highlighted in Section 3. Whilst the majority of the sub-Nyquist spectrum sensing algorithms were independently developed, comparisons are rarely (if at all) conducted. Such a comparison is intellectually and technically needed to benchmark such methods and give potential users an insight into the pros and cons of the available techniques.

In this paper, we evaluate the performance of a number of state-of-the-art sub-Nyquist spectrum sensing techniques outlining their complexities and limitations. We show that the non-compressive approach has numerous benefits, such as simplicity and low computational load, and delivers competitive performance compared with the compressive counterpart. DASP-based techniques can operate at arbitrary low acquisition rates at the expense of predetermined longer "*sensing time*", whereas compressive methods have a lower bound on the permissible rates.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a wide frequency range $\mathcal{B}\sim[0,B]$ composed of L non-overlapping subbands (a.k.a. channels) at predefined locations as in [3-6, 7]; for simplicity all subbands have an equal width of B_C and $B=LB_C$. At any time-instant or geographic location, the maximum number of concurrently active channels and their joint bandwidth are denoted by L_A and $B_A=L_AB_C$, respectively. The low spectrum utilisation premise implies $B_A \ll B$. A CR monitoring $\mathcal{B}$ processes a single-sided bandwidth of width $B=LB_C$ and the received signal at the CR is given by: $x(t) = \sum_{k=1}^{K} x_k(t)$ where $x_k(t)$ is the transmission over the k-th subband and $K \leq L_A$. We can assume that all CRs are inactive during the sensing operation (enforced by the MAC layer) and the incoming signal arises only from PUs and noise.

The task of the wideband spectrum sensing module is to scan the overseen bandwidth $\mathcal{B}$ and identify the active channel(s)/PU(s); hence Multiband Spectrum Sensing (MSS). It involves deciding between two hypotheses: $\mathcal{H}_{1,k}$ (k-th channel is active) or $\mathcal{H}_{0,k}$ (k-th channel is inactive). Whilst classical DSP sensing techniques impose sampling $x(t)$ uniformly at/above the Nyquist rate $f_{US} \geq f_{Nyq} = 2B$ [2, 3], the sub-Nyquist methods accomplish the sensing task with notably low sampling rates $\alpha \ll f_{Nyq}$. For simplicity and to compare various MSS techniques in an identical set-up, we consider the scenarios where all the active subbands are of similar power levels and the signal propagates via an Additive White Gaussian Noise (AWGN) channel.

3. SUB-NYQUIST WIDEBAND SPECTRUM SENSING

MSS typically relies on estimating the spectrum of the received signal [1-7, 10, 11]. A common approach is to measure the spectral energy in each overseen channel via $\mathfrak{T}_k(\hat{X}(f)) = \sum_{f_m \in \mathcal{B}_k} \left|\hat{X}(f_m)\right|^2$ where $\hat{X}(f)$ is the estimated spectrum of the incoming signal and $\mathcal{B}_k$ is the frequency range of the k-th subband. The number of frequency points featured in $\mathfrak{T}_k(\hat{X}(f))$ is denoted by S; it depends on the spectrum resolution and it can be $S = 1$. MSS can be formulated as a classical binary hypothesis testing problem:

$$\begin{aligned} &\mathcal{H}_{0,k} : \mathfrak{T}_k\big(\hat{X}(f)\big) < \gamma_k \\ &\mathcal{H}_{1,k} : \mathfrak{T}_k\big(\hat{X}(f)\big) \geq \gamma_k \quad \text{for } k = 1,2,\dots,L \end{aligned} \tag{1}$$

where γ_k is a predefined threshold value. Figure 1 depicts a block diagram of common MSS techniques. It is noted that the Modulated Wideband Converter (MWC) approach in [6] is distinct from spectral domain energy detection methods. It utilises the Continuous to Finite (CTF) algorithm to recover the signal's spectral support, i.e. $\mathbf{d} \in \{0,1\}^L$ where "0" and "1" represent an active and inactive subband respectively.

The performance of a spectrum sensing approach is commonly measured by the Receiver Operating Characteristics (ROC) that captures the relationship between the probabilities of false alarm $\mathrm{P}_{\mathrm{FA,k}} = \Pr\{\mathcal{H}_{1,\mathrm{k}} \,|\mathcal{H}_{0,\mathrm{k}}\}$ and detection $\mathrm{P}_{\mathrm{D,k}} = \Pr\{\mathcal{H}_{1,\mathrm{k}} \,|\mathcal{H}_{1,\mathrm{k}}\}$ in the system subbands [1, 2]. Since the CR objective is to maximise the opportunistic use of vacant subbands ($P_{DSO,k} = 1 - \mathrm{P}_{\mathrm{FA,k}}$) and minimise interference (i.e. $\mathrm{P}_{\mathrm{M,k}} = 1 - \mathrm{P}_{\mathrm{D,k}}$), an adequate metric to assess the sensing quality is given by:

$$\left\{P_{D,k}^{*}, P_{DSO,k}^{*}\right\} = \underset{P_{D,k}(\mathrm{i}) \in \mathbf{P}_{D,k},\, P_{DSO,k}(\mathrm{i}) \in \mathbf{P}_{DSO,k}}{\arg\max} \left[P_{D,k}(\mathrm{i}) + P_{DSO,k}(\mathrm{i})\right] \tag{2}$$

where $\mathbf{P}_{D,k} = \left[P_{D,k}(1), P_{D,k}(2), \dots, P_{D,k}(p)\right]$ and $\mathbf{P}_{DSO,k} = \left[P_{DSO,k}(1), P_{DSO,k}(2), \dots, P_{DSO,k}(p)\right]$ are the ROC probabilities vectors for all threshold values. The outcome of (2) is known as the ROC deflection point.

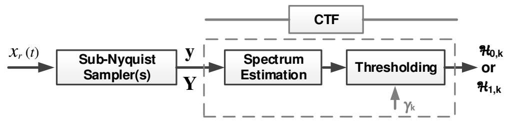

Figure 1. Block diagram of the addressed sub-Nyquist MSS methods.

It is important to note that the problem tackled here is spectrum sensing and not spectrum estimation. Thus, any frequency representation, not necessarily the signal's exact spectrum or Power Spectral Density (PSD), which facilitates unveiling the active subbands in $\mathcal{B}$ suffices. Below, we outline and compare a number of widely used sub-Nyquist multiband spectrum sensing methods.

3.1 DASP-based Sub-Nyquist Spectrum Sensing

Whilst uniform sampling DSP dictates $t_m = mT_{US}$ for $m = 1,2,..,N$ and $T_{US} \leq 1/f_{Nyq}$, DASP utilises NUS to collect data samples $x(t_m)$ at non-equidistant time instants $\{t_m\}_{m=1}^{M}$ generated randomly [9]. This is aimed at suppressing/eliminating the aliasing effect(s). Several NUS schemes exist, e.g. Random Sampling on Grid (RSG) where M out of N Nyquist samples are randomly selected with equal probability.

DASP-based MSS uses the following periodogram-type estimator with a small set of the signal nonuniform samples:

$$\hat{X}_{NUS}(f) = \sum_{j=1}^{J} \beta \left|\sum_{m=1}^{M} y(t_m) w_j(t_m) e^{-i2\pi f t_m}\right|^2 \tag{3}$$

where β is a scalar dependent on the sampling scheme and $\{y(t_m) = x(t_m) + n(t_m)\}_{m=1}^{M}$ are the signal samples contaminated with AWGN. They are collected within a time analysis window $\mathcal{T}_j = \left[t_j, t_j + T_0\right]$ where t_j is its initial time instant. The average sampling rate of (3) is: $\alpha = M/T_0$. The windowing function $w_j(t)$ is deployed to minimise spectral leakage where $w_j(t) = 0$ for $t \notin \mathcal{T}_j$. Recalling that spectrum sensing does not require determining the signal detailed spectral shape, it was shown in [10, 11] that (3) yields a frequency representation that facilitates MSS regardless of the value of α. The statistical characteristics of (3) illustrate that $\hat{X}_{NUS}(f)$ delivers reliable spectrum sensing routine for appropriately selected α and sensing time T_{ST}; the latter is the signal time observation window needed to make a decision on the channel activity $T_{ST} = cT_0$, $c > 1$.

Returning to (1), the non-compressive DASP-based sensing approach in [10, 11] inspects one frequency point per subband to establish its status, i.e. $S = 1$. This can be reinforced by maintaining relatively smooth spectrographs using a short time window. To ensure satisfying certain probabilities of detection and false alarm, prescriptive guidelines were derived for the DASP-based MSS approach and closed form formulas were presented showing that the sensing time is a function of: the sampling rate, Signal to Noise Ratio (SNR), maximum spectrum occupancy B_A and requested P_D and P_{FA}: $T_{ST} = \mathcal{F}(\alpha, \mathrm{P}_D, \mathrm{P}_{\mathrm{FA}}, B_A, SNR)$. Such guidelines clearly depict the trade-off between sensing time, sampling rate and achievable detection performance. Most importantly, these guidelines clearly illustrate that the sampling rate can be arbitrarily low at a predetermined additional sensing time and vice versa. Hence there is no lower bound on the sampling rate. Different NUS schemes have different properties and guidelines (see [9-11]).

3.2 Compressive Sub-Nyquist Spectrum Sensing

Given the low spectrum utilisation in CR networks, the incoming wideband signal $x(t)$ is inherently sparse in the frequency domain, e.g. by using Discrete Fourier Transform (DFT) basis/frame. Sparse implies that the signal can be represented by only few of its coefficients in the appropriate sparsifying basis/frame. Compressive sensing enables the processing of $x(t)$, e.g. to perform spectrum sensing, from few of its samples collected at remarkably low sub-Nyquist rates [8]. Due to the current immense interest in CS, new compressive MSS methods are regularly emerging. Given the space constraints, this paper by no means considers all the existing techniques; however a number of widely cited compressive approaches are addressed here.

3.2.1 CS-based Spectrum Estimation

In CS, the cognitive radio collects M sub-Nyquist samples using CS according to: $\mathbf{y} = \mathbf{\Phi x}$ where $\mathbf{y} \in \mathbb{C}^M$ is the samples vector, $\mathbf{x} \in \mathbb{C}^N$ is the discrete-time representation of $x(t)$ captured at/above the Nyquist rate and $\mathbf{\Phi} \in \mathbb{C}^{M\times N}$ is the measurement matrix such that $M < N$. For the DFT transform basis $\mathbf{\Psi} \in \mathbb{C}^{N\times N}$, the signal $\mathbf{x} = \mathbf{\Psi X}$ and $\|\mathbf{X}\|_0 \leq K_S$ where $K_S \ll N$. We have:

$$\mathbf{y} = \mathbf{\Upsilon X} \tag{4}$$

and $\mathbf{\Upsilon} = \mathbf{\Phi\Psi}$. CS allows the exact recovery of $\mathbf{X}$ from the $M < N$ measurements, e.g. $M = \mathcal{O}(K_S \log(N/K_S))$ suffices [8], leading to substantial reductions in the data acquisition requirements. Reconstructing $\mathbf{X}$ from (4) entails solving an optimisation problem with plethora of existing sparse recovery techniques such as convex-relaxation or greedy methods. They typically involve finding the pseudo-inverse of $\mathbf{\Upsilon}$ [8]. The CS average sampling rate is $\alpha = M/T_{ST}$ where T_{ST} is the signal time observation window. In practice, the compressed samples $\mathbf{y}$ can be obtained without the need to capture the Nyquist samples, refer to [8] for further details.

Here we pursue two MSS approaches that are based on spectrum estimation and use the DFT basis:

1) *CS Method 1* (*a.k.a. CS-1*): This technique is a natural direct application of CS to the multiband spectrum sensing problem [1, 5]. The coefficients $\mathbf{X} \in \mathbb{C}^N$ are obtained using a CS sparse recovery algorithm where $N = T_{ST} f_{Nyq}$ is the number of Nyquist samples. The estimated spectral points that belong to each channel are grouped to calculate $\mathfrak{T}_k\left(\widehat{X}(f)\right)$ in (1) . The resultant values are compared with a threshold value to reach a decision on the channel status.

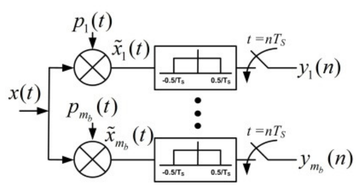

Figure 2. MWC sampler block diagram.

2) *CS Method 2* (*a.k.a. CS-2*): Unlike the previous, L DFT points are calculated in this approach ($\mathbf{X} \in \mathbb{C}^L$ instead), i.e. one DFT point per monitored subband. The time analysis window is $T_0 = L/f_{Nyq}$, and $\mathfrak{T}_k\left(\widehat{X}(f)\right) = |\widehat{X}(f_m)|^2$ such that f_m belongs to the k-th channel and is reasonably placed at its center [4, 7]. To improve performance, J CS spectral estimates are averaged; this emulates the scenario of J distributed CRs collaboratively overseeing $\mathcal{B}$.

3.2.2 Modulated Wideband Converter

The MWC data acquisition system which facilities sampling wideband signals at sub-Nyquist rates was introduced in [12]. The MWC is referred to by CS-MWC in the sequel. It is comprised of m_b bank of modulators and low-pass filters as exhibited in Figure 2. In the i-th branch, $i = 1,2,..,m_b$, the input signal $x(t)$ is multiplied by a periodic waveform $p_i(t)$ of period T_p. The modulated output is low-pass filtered with cut-off frequency $0.5/T_S$ and subsequently sampled at a sub-Nyquist sampling rate equal to $1/T_S$. The MWC configuration imposes: $1/T_p \geq B_C$, typically $T_p = T_S$ and $m_b \geq 4L_A$ which leads to:

$$\alpha_{MWC} \geq 4L_A B_C \tag{5}$$

assuming maximum spectrum occupancy. Equation (5) gives the lower bound on the operational sub-Nyquist sampling rates of MWC; $L_A \ll L$ implies $\alpha_{MWC} \ll f_{Nyq}$.

Mathematically, the analogue mixture boils down to an underdetermined linear system of equations which ties the known discrete-time Fourier transform to the unknown continuous-time spectrum. Algorithms from the CS field are then employed to identify the nonzero/zero entries in the underdetermined system, which reveals the spectrum support of the signal at resolution B_C and establishes the activity of the monitored subbands. Since only MSS is sought here, there is no need to fully reconstruct the signal and recovering $x(t)$ spectral support using the CTF block suffices as noted in [6]. We mention here the related Random Demodulator (RD) system devised for multitone signals [8] is not suitable for sensing multiband signals.

3.3 Compressive Versus Non-Compressive MSS

Below, we succinctly compare the considered sub-Nyquist multiband spectrum sensing approaches prior to evaluating their detection performances in Section 4.

3.3.1 Minimum Sub-Nyquist Rates

Whilst the sampling rates of DASP-based MSS can be arbitrarily low at a predetermined cost in terms of longer sensing time, MWC minimum sampling rate in (5) is $4B_A$. This renders MWC unsuitable for very low sampling rate or high spectrum occupancy. Other CS-based methods rely on the premise that the spectrum of the signal is sparse where generally $M \geq 2K_S$ and K_S is the number of non-negligible DFT coefficients, i.e. minimum α is proportional to $2B_A$.

3.3.2 Design Parameters

DASP-based spectrum sensing provides clear guidelines on the required sampling rate and sensing time to deliver particular sensing probabilities specified by the user. In practice, this will circumvent the need to set up and run extensive simulations to produces ROC plots, which can take considerably long time. There are no counter results in CS and only theoretical guarantees on the exact spectrum reconstruction are derived. Typically, they are asymptotic and proportional to the present noise energy; they do not give clear indications of the obtainable detection results. Thus the non-compressive approach has substantially lower design workload. Its guidelines for the trade-off between sampling rate, sensing time and detection probabilities allow a system designer to make a quick and informed decision on the required sampling and time resources.

3.3.3 Computational Complexity

The main feature of the DASP-based MSS is simplicity and low computational complexity, where DFT-type or optimised FFT-type operations are undertaken. Whereas, the CS-based techniques involve solving underdetermined sets of linear equations. This is usually computationally expensive even for new emerging sparse reconstruction algorithms. It is noted that MWC adopts a more practical approach to spectrum sensing compared with other CS methods. It utilises the CTF algorithm where the processed matrices are of size $m_b \times L$ in lieu of $M \times N$ as in *CS Method 1*; $N \gg L \geq m_b$ is the number of Nyquist samples in the observation window and it is typically very large.

3.3.4 Implementation Complexity

CS and DASP face similar implementation challenges where pseudorandom sampling sequences is commonly used as a compression strategy. Novel ADC architectures that support random sampling are emerging, e.g. [9, 14]. Certain CS approaches that do not utilise the aforementioned pseudorandom sampling were implemented and prototype systems were produced, e.g. MWC and RD. They however require complex specialised analogue pre-conditioning modules prior to the low rate sampling (see Fig. 2). Their subsequent processing is also sensitive to sampler mismodelling. Thus such solutions are inflexible and are high Size, Weight, Power and Cost (SWPAC). Designing flexible low SWPAC sub-Nyquist samplers remains an open research question.

4. EMPIRICAL PERFORMANCE EVALUATION

Consider a CR monitoring $L = 160$ subbands, each of width $B_C = 7.5$ MHz, and hence $\mathscr{B} \sim [0, 1.2]$GHz. The Nyquist sampling rate in this case is $f_{Nyq} = 2.4$ GHz. The maximum number of concurrently active subbands at any point in time is $L_A = 8$. Here, extensive set of numerical experiments for L_A active channels are presented to assess the performance of the considered sub-Nyquist techniques in terms of the delivered probabilities of detection and false alarm. QPSK or 16QAM transmissions with randomly selected carrier frequencies are used. The displayed P_D and P_{FA} are selected according to (2). The objective here is to gain an insight into the behavior of the MSS approaches for the available design resources and operation parameters, namely compression ratio α/f_{Nyq} (α is the sub-Nyquist sampling rate), SNR and sensing time T_{ST}. In all the presented plots, a large number of independent experiments are averaged to obtain the displayed probabilities, $T_0 = 0.2\,\mu s$, $T_{ST} = JT_0$, $\mathbf{\Upsilon}$ is a random partial Fourier matrix, subspaces pursuit is used for CS-1/CS-2 and RSG is the NUS scheme.

Motivated by the goal of furnishing savings on the data acquisition rate, Figure 3.a depicts P_D and P_{FA} for sub-Nyquist rates that achieve over 85% reductions on f_{Nyq}; $SNR = 0$ dB and $T_{ST} = 15\mu s$. It is noted that MWC condition in (5) is satisfied only for $\alpha/f_{Nyq} \geq 0.1$. It can be noticed from Figure 3.a that DASP-based MSS outperforms the compressive techniques, more noticeably the state-of-the-art MWC, for low sampling rates. In Figure 3.b, $\alpha/f_{Nyq} \in [0.15, 0.5]$ values are tested to examine the sub-Nyquist MSS methods response to higher sampling rates. Due to the prohibitively high memory and computations requirements of the enormous sensing matrices associated with *CS Method 1* for high α and/or T_{ST} , the sensing time is set to $T_{ST} = 3\mu s$. In Figure 3.b the compressive approaches deliver better detection performance compared with the DASP-based techniques only for higher sampling rates, e.g. for $\alpha/f_{Nyq} \geq 0.2$ in Figure 3.b. Simulations also illustrate that MWC delivers a competitive performance only when the operation sampling rate significantly exceeds the lower theoretical bound in (5). Therefore, Figure 3 evidently shows that the low-complexity non-compressive DASP-based approach has a superior sensing performance compared with other compressive CS-based techniques when substantial savings on the data acquisition rates are sought. This advantage becomes less visible as the sampling rate increases at the expense of significant surge in the incurred computational cost, especially for *CS Method 1*.

In Figure 4, the obtained probabilities of detection and false alarm are displayed for a changing sensing time whilst $\alpha/f_{Nyq} = 0.15$ and $SNR = 0$ dB. CS-1 was omitted because of its prohibitive computational complexity. It is apparent from the figure that DASP-based and CS-2 exploit the increase in the available sensing time to enhance their sensing capabilities. Whereas, MWC sensing probabilities remain nearly constant as the sensing time increases. This figure demonstrates that non-compressive and CS-2 approaches can effectively utilise or trade-off the sensing time for improved spectrum sensing performance and vice versa, unlike the original MWC.

The addressed sub-Nyquist MSS algorithms are tested for a varying SNR in Figure 5; $\alpha/f_{Nyq} = 0.15$ and $T_{ST} = 15\,\mu s$. This figure exhibits the DASP-based technique predominantly outperforming MWC and *CS Method 2* as the SNR increases. The MWC noticeably continues to perform poorly compared with other methods despite the increasing signal power; thus the sampling rate is the MWC limiting factor. We recall that the used sampling rate is 1.5 times the MWC theoretical minimum rate in (5). We note that the

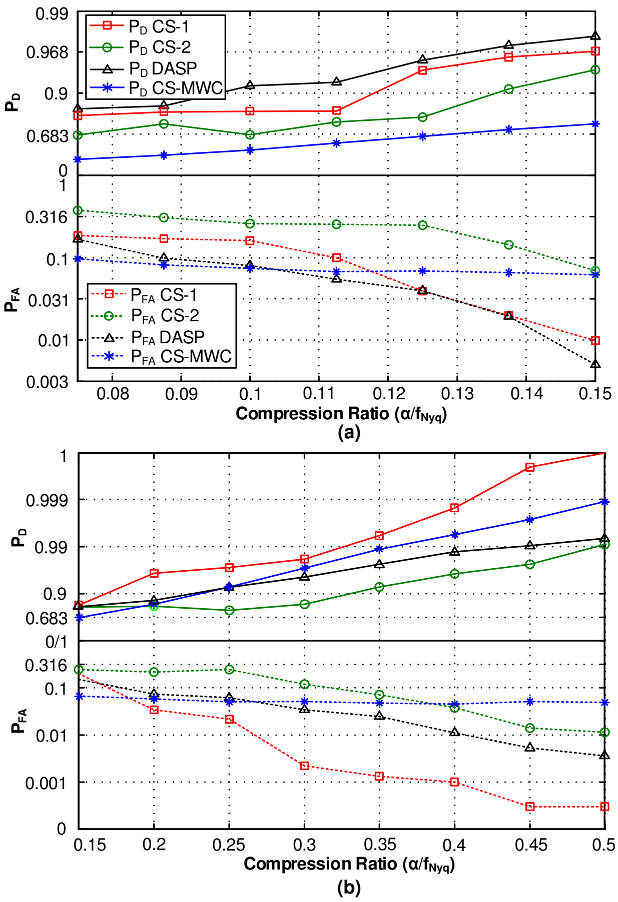

Figure 3. Sub-Nyquist MSS P_D and P_{FA} for various compression ratios; $SNR = 0$ dB. (a) $T_{ST} = 15\mu s$ and (b) $T_{ST} = 3\mu s$.

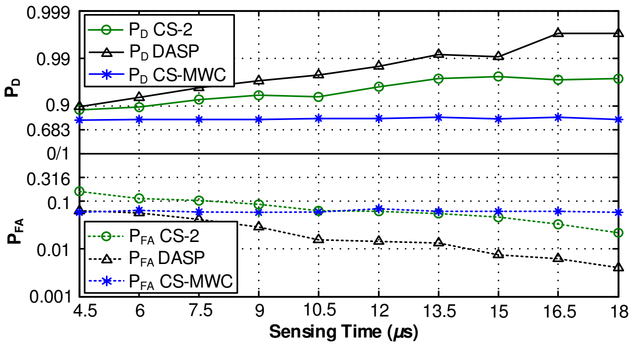

Figure 4. Sub-Nyquist MSS P_D and P_{FA} for various sensing time values; $\alpha/f_{Nyq} = 0.15$ and $SNR = 0$ dB.

ability of MWC to achieve low P_{FA} for low α/f_{Nyq}, SNR and T_{ST} in Figures 3, 4 and 5 is a by-product of the considerably low attained P_D in such conditions.

5. CONCLUSION

Whilst compressive spectrum sensing techniques benefit from the powerful mathematical tools originally developed

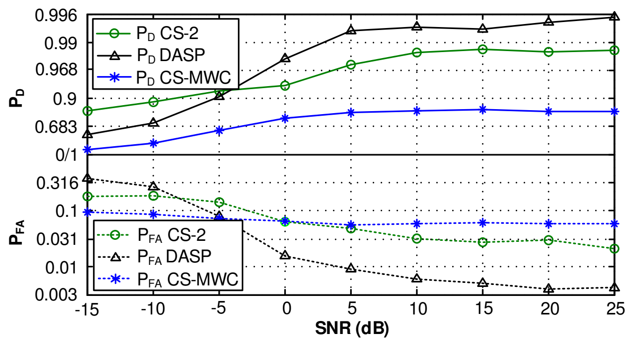

Figure 5. Sub-Nyquist MSS P_D and P_{FA} for various SNR values; $\alpha/f_{Nyq} = 0.15$ and $T_{ST} = 15\ \mu s$.

for exact signal reconstruction in the booming CS methodology, the non-compressive DASP-based approach offers simple effective solution to the tackled problem, i.e. wideband spectrum sensing, with low computational complexity, clear recommendations on how to utilise the provided means and most importantly superior performance for remarkably low sampling rates. This paper calls for a more comprehensive review of the available sub-Nyquist methods with detailed comparison(s) of their behaviors in terms of sensing probabilities.

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