BLIND FREE BAND DETECTOR BASED ON THE SPARSITY OF THE CYCLIC AUTOCORRELATION FUNCTION

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ABSTRACT

In this paper, we will firstly show that the Cyclic Autocorrelation function (CAF) is a sparse function in the cyclic frequency domain . Then using this property we propose a new CAF estimator, using Compressed Sensing (CS) technique with OMP algorithm [1]. This estimator outperforms the classic estimator used in [2]. Furthermore, since our estimator does not need any information, we claim that it is a blind estimator whereas the estimator used in [2] is clearly not blind because it needs the knowledge of the cyclic frequency. Using this new CAF estimator we proposed in the second part of this paper a new blind free bands detector. It assumes that two estimated CAF of two successive packets of samples, should have close cyclic frequencies, if a telecommunication signal is present. This new detector is a soft version of the detector already presented in [3]. This methods outperforms the cyclostationnarity detector of Dantawate Giannakis of [2].

***Index Terms*—** Cognitive Radio, Dynamic Spectrum Access (DSA), Spectrum sensing, Compressed sensing, Sparsity.

1. INTRODUCTION

To answer to the scare spectrum resource problem Cognitive Radio has been proposed as an efficient technique by accessing networks in an independent and dynamic way. In fact, most parts of the spectrum are allocated to licensed radio services (referred to as Primary Users (PUs)) that have exclusive access rights. However, Secondary Users (SUs) can still access opportunistically to the spectrum held by the PUs using spectrum sensing. There are many spectrum sensing techniques that enhance the SUs detection opportunities in to licensed bands, allowing the access to unused portions of the licensed spectrum. Classifying all the existing techniques is always difficult, because any sharing could be discussed. It is classical to have a first coarse sharing between collaborative and non-collaborative techniques. In these two classes there exist a lot of methods, it is not the objective of this paper to classify these techniques and to give their pros and cons. A lot of very good review papers already did it. The reader may refer to [4, 5, 6, 7]. The work of this paper belongs to the non-collaborative class. Furthermore we are interested in narrow band detection. Our objective is to find a robust technique (in terms of detection probability), fully blind, with short sensing time (small number of samples needed).

The most well known techniques for our problem are energy detection (ED) [8, 9], matched filtering (MF) detection [9], and cyclostationary detection (CSD) [2, 11], and all techniques which mixed them as in [12, 13]. Cyclostationary detection, which relies on the Higher-Order-Statistics (HOS), requires high computation time and sufficient signal information. Indeed, it is not robust when the sample size is small [14]. As for the MF, it is an optimal detection method but needs an exact knowledge of the transmitted signal and requires synchronization which is usually difficult to achieve. However in practice, SUs can't know anything about the PUs signal structure. On the other hand, ED is the simplest detection method but needs a perfect knowledge of the noise level. In fact, a small error on the estimation of the noise level is known to seriously impact the detection performance [8]. In this paper we are specifically interested in cyclostationnary techniques, we propose a new fully blind detection technique based on the sparsity of the Cyclic Autocorrelation Function (CAF).

The remaining part of the paper is organized as follows. In section 2, we present the system model adopted throughout this work. We introduce the background of the CAF, then we show its sparsity property in section 3. In section 4, we define the Cyclic Autcorrelation Vector (CAV), and we describe how it could be estimated using compressed sensing technique. Then the new blind detection algorithm using this estimation will be presented in section 5. Section 6 presents simulation results and discussions. Finally, Section 7 presents the conclusions of this study and makes some suggestions for future work.

2. SYSTEM MODEL

The spectrum sensing detection problem consists of collecting a set of N samples $y(0), y(1), ..., y(N-1)$ from a given frequency band B. Denote **y** by the vector formed by N samples, $\mathbf{y} = [y(0), .., y(N-1)]^t$. H_1 and H_0 denote the binary hypotheses that a primary user is present and absent, respectively.

The binary hypotheses (H_0, H_1) are defined in a way such that, under hypothesis H_1 and $k \in [0, .., N-1]$, the k^{th} collected sample, $y(k)$, is composed of a primary user signal sample $x(k)$, plus an additive Gaussian noise sample $n(k) \sim \mathcal{N}(0, \sigma_n^2)$ where $\mathcal{N}(m, \sigma^2)$ denotes the normal distribution with mean m and variance σ^2. Under hypothesis H_0, the k^{th} sample, $y(k)$, consists of the additive Gaussian noise sample $n(k)$. Hence, we can write:

$$\begin{cases} H_0: \; y(k) = n(k) \\ H_1: \; y(k) = x(k) + n(k) \end{cases}$$

Hence, the performance of any spectrum sensing method is indicated by two probabilities: the detection probability, P_d $(P(H1/H1))$, and the probability of false alarm, P_{fa} $(P(H1/H0))$.

3. CYCLIC AUTOCORRELATION FUNCTION AND SPARSITY

Since the pioneering work of Gardner in the 75's [16], it is well known that every telecomunications signal is a cyclostationnarity signal and its Cyclic Autocorrelation Function (CAF) has cyclic frequencies. This cyclostationnarity property comes from, for example, the symbol frequency, the Guard Interval for OFDM modulation... It is shown in [10] that the expression of the CAF, $R(\alpha, \tau)$ of a linearly modulated signal, with T_s the symbol period, and τ a given delay, is null except when α takes an integer multiple of $\frac{1}{T_s}$:

$$R_{yy}(\alpha, \tau) = \begin{cases} \frac{\sigma_d^2}{T_s} e^{-j2\pi\alpha\epsilon} \int_{-\infty}^{\infty} g(t - \frac{\tau}{2}) g^*(t + \frac{\tau}{2}) e^{-j2\pi\alpha t} dt, \\ 0 \text{ for } \alpha \neq \frac{k}{T_s}, \; k \in Z \end{cases} \tag{1}$$

with ϵ an unknown delay, σ_d^2 the power of a symbol at the emission side, $g(t)$ the temporal impulse response of the transmission filter, and $g^*(t)$ is the complex conjugate of $g(t)$. It is clear from this expression that the CAF is a sparse function in the cyclicfrequency domain.

We define the Cyclic Autocorrelation Vector (CAV) of the signal $y(t)$ as a particular vector issued from the CAF for a fixed delay $\tau = \tau_0$, and over a given cyclic frequency domain $[\alpha_{min}, \alpha_{max}]$. The CAV is then given by:

$$\begin{aligned} \mathbf{r}_{yy}^{(\tau_0)}(\alpha) = & [R_{yy}(\alpha_{min}, \tau_0), R_{yy}(\alpha_{min} + \delta_\alpha, \tau_0), \\ & \cdots, R_{yy}(\alpha_{max}, \tau_0)]^t \end{aligned} \tag{2}$$

with δ_α is the frequency resolution step.

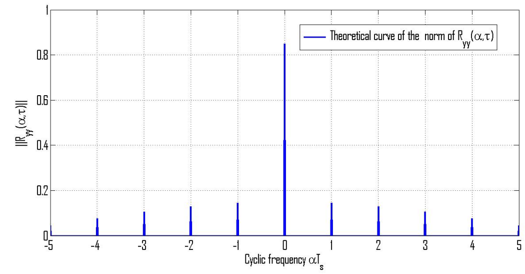

Fig. 1. CAV theoretical curve for a BPSK: the sparse property appears clearly

This function is represented in Figure1 in which the sparse property appears clearly.

4. CYCLIC AUTOCORRELATION VECTOR ESTIMATION

4.1. Classical Cyclic Autocorrelation Vector estimation

A classical estimation of the CAF, at a given point (α, τ), of a signal $y(t)$, $R_{yy}(\alpha, \tau)$ can be estimated using the unbiased estimator, given by equation 3 and used in [2]:

$$\hat{R}_{yy}^{\mathbf{cl}}(\alpha, \tau) \cong \frac{1}{N} \sum_{k=0}^{N-1} y(kT_e) y(kT_e + \tau) e^{-j2\pi\alpha k T_e} \tag{3}$$

Where $\frac{1}{T_e}$ is the sampling frequency.
The classical estimated CAV, using equation 3 is then given by expression (4):

$$\begin{aligned} C\hat{A}V_{classic} = \hat{\mathbf{r}}_{yy}^{(\tau_0)}(\alpha) = & [\hat{R}_{yy}^{\mathbf{cl}}(-\alpha_{max}, \tau_0), \\ & \hat{R}_{yy}^{\mathbf{cl}}(-\alpha_{max} + \delta_\alpha, \tau_0), \cdots, \hat{R}_{yy}^{\mathbf{cl}}(\alpha_{max}, \tau_0)]^T \end{aligned} \tag{4}$$

Where $\delta_\alpha = \frac{2 \cdot \alpha_{max}}{\widetilde{N}}$ is the frequency resolution step and $\widetilde{N}$ the CAV dimension.

We may notice that $\mathbf{r}_{yy}^{(\tau_0)}(\alpha)$ could also be estimated using the FFT operator on the product $y(kT_e) \cdot y(kT_e + \tau)$. In fact, we define

$$f_\tau(kT_e) = y(kT_e) y(kT_e + \tau) \tag{5}$$

Let be $\mathbf{f}_\tau$ the following vector:

$$\mathbf{f}_\tau = [f_\tau(0), f_\tau(1 \cdot T_e), \cdots, f_\tau(\widetilde{N} \cdot T_e - 1)]^T \tag{6}$$

Therefore vector $\hat{\mathbf{r}}_{yy}^{(\tau_0)}(\alpha)$ which represents the CAV estimation of the signal $y(t)$,in the cyclic domain $[-\alpha_{max}, \alpha_{max}]$ is simply the DFT of the vector $\mathbf{f}_{\tau_0}$ multiplied by $\frac{1}{\widetilde{N}}$:

$$\hat{\mathbf{r}}_{yy}^{(\tau_0)}(\alpha) = \frac{1}{\widetilde{N}} DFT(\mathbf{f}_{\tau_0}) \tag{7}$$

This observation will be used in the next section

4.1.1. Compressed Sensing Cyclic Autocorrelation Vector estimation

In recent years sparse approximation approach has been detailed through many fields in signal processing [18, 19, 20, 21], whose theoretical aspects focus more specifically on the so-called *compressed sensing* [15, 17]. In this section, we will describe the way to estimate the CAV by Compressed Sensing technique.

Sparse approximation consists of finding a signal or a vector with sparseness property, that is having a small number of nonzero elements, that satisfies (approximately) a system of equations. For example, consider a linear system of equations $\mathbf{y} = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is an n−by−$\widetilde{N}$ matrix with $n < \widetilde{N}$. Since $\mathbf{A}$ is over complete ($n < \widetilde{N}$), therefore this problem does not have a unique solution. Among all the possible solutions, if the true one is known *a priori* to be sparse then it happens that the sparsest, i.e. the solution $\mathbf{x}$ containing as many as possible zero components and satisfying $\mathbf{y} \cong \mathbf{A}\mathbf{x}$, is close to the true solution.

Based on the sparse property of the CAV in the cyclic frequencies domain, we proposed in [1] a method that uses compressed sensing technique (sparse reconstruction method) in order to estimate the CAV. We showed that there are many advantages in using such a method, such as obtaining a smaller Mean Square Error between the theoretical and the estimated curve when applying sparse reconstruction technique, rather than using the classical estimator of the CAV based on (3). Also with compressed sensing we showed that its possible to make good estimation of the cyclic frequency with few samples (short observation time), without the need of an *a priori* information over the cyclic frequency contrary to the classical method that requires the knowledge of the value $\alpha = \frac{1}{T_s}$ in order to estimate the CAF at $\alpha = \frac{1}{T_s}$.

We reformulate the equation 7 in order to be able to apply Compressed Sensing.

Let be $\mathbf{b}^{(\tau_0)}$ the vector build with n first elements of $\mathbf{f}_{\tau_0}$ Let be $\mathbf{A}$ the matrice build with $\widetilde{N}$ frequencies with a step δ_α. This matrice is also a submatrice of the Fourier matrice (n first lines of $\mathbf{F}^*$). $\mathbf{F}^*$ being the complex conjugate of the square Fourier matrice of dimension $\widetilde{N}$.

The problem becomes therefore the following:

$$\mathbf{A}\mathbf{r}^{(\tau_0)} = \mathbf{b}^{(\tau_0)} \tag{8}$$

To solve equation 8 we proposed in [1], for complexity reasons, to use the *Orthogonal Matching Pursuit* (OMP) algorithm [22].

Figure 2 show the theoretical CAV curve as well as the estimated CAV obtained by the classical estimator and by the CS estimator. Both estimators recovers correctly the cyclic frequencies. However, having a look on Figure 3, which is a zoom on the previous Figure, shows clearly that the CS estimator has no estimation noise whereas the classical estimator has.

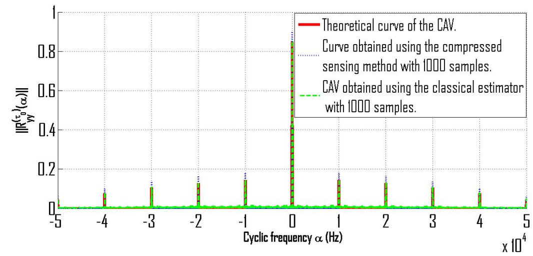

Fig. 2. CAV estimation thanks to Compressed Sensing

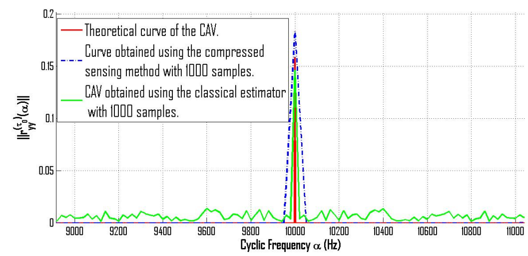

Fig. 3. Zoom on Figure 2, which highlights the estimation noise of the classical estimator

5. THE NEW BLIND DETECTOR

We propose, in this section, an improved version of the detector initially presented in [3]. The main idea behind this detector is: two consecutive set of small number of samples should have exactly the same cyclic frequencies if the hypothesis presence of signal is true. It is called in the following: Two Sets Compressed Sensing (TSCS) detector. A similarity criteria on the cyclic frequencies position is performed on the consecutive sets, in fact we compute the indexes difference δ_i between cyclic frequencies of the two sets s_1 and s_2. This is performed for several delays τ (over M values of τ), then an average on these similarities is computed (equation 9). If it is below a predefined threshold k, then the hypothesis H_1 is decided. The ROC curves are obtained by varying the index k from 1 to $\widetilde{N}$. It is evident that smaller the value of k is, smaller is P_{fa} and inversly. In other terms for each value of k we obtain a set (P_d, P_{fa}), that belongs to the ROC curve for a given SNR.

$$\bar{\delta} = \frac{1}{M}\sum_{i=1}^{M}\delta_i \tag{9}$$

The algorithm of the TSCS detector is given below:

$\mathbf{s}_1 \leftarrow [y_1(0), ..., y_1(n_s - 1)]^T$
$\mathbf{s}_2 \leftarrow [y_2(0), ..., y_2(n_s - 1)]^T$
for $i = 1$ to M **do**
  $\mathbf{b}_1^{(\tau_i)} \leftarrow n_s$ elements of $\mathbf{f}_{\tau_i}^{(1)}$
  $\mathbf{b}_2^{(\tau_i)} \leftarrow n_s$ elements of $\mathbf{f}_{\tau_i}^{(2)}$

$\hat{\mathbf{r}}_1^{(\tau_i)} \leftarrow \text{OMP}(A, \mathbf{b}_1^{(\tau_i)})$
$\hat{\mathbf{r}}_2^{(\tau_i)} \leftarrow \text{OMP}(A, \mathbf{b}_2^{(\tau_i)})$
$index_1 \leftarrow index(max(|\hat{\mathbf{r}}_1^{(\tau_i)}|))$
$index_2 \leftarrow index(max(|\hat{\mathbf{r}}_2^{(\tau_i)}|))$
note: $index_1$ and $index_2$ are chosen without taking into account the zero cyclic frequency.
$\delta_i \leftarrow |index_1 - index_2|$
end for
$\bar{\delta} \leftarrow \frac{1}{M} \sum_{i=1}^{M} \delta_i$
if $\bar{\delta} < k$ **then**
H_1 is decided
else
H_0 is decided
end if

6. SIMULATIONS AND RESULTS

In this section, we first compare our new proposed TSCS detector with its original non soft version proposed in [3]. For fair comparisons, we use the same total number of samples for both detection methods ($2n_s = 400$ samples), we also use the same lag set τ_i ($M = 5$) and the same value of $\widetilde{N} = 512$. We used a simple BPSK modulation in all the simulations. For a $SNR = 0$ dB, the result is shown on Figure 4. The observed result is clear; by using a soft desicion the new TSCS outperfoms its original version proposed in [3] (we note that the TSCS does not require any additional complexity compared to the original method of [3]). Then we compare our new proposed TSCS detector to the classical Dantawate and Giannakis (DG) detecor [2] using the same total number of samples for both detection methods ($2n_s = N' = 400$ samples) and the same set of delays ($M = 5$). For the same $SNR = 0$ dB, the result is also presented on Figure 4. The conclusion is that the TSCS detector outperforms the DG detector. Furtheremore it does not need the knowledge of the cyclic frequencies.

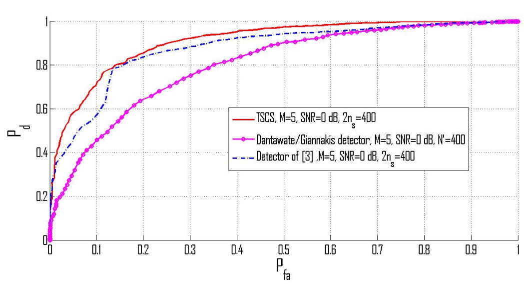

Fig. 4. ROC curves for TSCS and DG detectors for 400 samples and a SNR of 0 dB.

Figure 5 shows the detection probability versus the samples number for the two detectors with and without a Rayleigh channel (each sample is multiplied with a coefficient that follows a Rayleigh distribution of variance 1 and these coeficients are i.i.d.), in the same conditions ($SNR = 0$, P_{fa} equal to 10% and same delays number ($M = 5$). For the same detection probability (horizontal line) the gain in observation time (sample number) is obvious for the TSCS detector. As expected, for both detectors the performances decrease with the Rayleigh channel. Nevertheless, in both conditions (with and without channel) our TSCS detector clearly outperforms the DG detector.

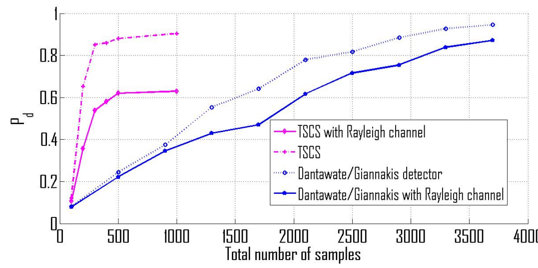

Fig. 5. Detection probability P_d for a fixed false alarm of 10% and $SNR = 0$ dB, versus the samples number, with and without the propagation channel.

6.1. complexity analysis

OMP complexity is equal to $O(l_1.l_2.l_3)$, with l_1, l_2, et l_3 the dictionnary number of lines, the atoms number, and the iteration number respectively. Because OMP is used twice in the TSCS detector, then the complexity is given by $O(2.n_s.M.S.\widetilde{N})$, with S the OMP iterations number. Practically S is equal to 3.

The DG detector complexity is given by $O(M.N'.(L+1)+4.M.L^2+8.M^3+6.M^2+2.M) \cong O(M.N'.(L+1)+4.M.L^2)$, where L is the length of the spectral window and N' the samples number.
Table 1 shows the complexity comparison (operation number) versus the observation time (samples number) without using a propagation channel. We conclude that the TSCS detector is more complex (around 3.8 times) than the DG detector but needs smaller number of samples or observation time (around 9 times) at the point (P_d,P_{fa})=(0.85,0.1), see Figure 5. Furthuremore our TSCS detector is a blind detector wheras DG detector needs the cyclic frequency knowing.

7. CONCLUSION

In this paper we applied compressed sensing technique to estimate the sparse CAF in the cyclic frequencies domain. We showed that the compressed sensing estimator outperforms the classical estimator. We also noticed that with compressed sensing we can estimate the CAV blindly without the need to know the cyclic frequency of the transmitted signal, in contrary to the classical method.

	Our TSCS Detector	D G Detector
Complexity	$O(2 \cdot M \cdot n_s \cdot S \cdot \widetilde{N})$	$O(M \cdot N'(L+1) + 4 \cdot M \cdot L^2)$
Samples number for $(P_{fa}, P_d) = (0.1, 0.85)$	300	2695
Log(Operations number)	6.36	5.77
a priori information	Blind detector	$\alpha_f = \frac{1}{T_s}$

Table 1. Complexity comparison of the two detectors in the same conditions $(P_{fa}, P_d) = (0.1, 0.85)$.

Using this blind estimator we have proposed a new detector, called TSCS, which outperforms the classical Dantawate/Giannakis detector of [2] for the same number of samples and moreover it is a blind detector.

Our future work aims at using this estimator to derive other blind detectors.

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