ASYNCHRONOUS BANK OF FILTERS FOR SPARSE SIGNAL DECOMPOSITION

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ABSTRACT

Asynchronous signal processing addresses the need for analog devices that provide efficient representation and reconstruction of data at a low–power consumption. In this paper we propose an asynchronous filter–bank decomposer for sparse signals commonly found in biomedical applications. It is based on a modified version of the asynchronous sigma delta modulator (ASDM), a non–linear feedback system that maps the amplitude of a bounded signal into a binary signal. Signal reconstruction requires the solution of an integral equation that depends on the zero–crossing times of the ASDM output. Letting the input be the derivative of the signal, the integral equation is reduced to a first–order difference equation of which the input is a function of the zero–crossing times and a scale parameter of the ASDM. Using a bank of filters to determine the corresponding scale parameters we obtain a decomposer that samples non–uniformly and that has a recursive reconstruction. Thus the range of frequencies is set by the bank of filters and the width of local windows is set by the characteristics of the derivative of the signal at those frequencies. To illustrate the procedure we consider actual signals.

Index Terms— Asynchronous signal processing, sigma delta modulator, sparse signals, bank of filters

1. INTRODUCTION

An increasing number of wireless sensors and remote health monitoring devices call for the evolution of conventional signal processing algorithms. Traditional synchronous methods are not sufficient to address emerging needs of low power consumption and efficient representation of the sensory information. As an alternative, a large body of research has been established in the area of asynchronous digital signal processors as well as in asynchronous analog to digital converters.

As the first step in analog to digital signal representation, sampling methods constitute an important part of the innovations. It has been shown that structured data, e.g. sparse signals, can be efficiently sampled at far below the Nyquist rate [1, 2]. Non–uniform sampling suits sensor network applications in emerging biomedical devices [3]. Furthermore, many biological signals are bursty by nature and only need to be sampled in the regions where data is significant. Speech signals for instance, can also be considered as bursty since utterances are interrupted by periods of silence. In many biomedical applications, such as in brain–computer interfacing, implementing non–uniform samplers asynchronously is advantageous due to the small size of the devices. This is not only because these clock–free designs are free from aliasing and consume less power, but also for health reasons: high–frequencies due to the clock could harm the patient. A well-known non–uniform sampling scheme is level crossing (LC) [4] which has been used for analog processing [5]. The sample values in LC are collected only when a specified quantization level is attained, thus making it advantageous in biological and sensor applications where the significant information in the signal is sparse. A drawback of LC sampling, however, is that a set of quantization levels needs to be specified *a-priori* and that the sampling times and the corresponding amplitudes must be kept. Furthermore, only a multilevel reconstruction is possible from the LC sampling. However, the LC sampler is not hampered by aliasing or quantization error and is employed in analog processing using digital methods. In [6] a time-encoding method using asynchronous sigma delta modulators has shown to be equivalent to an LC sampler with quantization levels set as local estimates of the signal average.

In this paper we investigate a new signal–dependent non–uniform sampling and reconstruction approach based on the asynchronous sigma delta modulator (ASDM) — a non-linear feedback system capable of mapping the amplitude of its bounded input into a binary output signal. The ASDM is a low–power device and the zero-crossing times of its output provide a computationally inexpensive reconstruction method. Although there has been considerable research conducted on non–uniform sampling, it typically relies on the assumption that the analyzed signals are band–limited. We propose a bank of filters structure to allow to analyze in desired frequency ranges, independent of whether the signal is band–limited or not. Furthermore, we modify the original structure of the ASDM [7] so as to obtain a recursive equation that permit us to obtain non–uniform samples in the analysis part of the procedure. Zero–crossing times and scale parameters of the modified ASDM in the synthesis part of the procedure provide the regeneration of the samples which can be interpolated to reconstruct the original signal.

The details of the proposed sampler is given in Section 2, while Section 3 describes the overall sampling and reconstruction set–up which includes the proposed sampler, an iterative estimator and a polynomial interpolator. In Section 4, sparse signal illustrations are considered to highlight the biomedical applications where sparse signals are typically processed. However, the principles presented apply to processing other signals as well.

2. ASYNCHRONOUS SAMPLER

The asynchronous sigma delta modulator (ASDM) is a nonlinear feedback system consisting of an integrator and a Schmitt trigger that maps the amplitude of a bounded signal into a binary signal. The model shown in Fig. 1 has been used for analyzing the ASDM [7]. Unlike the conventional delta modulator there is no clock involved in the ASDM and therefore the input signal is not affected by magnetic interference and high power consumption issues related to clock switching. Also the ASDM is robust to noise as voltage fluctuations due to noise is desensitized by the hysteresis of the Schmitt Trigger. The working characteristics and stability of the modulator have been studied [8] and its implementation as a time-encoder has been used in various biomedical applications [9].

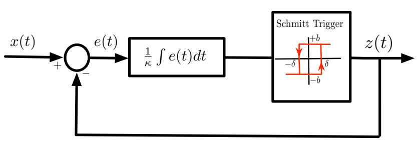

Fig. 1. Model for analyzing ASDM

For a bounded signal $x(t)$, $|x(t)| \leq c$, the following connection between the signal and the zero-crossing times $\{t_k\}$ of the binary output $z(t)$ has been shown [8]:

$$\int_{t_k}^{t_{k+1}} x(\tau)\, d\tau \;=\; (-1)^k[-b\,(t_{k+1}-t_k) \;+\; 2\kappa\delta] \qquad (1)$$

where $b > c$ is the amplitude of $z(t)$, δ is a threshold value used in the Schmitt trigger, and κ is a scale parameter that depends on the maximum frequency of the input signal. For simplicity we let $b = 1$ — the signal is normalized so that its $c < 1$ — and $\delta = 0.5$. As shown in [10], the scale parameter κ is bounded by the maximum frequency f_{max} of $x(t)$:

$$\kappa \leq \frac{1-c}{2f_{max}} \qquad (2)$$

this is so as to guarantee that $\{t_k\}$ satisfy the condition [8]

$$\max_k(t_{k+1}-t_k) \leq \frac{1}{2f_{max}}.$$

Because for sparse–on–time signals is not clear the range of frequencies it is not possible to determine values for κ. We will thus consider a joint time–frequency latticing with known frequency ranges and time–windows set by the corresponding κ for each of these ranges, leading into a bank of filters implementation as shown later.

The encoding of the zero–crossing times of the binary output of the ASDM for biomonitoring has been shown in [9], and recently we have shown that a noticeable compression of the information can be achieved using multi-level approximations [2]. The main challenge in non–uniform sampling is in the reconstruction. Depending on the sparseness of the sampling times in the LC reconstruction matrices tend to be ill–conditioned [11] and the methods are computationally expensive and not exact. Using the ASDM, we only need to keep the zero–crossing times to reconstruct the signal by approximating the integral equation 1[2]. Moreover, the sampling nature of the ASDM is not clear.

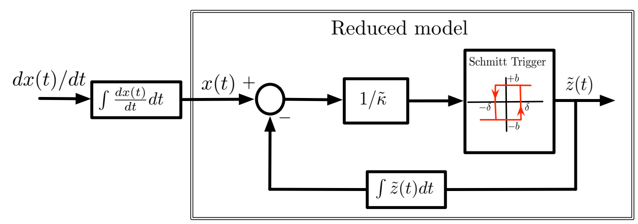

Fig. 2. Model for modified–ASDM

To see the sampling nature of the ASDM and how to solve the integral equation, consider the input of the ASDM to be the derivative of the signal, $dx(t)/dt$, rather than the signal itself. For this input equation (1) becomes

$$\int_{\tilde{t}_k}^{\tilde{t}_{k+1}} \frac{dx(\tau)}{d\tau}\, d\tau \;=\; (-1)^k[-(\tilde{t}_{k+1}-\tilde{t}_k) \;+\; \tilde{\kappa}] \qquad (3)$$

This is equivalent to extracting the integrator from the feedforward loop in the model of Fig. 1 to obtain the equivalent feedback system shown in Fig. 2. We thus obtain the following recursive equation:

$$x(\tilde{t}_{k+1}) - x(\tilde{t}_k) \;=\; (-1)^k[-(\tilde{t}_{k+1}-\tilde{t}_k) \;+\; \tilde{\kappa}] \qquad (4)$$

with a value $x(\tilde{t}_0)$ at the initial time $\tilde{t}_0$ of the process. Equation (4) iteratively returns sample values at sampling instants $\{\tilde{t}_k\}$ given by the ASDM for the input $dx(t)/dt$. It also gives a formulation for the recovery of the signal from $\{\tilde{t}_k, \tilde{\kappa}, x(\tilde{t}_0)\}$.

The recursive procedure is simple and depends on the derivative of $x(t)$. The iteration accuracy depends on the quantization of the time instants [12]. The stability conditions of this model are obtained as in the original model from the derivative as follows:

- The amplitude bound is now

$$c_d = \max \left| \frac{dx(t)}{dt} \right| < 1.$$

- The scale parameter should then satisfy

$$\tilde{\kappa} \leq \frac{1 - c_d}{2\tilde{f}_{max}}$$

where $\tilde{f}_{max}$ is the maximum frequency of $dx(t)/dt$.

In the case of sparse-on-time signals, this creates the same problem as before because the maximum frequency of the derivative of the signal is not known *a priori*. Furthermore, we need to connect the value of c_d with the signal rather than its derivative. In this modified setup we still consider the parameters b and δ to have the values of 1 and 0.5.

The bound on the amplitude of the derivative c_d, can be associated with the bound c of the signal itself. If we let the bound on $x(t)$ be c, and assume $x(t)$ is continuous we have

$$\left| \frac{dx(t)}{dt} \right| \leq \lim_{T \to 0} \frac{|x(t_i + T)| + |x(t_i)|}{T} = \lim_{T \to 0} \frac{2c}{T}$$

Letting $T \leq 1/(2f_{max})$, where f_{max} is the maximum frequency of $x(t)$ we have

$$c_d \leq \frac{2c}{T} \tag{5}$$

Again, the lack of knowledge of f_{max} is an issue to be addressed by the proposed bank of filters.

The dependence of the recursion on $\tilde{\kappa}$ can be eliminated by considering evaluating (1), for $b = 1$ and $\delta = 0.5$, in two consecutive time intervals giving the following relation:

$$\int_{t_k}^{t_{k+2}} x(\tau)\, d\tau = t_{k+2} - 2t_{k+1} + t_k$$

and as such when replacing the signal by its derivative as the input in the ASDM we have

$$x(\tilde{t}_{k+2}) - x(\tilde{t}_k) = \tilde{t}_{k+2} - 2\tilde{t}_{k+1} + \tilde{t}_k$$

This not only eliminates the value of $\tilde{\kappa}$ in the calculations but reduces the number of sample values $\{x(\tilde{t}_k)\}$. For dense sampling, the additional values obtained before are not needed to recover the $x(t)$ using polynomial interpolation. This is also improves the compression.

Finally, inputing $dx(t)/dt$ into the modified ASDM is equivalent to inputing $x(t) - x(t_0)$ by putting the differentiator and the integrator together in Fig. 2. However, the bound c_d and the value of $\tilde{\kappa}$ depend on the derivative, and values for these parameters will be available once we set a maximum frequency for the signal which will be done by passing the signal through a filter of known bandwidth. Interestingly enough, an earlier realization of the asynchronous sigma delta modulator [13] has an integrator only in the feedback loop as shown in Fig. 3 which is similar to our reduced model. An example illustrating the non-uniform sampling is presented later in section 4.

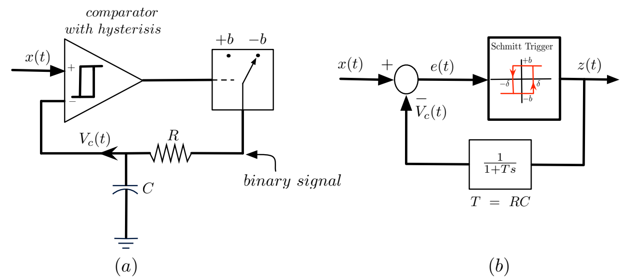

Fig. 3. (a)Asynchronous Delta-Sigma Modulator, (b) Model used for analyzing ADSM

3. BANK OF FILTERS DECOMPOSER

As indicated the proposed asynchronous sampler suffers from the lack of knowledge of the frequency content of the signal or its derivative. We thus propose a joint time–frequency lattice where the frequency ranges are given and the time–windows, connected with the scale parameters, depend on the the maximum of these ranges and on the signal being processed. This is accomplished by using a bank of filters where the bandwidths are set so that the desired frequencies are covered and that the filters constitute an all–pass filter when considered together. The schematics of the analysis and synthesis are shown in Fig. 4. The cutoff frequency of the filter determines the values of $\tilde{\kappa}_i$ and c_{di} for the ith filter. As indicated before, we use equation 6 in the recovery as it requires fewer samples which are sufficient to recover the signal using polynomial interpolation.

Once the amplitude of the input signal is normalized using equation (5) and the maximum frequency of the input is known, Modified-ASDM returns non-uniform samples drawn at a rate proportional to change in the signal. As it is discussed in Section 2, the ASDM requires the maximum frequency of the signal to adjust its parameter κ. The effect of different κ values reflects in windows of different width when computing 6 as indicated in [10, 2]. Also, here we again avoid the derivative as input and reformulate the problem with respect to the signal itself. This can be done by adding a zero to each of the filters to make $x(t)$ the input rather than its derivative (see Fig. 5). We also include an extra branch to estimate the dc-bias, which is lost when the derivative is calculated, if the signal has any.

After filtering the input, we set the

$$\tilde{\kappa}_i = \frac{2\pi(1 - c_d)}{\Omega_i}, \; i = 0, 2, .., M - 1$$

where Ω_i is the cut-off frequency of the i^{th} bandpass-filter, and M is the number of branches. Assuming the time zero–crossings and the scale parameters in each branch are represented and transmitted with high precision, the iterative estimator reconstructs the sample values according to (4). Be-

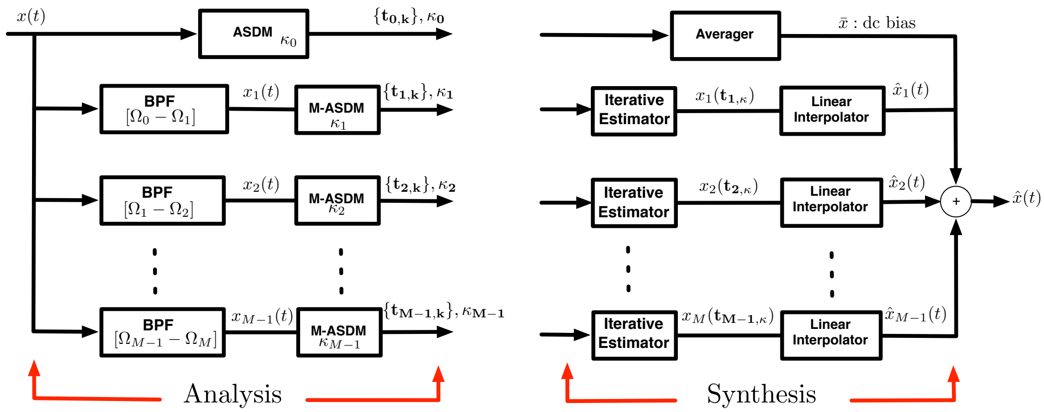

Fig. 4. Asynchronous Decomposer

cause of the distribution of the samples, the density depend on the change in the signal, first order polynomial interpolator is shown to be a good fit for the samples values. The number of zero–crossing $\tilde{t}_{ik}$ can be reduced by noticing that whenever the signal becomes constant the output of the MASDM becomes periodic, and so if this is detected only an initial value is needed for these segments.

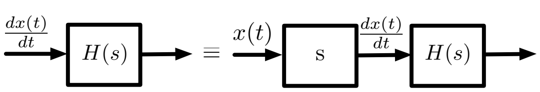

Fig. 5. The equivalent filters

In addition to being signal dependent, low-power sampling technique, the advantage of this overall scheme is that it does not require the knowledge of the maximum frequency of the input. It provides a complete description of the signal without knowing its bandwidth. The illustration of the method on synthetic and sparse data is provided in the next section.

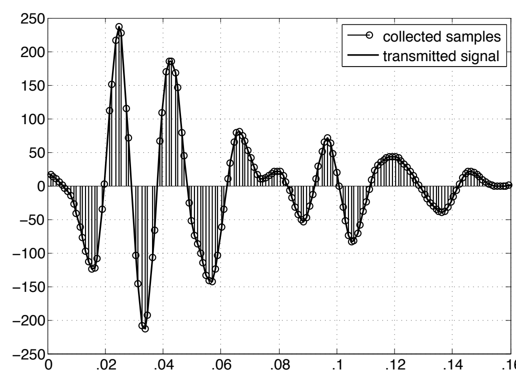

Fig. 6. Asynchronous Sampler

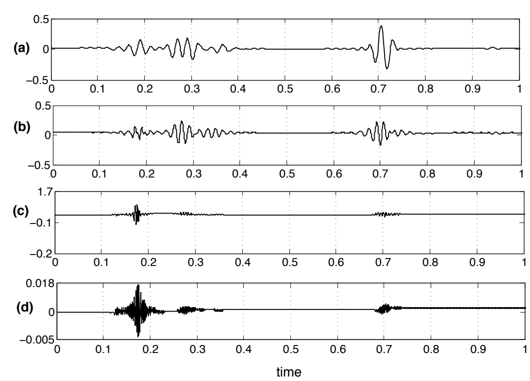

Fig. 7. Reconstructed signals corresponding to outputs of bandpass filters.

4. SIMULATIONS

To illustrate the performance of the modified ASDM as an asynchronous sampler, we use it to sample a low–changing component of an EEG signal. The scale parameter is chosen from the maximum frequency of this signal. The samples of the signal obtained non–uniformly are given by the asynchronous sampler.

For the second simulation we considered the same EEG signal in the bank of filters structure. The bank of filters consists of four band–pass filters, and an ASDM that computes the dc component of the signal — adding to all-pass filter covering the bandwidth of interest of the signal. The bandwidth of the signal was 50 Hz, and the bandpass filters covered each one-fourth of this bandwidth. Figure 7 displays the reconstructed signals corresponding to the outputs of the four band–pass filters. The dc component was found to be zero. A linear interpolator was used to get analog reconstructed signal from the outputs of the iterative estimators. Figure 8 shows the reconstructed signal as a sum of the reconstructed signals in Fig. 7. The error of the reconstruction is shown at the bottom.

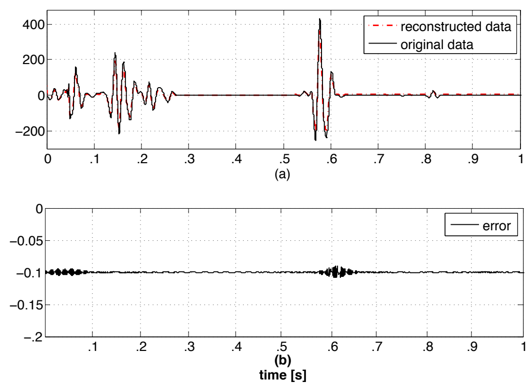

Fig. 8. Reconstruction by linear interpolator of the overall signal.

For the final simulation we consider a 0.2–second phonocardiographic recordings of heart sounds sampled at 4 kHz. The upper plot in Fig. 9 displays the original signal and the lower 4 plots display the reconstructed signals corresponding to each of the bandpass filter outputs, with the reconstruction error show on the right side.

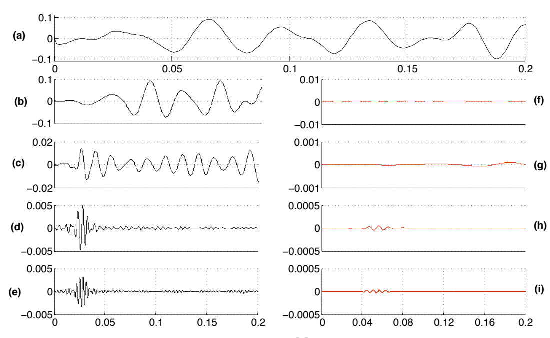

Fig. 9. (a) Original signal; (b)-(e) reconstructed filter outputs and (f)-(i) associated errors.

5. CONCLUSIONS

In this paper we presented an asynchronous bank of filters approach to the sampling and reconstruction of non–stationary signals. The approach is to lattice the time–frequency plane by selecting frequency ranges to determine the scale parameters of modified ASDMs. By changing the structure of the ASDM we obtain a recursive way to obtain the sample values as well as to reconstruct the analog signal. The resulting analysis and synthesis parts of the procedure resemble wavelet decomposers. To illustrate our approach we considered biomedical recordings.

6. REFERENCES

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