NON–STATIONARY DECOMPOSITION USING THE DISCRETE LINEAR CHIRP TRANSFORM (DLCT) FOR FM DEMODULATION

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ABSTRACT

In this paper, we consider FM demodulation as an application of the decomposition of non–stationary signals. Non–stationary signal decomposition can be done using either the empirical mode decomposition (EMD) or the Discrete Linear Chirp Decomposition (DLCT) methods. These methods decompose non-stationary signals using local time-scale signal characteristics. While the EMD decomposes the signal into a number of intrinsic mode functions (IMFs), the DLCT obtains a parametric model based on a local linear chirp model. Analytically the DLCT considers localized zero–mean linear chirps as special IMFs. The DLCT is a joint frequency instantaneous–frequency orthogonal transformation that extends the discrete Fourier transform (DFT) for processing of non–stationary signals. FM demodulation is commonly done by computing the signal derivative to convert it into an amplitude demodulation. We will show that the demodulation can be approached with the EMD and the DLCT and that the second method provides better results. The performance of the DLCT and the EMD are illustrated and compared when used as an FM demodulation scheme in software defined radio.

***Index Terms*—** EMD, DLCT, FM demodulation, Hilbert–Huang spectrum

1. INTRODUCTION

The processing of non–stationary signals in practical applications is complicated by the continuous variation of local time–scale signal characteristics. Over the years, many approaches have been proposed to represent such signals: from wavelets to time–frequency distributions to the more recent empirical mode decomposition and Hilbert spectrum.

Using the concept of intrinsic mode functions (IMFs), or functions with a point–wise zero–mean envelope and matching extrema and zero–crossings, Huang [1] proposed the empirical mode decomposition (EMD) algorithm that can be connected through the Hilbert transform to a time–frequency spectrum where the instantaneous frequency (IF) of each of the components is obtained. Although EMD is signal–dependent and intuitive, it is not optimal and it does not consider orthogonal basis functions.

Locally a signal can be well–approximated as a sum of chirps with polynomial instantaneous–frequencies. By decreasing the local support, linear chirps are sufficient to consider. The symmetry of the envelopes of zero–mean linear chirps and the matching of their extrema and zero–crossings make them possible IMFs. In [2] we propose the discrete linear chirp transform (DLCT) for local approximation of non–stationary discrete signals using linear chirps. The linear chirp parameters permits us to obtain the IF of each of the components, just like the Hilbert spectrum does for each of the IMF components. Because of its parametric approach, the DLCT approach does not suffer from some of the drawbacks of the EMD and it is completely analytic. Given the commonalities between the EMD and the DLCT it is appropriate to compare their results in the demodulation of FM signals. Using more sophisticated procedures such as the polynomial Fourier transform [11] and the ensemble EMD [12] would require higher computational cost and complexity.

The Hilbert transform applied to each IMF component gives the global time–frequency distribution of the underlying signal [3, 4]. The application of the Hilbert–Huang Transform (HHT) to audio and speech signals have been presented in [5, 6]. While in [7, 8, 9] the HHT is used to estimate the IF of biomedical signals such as EEG and ECG. However, the estimated IF obtained through the EMD is not clear because of the rough approximation of the IMFs. Given that the DLCT provides a parametric representation of the signal, the estimation of the IF of the components is improved. Here we apply this method to estimate the IF of Frequency Modulated signals which by definition is the message signal. Thus this approach is considered as an FM demodulator which may be used in software defined radio systems.

2. FM DEMODULATION

A frequency modulated (FM) signal is the prototypical example of a non–stationary signal as its frequency varies with time depending on the message. For a message $m(t)$ the FM modulated signal is given as

$$f(t) = \cos(\omega_c t + k_f \int_0^t m(\tau) d\tau) \quad (1)$$

where ω_c is the carrier frequency and k_f is the modulation index. The instantaneous frequency of the FM signal is

$$IF(t) = \omega_c + k_f m(t)$$

or the derivative of the argument of the cosine. In general, $k_f m(t) << \omega_c$ and also the value of k_f and the amplitude of the message determine the bandwidth of the FM signal. Wide–band FM is the more interesting and challenging case which we consider here. Notice that if we are able to estimate the instantaneous frequency of $f(t)$ we would obtain the message. But the message can also be obtained by computing the derivative of the signal and using an envelope detector [10], a procedure used in commercial AM.

This can be seen from the derivative of $f(t)$ which gives

$$\begin{aligned} \frac{df(t)}{dt} &= -(\omega_c + k_f m(t)) \sin(\omega_c t + \gamma(t)) \text{ where} \\ \gamma(t) &= k_f \int_0^t m(\tau) d\tau \end{aligned} \quad (2)$$

an AM-FM signal, i.e., it has a time–varying amplitude modulating an FM signal. Since $k_f m(t) << \omega_c$ to obtain the time–varying amplitude we demodulate (2) by multiplying by $\sin(\omega_c t)$ given that ω_c is known to obtain

$$\begin{aligned} \frac{df(t)}{dt} \sin(\omega_c t) &= f_1(t) + f_2(t) \\ f_1(t) &= -0.5(\omega_c + k_f m(t)) \cos(\gamma(t)) \\ f_2(t) &= 0.5(\omega_c + k_f m(t)) \cos(2\omega_c t + \gamma(t)) \end{aligned} \quad (3)$$

Thus demodulating the derivative of the FM signal $\sin(\omega_c t)$ we obtain components $f_1(t)$ and $f_2(t)$ with the same time–varying amplitude. Each of these can then be processed either with an envelope detector or Hilbert transform to obtain the message. In the Hilbert transform method, we calculate the analytic signal of $f_2(t)$ and then the message can be extracted from the absolute part of the analytic signal. The condition $k_f m(t) << \omega_c$ makes the upper envelop be related to $k_f m(t)$ from which we can obtain the message $m(t)$. However, we need to obtain the two components of the sine demodulated signal. When ω_c and $\gamma(t)$ are not close, this can be done using the EMD or the DLCT. However, whenever ω_c and $\gamma(t)$ are close, the EMD method could fail while DLCT does not.

3. DISCRETE LINEAR CHIRP TRANSFORM

3.1. Chirps as IMFs

A non–stationary signal can be decomposed into a finite number of IMF functions with meaningful instantaneous frequencies. We would like these IMF functions to be independent of each other and to satisfy the following two conditions:

- The number of extrema and the number of zero crossings must either equal or differ at most by one.
- The instantaneous mean of their envelope, defined by the IMFs maxima and minima, must be zero.

A chirp function

$$c(t) = Ae^{j\phi(t)} \qquad 0 \leq t \leq T \quad (4)$$

where $\phi(t)$ is a polynomial in t

$$\phi(t) = \sum_{k=1}^{\infty} \beta_k t^k$$

can be considered an IMF. Indeed, its envelope is symmetric so that its instantaneous mean is zero, and by adjusting the value of T the number of extrema and of zero crossings are made to match.

Given the complexity of using higher order chirps, we consider linear chirps with an appropriate support as IMFs. A local representation — mandated by T — of a signal is then

$$x(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f, \beta) e^{j(2\pi f t + \beta t^2)} df d\beta, \ \ 0 \leq t \leq T \quad (5)$$

or an infinite sum of linear chirps with constant magnitude $X(f, \beta)$, and instantaneous frequency (IF)

$$IF_{f,\beta}(t) = 2\pi f + 2\beta t.$$

3.2. Discrete Linear Chirp Transform

For computer implementation, a discrete version of the chirp representation given in (5) is needed. For $x(n)$, a discrete–time signal of finite support $0 \leq n \leq N-1$, the following discrete linear chirp transform (DLCT) and its inverse have been proposed [2] :

$$\begin{aligned} X(k, m) &= \sum_{n=0}^{N-1} x(n) \exp\left(-j\frac{2\pi}{N}(mCn^2 + kn)\right), \quad (6) \\ 0 &\leq k \leq N-1, \quad -\frac{L}{2} \leq m \leq \frac{L}{2} - 1 \\ x(n) &= \sum_{m=-L/2}^{L/2-1} \sum_{k=0}^{N-1} \frac{X(k,m)}{LN} \exp\left(j\frac{2\pi}{N}(mCn^2 + kn)\right) \\ 0 &\leq n \leq N-1, \ \ C = 2\Lambda/L \quad (7) \end{aligned}$$

To obtain a discrete transformation the chirp rate, that in a continuous chirp has no bounds, i.e., $-\infty < \beta < \infty$, is now bounded. That is, we consider a range $[-\Lambda, \Lambda]$, $0 < \Lambda < \infty$, and discretize these values to $mC = 2\Lambda m/L$, $-\frac{L}{2} \leq m \leq \frac{L}{2} - 1$. The value of N is taken as the width of a window chosen so that the matching of extrema and zero–crossing condition for the chirps to be IMFs is satisfied. Notice that the DLCT is a generalization of the discrete Fourier transform (DFT), $X(k, 0)$ is the DFT of $x(n)$.

If for the signal $x(n)$ we can identify from its DLCT $X(k, m)$ the number of components and the corresponding chirp–rate and frequency parameters, we then have a linear–chirp approximation of $x(n)$. Indeed, using equation (7) we obtain the following Parseval type of relation between the energy in the two domains:

$$\sum_{n,m} |x_m(n)|^2 = \sum_{m,k} \frac{|X(k,m)|^2}{N^2}$$

It can then be shown that for each value of the discretized chirp–rate $2\Lambda m/L$, $-L/2 \leq m < (L/2) - 1$ we have that

$$x_m(n) = \sum_{k=0}^{N-1} \frac{X(k,m)}{N} \exp\left(j\frac{2\pi}{N}(Cmn^2 + kn)\right) \qquad (8)$$

equals $x(n)$. That is, the inverse DLCT is the average over all values of m: replacing $X(k, m)$ from (5) in (6) gives

$$\begin{aligned} x_m(n) &= \sum_{\ell=0}^{N-1} \frac{x(\ell)}{N} \exp\left(j\frac{2\pi}{N}Cm(n^2 - \ell^2)\right) \\ &\quad \times \underbrace{\sum_{k=0}^{N-1} \exp\left(j\frac{2\pi}{N}k(n-\ell)\right)}_{N\delta(n-\ell)} = x(n). \end{aligned}$$

And thus, we have that

$$\sum_{n,m} |x_m(n)|^2 = \sum_{n} L|x(n)|^2$$

As such, the energy concentration is indicated by the peak values of $|X(k, m)|^2$ as a function of k and m. Considering the region in the joint chirp–rate frequency plane where these peak values occur, we should find the values of the chirp–rates and frequencies that can be used to approximate the given signal locally as a sum of linear chirp components

$$x(n) = \sum_{i=1}^{P} a_i \exp\left(j\frac{2\pi}{N}(\beta_i n^2 + k_i n) + j\phi_i\right) \qquad (9)$$

where a_i, β_i, k_i, and ϕ_i are amplitude, chirp rate, frequency, and phase of the ith linear chirp, respectively.

The determination of the range of chirp–rates and frequencies where the energy of the signal is the most significant is analogous to masking. Selecting the frequency band $[f_1, f_2]$ and the chirp–rate range $[\beta_1, \beta_2]$ as in Fig. 1 one can determine by the number of peaks the number of components and for each the frequency and chirp–rate — thus obtain a parametric representation of the signal components. This equivalent to masking in the joint time–frequency plane using a time–varying filter with the desired frequency band $[f_1, f_2]$ but a shape determined by the chirp–rate range $[\beta_1, \beta_2]$. See Fig. 1.

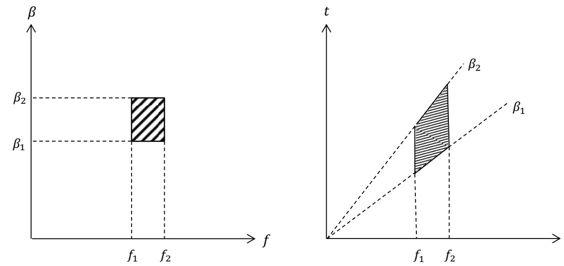

Fig. 1. Joint chirp–rate β and frequency f masking in chirp–rate frequency plane (left) and corresponding time–varying mask in time–frequency plane (right).

4. COMPARISON OF EMD AND DLCT

The EMD is a technique that decomposes a non–stationary signal into a small number of components, and it has been proven to be useful in a wide range of applications. However, in situations where the instantaneous frequencies are not well separated in the time–frequency plane it does not perform well. The DLCT on the other hand, providing a parametric representation of the signal is more robust. To illustrate the behavior of the two algorithms we consider two examples one with simulated data and the other with real data.

To test the IF estimation, consider the addition of two sinusoidal chirps corrupted with additive Gaussian noise giving a signal to noise ratio SNR= 5 dB:

$$\begin{aligned} x(n) &= \cos\left(\frac{\pi}{10}n + \frac{\pi}{6400}n^2 + 8\sin\left(\frac{\pi}{200}n\right)\right) \\ &\quad + \cos\left(\frac{2\pi}{5}n + \frac{0.18\pi}{1024}n^2 + 8\sin\left(\frac{\pi}{50}n\right)\right). \end{aligned}$$

Figure 2(a) depicts the signal $x(n)$, while the actual IFs of the two individual components are shown in Fig. 2(b). Figures 2(c) and (d) give the estimated IFs of $x(n)$ using the DLCT and the EMD decomposition algorithms, respectively. The performance of the EMD as an IF estimator is very much affected by the presence of the noise.

The performance of decomposition and representation of speech signals using the EMD and the DLCT is explored. Our experiment is conducted using a speech segment corresponding to "*among them are canvases by a young artist*" sampled at 16 kHz. The speech signal is divided into blocks of 218 msec to capture its local characteristics. The DLCT decomposes the speech segment into 5 components $s_i(t)$ as shown in

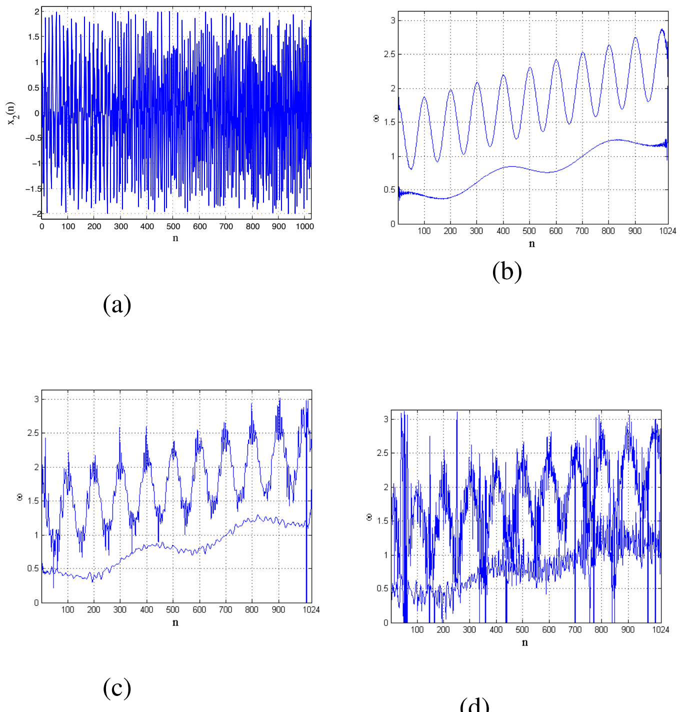

Fig. 2. Synthetic signal $x(n)$ with additive noise (SNR=5dB): (a) noiseless signal $x(n)$; (b) IF of $x(n)$; (c) estimated IF of $x(n)$ using DLCT; (d) estimated IF of $x(n)$ using EMD.

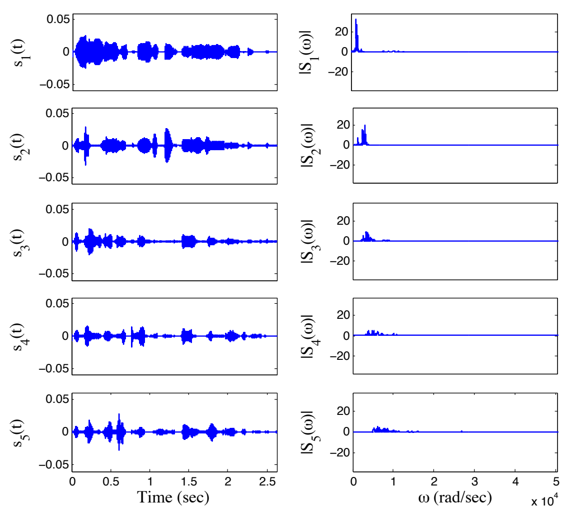

Fig. 3. Speech segment decomposition using DLCT: components $\{s_i(t)\}$, $i = 1, \cdots, 5$ (left) and their corresponding spectra (right).

Fig. 3. The SNR for the reconstructed signal is 26.08 dB. Notice that each of the $s_i(t)$ display the characteristics of IMFs and that they do not overlap in frequency, as indicated by their spectra.

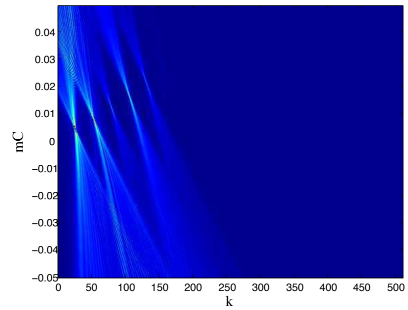

Fig. 4. Magnitude of the DLCT for one block of the segment of speech. Peaks indicate 5 different linear chirps with the corresponding chirp–rates and frequency parameters for each indicated by the peak.

The decomposition includes the calculation of the DLCT for each block. Figure 4 shows the two–dimensional plot of the DLCT magnitude for one of the blocks. The proposed algorithm follows the peaks displayed in the magnitude plots and masks them in the frequency chirp–rate plane — this method is thus a time–varying bank of filters.

For comparison, we decompose the same speech signal using the EMD. The speech segment is windowed into blocks of the same width as in the above DLCT experiment. The EMD decomposes the speech signal also into 5 IMFs as shown on the left in Fig. 5. However, the reconstructed signal gives a SNR=18.21 dB not as good as the reconstructed signal from the DLCT method. Also the frequency spectrum of each of the five components has overlapping bandwidths (see Fig. 5 on the right) different from the spectral representation obtained with the DLCT (see Fig. 3 on the right).

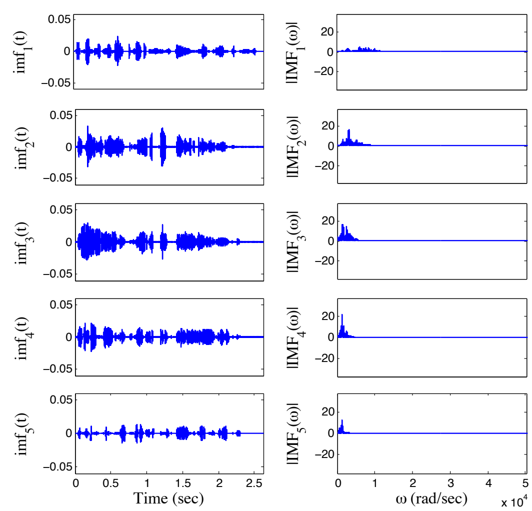

Fig. 5. Speech decomposition using EMD: five IMFs (left) and their corresponding spectra.

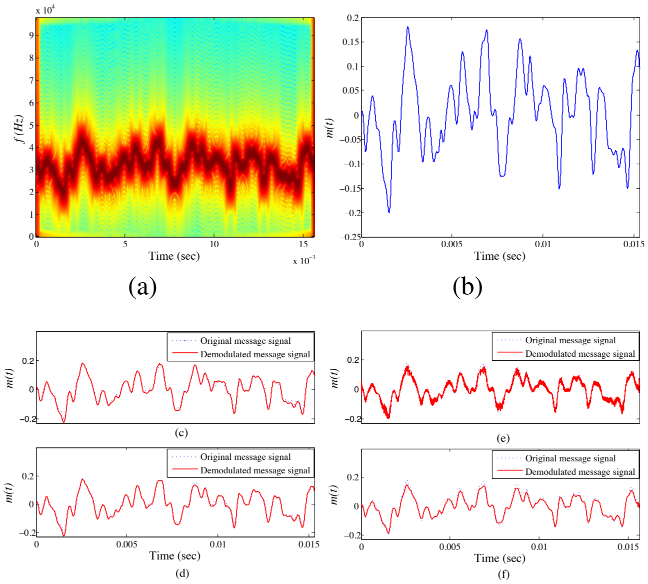

Fig. 6. (a) Spectrogram of the FM modulated signal; (b) *handel* message; (c) demodulation with EMD using envelope detector; (d) demodulation with DLCT using envelope detector; (e) demodulation with EMD using Hilbert transform; (f) demodulation with DLCT using Hilbert transform.

5. SIMULATION OF DEMODULATION OF FM SIGNAL

Finally, we conduct the experiment of demodulating an FM signal. The spectrogram of the FM signal is shown in Fig. 6(a). The message given in Fig. 6(b) is a segment of MATLAB's *handel* signal. Figures 6(c) and (d) show the demodulated signal using the envelope detector method after securing the higher frequency component $f_1(t)$ by the EMD and the DLCT. The SNR of the detected signal is 17.41 dB for the EMD case while it is 18.81 dB for the DLCT case. Now, if we demodulate the signal with the Hilbert transform method, then we obtain the message signal with SNR 14.94 dB for the EMD case and 17.44 dB for the DLCT case as given in Figs. 6(e) and (f).

6. CONCLUSION

In this paper, we compare the DLCT and the EMD methods for the demodulation of FM signals. Different from the EMD, the basis used in the DLCT are orthogonal and optimal in a mean–square sense. The DLCT uses linear chirps as the IMFs and provides a parametric estimation of the instantaneous frequency of the signal components, which is not affected by the closeness of the frequency of the components. The DLCT provides a joint chirp–rate instantaneous–frequency masking — or a time–varying mask in the time–frequency domain– that permits us to separate the signal according to the magnitude of the DLCT. The advantage of the DLCT over the EMD is illustrated using FM demodulation or instantaneous–frequency estimation application.

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